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Markov-based Redistribution Policy Model for Future Urban Mobility Networks*

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Abstract— **In this paper we present a Markov-based urban transportation model that captures the operation of a fleet of taxis in response to incident customer arrivals throughout the city. We consider three different evaluation criteria: (1) minimizing the number of transportation resources for urban planning; (2) minimizing fuel consumption for the drivers; and (3) minimizing customer waiting time increase the overall quality of service. We present a practical policy and evaluate it by comparing against the actual observed redistribution of taxi drivers in Singapore. We show through simulation that our proposed policy is stable and improves substantially upon the default unmanaged redistribution of taxi drivers in Singapore with respect to the three evaluation criteria.**

I. Introduction

Understanding how to optimize transportation is critical for urban planning. In this paper we describe the operation of a fleet of service agents (taxis) in response to incident requests (arriving customers) in a city. We establish a theoretical Markov-based framework that describes an urban transportation network. We assume that we have a road network with discrete pickup and drop-off locations corresponding to designated points in the city. The arrival rates of customers at each location are known. A vehicle with a person on board will drive to the customer's goal destination. If a customer is waiting at this location the vehicle picks up the customer; if there are no waiting customers, the vehicle goes to a different location according to a redistribution policy. Our goal is to compute a solution in the form of the required number of vehicles in the system and their redistribution policy. We encode the solution as a scalable optimization problem and present a practical redistribution policy.

We then consider the solution with respect to three seemingly different optimization criteria. The first criterion considers the customers, whose end goal is to minimize the time spent waiting for a taxi. The second criterion considers the urban planning authority whose goal is to minimize the number of vehicles in the road network. The third criterion considers the cost and environmental implications of fuel consumption. Next, we leverage data from a fleet of 16,000 taxis in Singapore to create a realistic model of taxi fleet operation in Singapore, show how we can learn and interpret the current default behavior of taxi drivers within our framework, and prove that the current behavior is sub-optimal with respect to the evaluation metrics. We evaluate our proposed practical redistribution policy through simulation and show that it performs favorably in light of all three of these optimization criteria and improves substantially upon current ground truth behavior employed collectively by the taxis operating in Singapore.

A. Related Work

Urban mobility has been an active area of research since the turn of the century. In the US, the annual congestion cost is projected to grow to $133 billion by 2015 [6]. Not surprisingly, social and municipal trends are changing in favor of a modernized system of public transportation, and the recent volume of research in the subject reflects this.

The Dynamic Traffic Assignment problem (DTA) dates back as early as [8] and [4]. Generally speaking, the objective of DTA problems is to optimize traffic flow while accounting for congestion effects. DTA models commonly differ in the representation of the supply and demand processes. A thorough survey can be found in [13]. For example, the problem is grounded in continuous time control theory in [4], while [3] presents a variational inequality formulation. A mathematical programming approach in used in [8], [16]. Another recent work [16] models the problem as a linear program, and a simulation-based approach in [1] presents an offline model for estimation of supply and demand.

Mobility-on-Demand (MOD) is an emerging paradigm for handling traffic congestion. In MOD systems, the goal is to provide users with on-demand rental facilities of convenient and efficient modes of transportation [9]. Load balancing in MOD systems is similar to the Pickup and Delivery problem (PDP), whereby passengers arriving into a network are transported to a delivery site by vehicles. For a review of the state of the art see [2], [10] and the references therein. Autonomous load rebalancing in MOD systems has recently been studied in [11] and [12], where a fluid model was used to represent supply and demand.

As well as system-level traffic flow optimizations, socially motivated criteria have also been considered. Recent work on traffic planning explored optimizing a drivers's route subject to congestion [7]. Social optimum planning models for computing vehicle paths are presented in [14], [15].

Our work extends to encompass a broad scope of evaluation criteria. We consider the interplay between global optimization criteria typical of related studies of DTA and MOD

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systems, as well as social optimization criteria as motivated by recent studies of congestion-aware traffic systems.

