UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probability and Random Processes

Discussion Notes: Week 2
Fall 2007

Reading: Berstsekas & Tsitsiklis, §1.3, §1.4, §1.5 (no Bayes yet)

Key Stuff to Remember:

- **Conditional Probability:** $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$ for $A, B \subseteq \Omega$ and $B \neq \emptyset$.
- **Total Probability:** Given *disjoint* $A_1, \ldots, A_n$ that partition the sample space ($\bigcup_{i=1}^n A_i = \Omega$), for any event B we have:

$$\begin{aligned}\mathbb{P}(B) &= \mathbb{P}(A_1 \cap B) + \cdots + \mathbb{P}(A_n \cap B)\\ &= \mathbb{P}(A_1)\mathbb{P}(B \mid A_1) + \cdots + \mathbb{P}(A_n)\mathbb{P}(B \mid A_n)\end{aligned}$$

- **Independence:** Events $A_1, \ldots, A_n$ are independent if

$$\mathbb{P}(\bigcap_{i\in S} A_i) = \prod_{i\in S} \mathbb{P}(A_i)$$

for every subset S of $\{1, 2, \ldots, n\}$.

Problem 2.1
Consider a single throw of two fair, six-sided dice.

(a) Calculate $\mathbb{P}(\{$"the two faces are the same"$\})$.

(b) Calculate $\mathbb{P}(\{$"the two faces are the same"$\}|\{$"sum ≤ 3"$\})$.

Problem 2.2
(Bertsekas 1.21) Two out of three prisoners are to be released. One of the prisoners asks a guard to tell him the identity of a prisoner other than himself that will be released. The guard refuses with the following rationale: at your present state of knowledge, your probability of being released is 2/3, but after you know my answer, your probability of being released will become 1/2, since there will be two prisonners (including yourself) whose fate is unknown and exactly one of the two will be released. What is wrong with the guard's reasoning?

Problem 2.3
Jane has 3 children, each equally likely to be either sex, independently. We define the following events:

$$\begin{aligned}A &= \{\text{“all children are the same sex”}\}\\ B &= \{\text{“there is at most one boy”}\}\\ C &= \{\text{“the children include a boy and a girl”}\}\end{aligned}$$

(a) Show A is independent of B and B is independent of C.

(b) Is A independent of C?

(c) What if boys and girls are not equally likely?

(d) Do these hold if Jane has four children?

Problem 2.4
You have five coins: two double-headed, one double-tailed, and two normal. You pick one at random and toss it behind your back.

(a) What is the probability that the *lower* face is heads?

(b) You then open your eyes and see that the upper face is heads. Now what is the probability that the lower face is a head?

(c) You shut your eyes again, toss the coin, and open your eyes to see heads again. Now what is the probability that the lower face is a head?

Problem 2.5
(Galton's Paradox) You flip three fair coins. At least two are alike, and it is an evens chance that the third is a head or a tail. Therefore $\mathbb{P}(\{\text{"all alike"}\}) = 1/2$. Do you agree?