This paper is organized as follows. Section II presents the problem formulation and Section III outlines the solution. Section IV describes the challenges in creating a realistic urban mobility model and presents the model formally. Section V formulates the optimization problem and provides an algorithm for a practical redistribution policy. We address stability of the policy and discuss the trade-offs between complexity and accuracy. Section VI discusses the optimization criteria and provides metrics for evaluating policies accordingly. Finally, Section VII describes experiments with our proposed policy and discusses the results.

II. Problem Statement

We consider a pickup and delivery problem (PDP) on an undirected graph. There are m nodes in the network, subject to incident request arrivals. The graph is patrolled by n mobile agents (taxis) that traverse it along its edges and service requests (customers) as they arrive. Requests arrive according to a Poisson process with an arrival rate of λ requests per time unit τ and are distributed among nodes according to an arrival distribution $\alpha = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_m]$. Thus a request will arrive at each node with an arrival rate of $\lambda \alpha_i$. The destination of incident requests is determined by a *request transition matrix* D, where $d_{i,j}$ is the probability that a request arriving at node i is destined for node j. Since each row of D is a probability distribution over a node, we require that the rows sum to 1, i.e. D is a Markov chain.

When a vehicle arrives at a node and encounters a request it must service that request and when a vehicle delivers a request to its destination node it is immediately available to service new requests. A vehicle that does not encounter a request transitions according to a *redistribution policy* transition matrix P. Each row of P is a probability distribution over a node, i.e. P is also a Markov chain.

We consider the system to evolve according to a single *system transition matrix* S so that for $X \sim h_i$

$$s_{i,j} = \Pr(X_{k+1} = j \mid X_k = i). \tag{1}$$

Thus the system evolves according to D when a request is being serviced and according to P when a request is not being serviced. Denoting by β_i the probability that a vehicle leaving node i is servicing a request, we express (1) as

$$s_{i,j} = \beta_i \Pr(X_{k+1} = j|X_k = i) + (1-\beta_i)\Pr(X_{k+1} = j|X_k = i). \tag{2}$$

We introduce the $m \times m$ matrix $B = diag\left([\beta_1 \ \beta_2 \ \dots \ \beta_m]\right)$ so that (2) can now be expressed in matrix form as

$$S = BD + (I_m - B)P \tag{3}$$

where I_m is the identity matrix of size m.

The stationary distribution of a Markov chain P is a vector q such that $qP = q$. For convenience we define the function

$$\pi : P \longmapsto q \mid qP = q. \tag{4}$$

In steady state the system will exhibit a stationary distribution $\phi = \pi(S)$ of agents among nodes. Given a number of vehicles n and a stationary distribution ϕ, in steady state we expect $n\phi_i$ vehicles to arrive at node i at a given time, and to find each vehicle located at node i with probability ϕ_i.

III. Optimization Setup

Informally, we want to ensure stability in steady state. We understand system stability to mean the condition whereby the steady state service rate at each node in the system exceeds the steady state arrival rate at that node. The solution space is the number of taxis n and the redistribution policy P. The objective of the problem is therefore to find a solution (i.e. determine the number taxis n and a policy P) such that the overall system is stable.

We formalize the problem as follows.

$$\begin{aligned} \text{Find} \quad & n, P \\ \text{s.t.} \quad & n\phi_i > \lambda \alpha_i, \ \forall i && (5) \\ & n \leq n_{max} && (6) \\ & 0 \leq p_{i,j} \leq 1, \ \forall i && (7) \\ & \textstyle\sum_j p_{i,j} = 1, \ \forall i. && (8) \end{aligned}$$

The first constraint (stability constraint) states that the service rate at each node must be greater than the arrival rate at each node. The second constraint states that the solution space is physically bounded by some maximum number of taxis. The last two constraints ensure that P is a valid Markov chain.

A. Hastings-Metropolis Algorithm

The solution space of the optimization problem is the number of taxis n and the redistribution policy P, while the stability constraint of the problem is specified as a function of the stationary distribution of the system transition matrix $\phi = \pi(S)$. This presents a computational challenge since the transformation from a Markov chain to its stationary distribution is non-linear. Thus it is infeasible to consider linear programming methods to find P directly.

Instead we propose a different approach, employing the Hastings-Metropolis algorithm [5]. The HM algorithm is a Markov chain Monte Carlo method that, given a stationary distribution, can be used to construct a Markov chain with that stationary distribution. Using this method we may simplify a potential optimization problem to that of finding a desired stationary distribution and generating the policy using HM. For convenience we denote the HM algorithm as

$$H : q \longmapsto P \mid qP = q. \tag{9}$$

However, using the HM algorithm poses another challenge: by solving for the stationary distribution, we cannot enforce the zero constraints of the target redistribution policy matrix (in other words we cannot enforce sparsity), which implies an underlying undirected clique network. In the following section, we show that an undirected clique model is highly restrictive in terms of the kind of transportation network that it can describe. We present a model for a real urban mobility network, and discuss the steps taken to ensure that the model is realistic while still complying with the Markov framework. We then present a mechanism by which

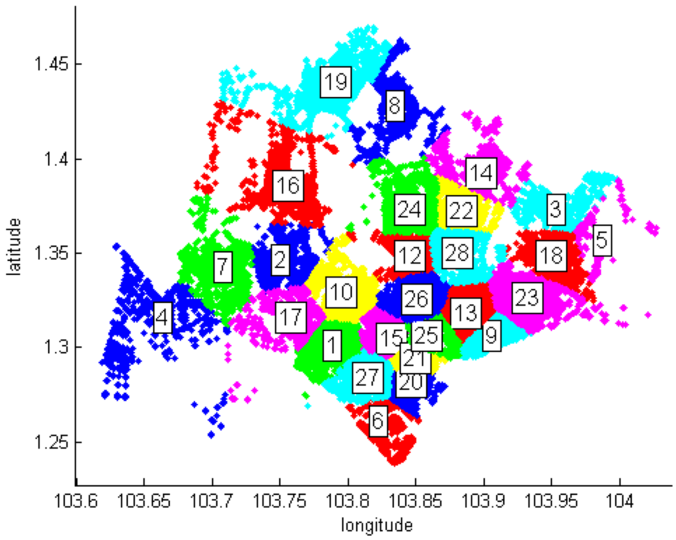


Fig. 1: The 42,000 Singapore nodes used in this study, color-coded according to their k-means clustering.

the HM optimization can be set up to handle a sparse network while maintaining a computational complexity that is independent of the degree of precision of the model.

IV. Modeling Urban Mobility

In this study we use transportation data from Singapore. The dataset is one month (August 2010) of taxi data from a fleet of 16,000 taxis. This data amounts to approximately 500 million data points at 42,000 GPS locations in Singapore. Each record contains the taxi and driver ID, time stamp, GPS coordinates, and status of operation. We partitioned the space of nodes from the Singapore taxi dataset with a k-means clustering into 27 regions (Fig. 1). The clustering was based on previous work, and the number of clusters chosen such that the clustering aligns well with the postal regions of Singapore. We also derived an extensive set of statistics for these regions from the Singapore taxi data.

A. Extended Network

A true Markov chain is a discrete probabilistic state machine, meaning that each transition occurs in one time step. This presents a challenge, as we can assign a real travel time to correspond to each transition, but in the current formulation there is no way to enforce varying travel times. We present an extended framework that captures this information. First, we derive a *base network* that considers each cluster as an individual *base node*. The base network is an undirected clique graph $G = (V,E)$ of size m. The Singapore dataset was used to calculate the average travel time $t_{i,j}$ from cluster i to cluster j. Trips within clusters are also considered, so there is no requirement that $i \neq j$. Also note that in general $t_{i,j} \neq t_{j,i}$, which reflects traffic inhomogeneity caused by congestion throughout the day. A discretization parameter τ specifies the the shortest travel time between 2 nodes represented by a single transition in the extended model. A travel time matrix T encodes the discretized travel times, where $\tau_{i,j} = \max\left\{\mathrm{round}(t_{i,j}/\tau), 1\right\}$.

We use this information to derive the *extended network* G'. Starting with the base network G, for each pair of nodes $i, j \in V$ we remove the edge connecting i and j by setting $G'_{i,j} = 0$ and create two *auxiliary connections* between i and j, one for each direction of travel. Each such auxiliary connection consists of $\ell = \tau_{i,j} - 1$ *auxiliary nodes* $x_1,\ x_2,\ \ldots x_\ell$. New edges are added by daisy-chaining node i to node j through the ℓ auxiliary nodes by setting $G'_{i,x_1} = G'_{x_1,x_2} = \ldots = G'_{x_\ell,j} = 1$. We refer to the first auxiliary node x_1 as the *proxy node* of i to j, denoted by $\theta(i,j)$. If $\tau_{i,j} = 1$, node i is simply connected to node j. Finally, for $i = j$, a single auxiliary connection is created in this way, representing the average travel time within cluster i. The resulting extended network is a sparse network $G' = (V',E')$ of size m'.

We make the assumption that customers arrive only at and travel to base nodes and are not picked up on the side of the road. Although in reality taxi drivers may divert from any prescribed policy, this is a reasonable model corresponding to the intuition of base nodes representing taxi stations for example. As a result, the extended arrival distribution is simply given by $\alpha' = [\alpha_1\ \alpha_2\ \ldots\ \alpha_m\ 0\ \ldots\ 0]$. This greatly simplifies the optimization problem since we do not need to satisfy stability constraints at auxiliary nodes.

B. Extended Redistribution Policy

Given an $m \times m$ redistribution policy P for the base network, we extend this policy to an $m' \times m'$ policy matrix P' that incorporates the transitional logic of P while still maintaining the mathematical properties of a Markov chain. We derive P' as follows. For each $p_{i,j}$ corresponding to a connection $g_{i,j} = 1$ in the base network, there is a corresponding auxiliary connection consisting of a set of $\ell = \tau_{i,j} - 1$ auxiliary nodes $G'_{i,x_1} = G'_{x_1,x_2} = \ldots = G'_{x_\ell,j} = 1$ in the extended network. We set $p'_{i,\theta(i,j)} = p_{i,j}$, i.e. the proxy node of i to j serves to ensure that the probability of transition from i eventually leading to j remains the same. We set $p'_{x_1,x_2} = \ldots = p'_{x_\ell,j} = 1$, i.e. once the taxi is en route from node i to j (corresponding to a single discrete transition in the base network), it will arrive at node j with probability 1 in exactly $\tau_{i,j}$ transitions. For convenience we denote this transformation from a base transition matrix to an extended transition matrix by $P' = extend(P)$. (The same transformation also applies to the customer transition matrix D.) The following example illustrates extending a simple base network with $m = 2$ to 3 nodes.

$$T = \begin{bmatrix} 1 & 1 \\ \mathbf{2} & 1 \end{bmatrix},\ P = \begin{bmatrix} 0.5 & 0.5 \\ \mathbf{0.2} & 0.8 \end{bmatrix},\ P' = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.8 & \mathbf{0.2} \\ \mathbf{1} & 0 & 0 \end{bmatrix}.$$

In this example the discretized travel time from node $i = 2$ to node $j = 1$ is $\tau_{2,1} = 2$. A single auxiliary node is created corresponding to $\tau_{2,1} - 1 = 1$. In extending a policy P to P' we maintain the transition probabilities from every node i to the corresponding eventual destination node j in the base network as transitions to the proxy node $\theta(i,j)$ of i to j. So, since $\tau_{2,1} > 1$, we set $p'_{2,1} = 0$ and set $p'_{2,3} = p_{2,1}$ instead. Once a taxi has made the transition to node 3, it transitions to node 1 with probability 1.

C. Extended Stability Condition

Extending the network by incorporating travel times yields an extended system transition matrix S'. However the associated stationary distribution $\phi' = \pi(S')$ does not relate linearly to ϕ, and depends on the varying ingoing travel times from each node $j \neq i$ to i. Thus a stable policy for the base network may not be stable for the extended network. To ensure that the stability condition is correctly translated, we introduce a scaling vector ζ that accounts for the travel times between nodes in the extended network by considering transitions through auxiliary nodes en route to node i as counting towards the fraction of time that the vehicle spends at node i in steady state. We denote by $Z \in V'$ the set of auxiliary nodes that lead to base node i. Then ζ is given by:

$$\zeta_i = \frac{\phi'_i}{\phi'_i + \sum_k \phi'_{Z_k,i}}. \tag{10}$$

Then the stationary distribution of the base network ϕ is given by $\phi = [\phi'_1/\zeta_1 \ \phi'_2/\zeta_2 \ \dots \ \phi'_m/\zeta_m]$, and the stability constraint (5) becomes

$$n\zeta_i\phi_i > \lambda\alpha_i, \ \forall i \tag{11}$$

V. Practical HM Policy

The solution space of the optimization problem is the number of taxis n and the redistribution policy P. To simplify the solution, we fix n constant. This eliminates variable product terms in the stability constraint and allows us to formulate the problem as a linear program. Since n is discreet we solve successive linear programs for increasing values of n until a feasible solution is found and the system is stable. Stability is determined experimentally by running a series of bootstrap simulations to check if the request queues remain bounded. We denote by n_{min} the minimum number of taxis that admits a feasible solution that is stable. We set up the linear program as follows:

$$\begin{aligned} \text{Minimize} \quad & 0 \text{ (i.e. find } q) \\ \text{s.t.} \quad & n\zeta_i q_i > \lambda\alpha_i, \ \forall i && (12) \\ & 0 \leq q_i \leq 1, \ \forall i && (13) \\ & \textstyle\sum_i q_i = 1 && (14) \\ & n \geq n_{min}. && (15) \end{aligned}$$

Algorithm 1 describes the procedure for calculating the practical HM redistribution policy P_{HM}. There is no cost function to minimize n, i.e. the optimization problem reduces to a search problem to find a stationary distribution q that satisfies the stability condition. The stability condition is evaluated by running a number of bootstrap simulations with redistribution policy P_{HM} and checking the stability of the system transition matrix S' at the end of the simulation, as dictated by $n\zeta_i\phi_i > \lambda\alpha_i, \ \forall i$. n_{min} is then given by the smallest n that yields experimental stability for S'. The practical HM policy is obtained by applying the Hastings-Metropolis transformation to q giving $P_{\mathsf{HM}} = H(q)$.

Algorithm 1 Practical HM Policy Optimization Algorithm

Data: n: proposed number of taxis
λ: customer arrival rate
α: customer arrival distribution
D: customer transition matrix
ζ: stationary distribution scaling vector
n_{min}: minimum required number of taxis (optional)
Result: P_{HM}: redistribution policy

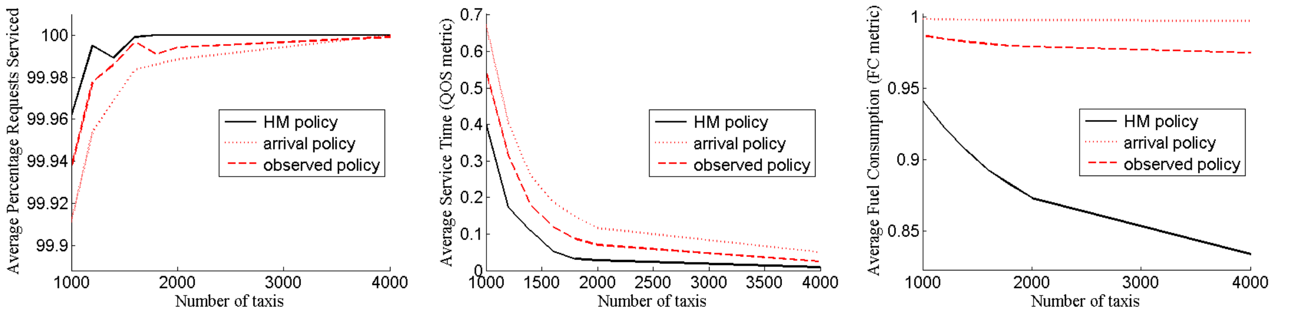
find q s.t. $n\zeta_i q_i > \lambda\alpha_i, \ \forall i$
$P := H(q)$
$P' := extend(P)$
$\phi' := \pi(S')$ {simulation}
$\phi := [\phi'_1/\zeta_1 \ \phi'_2/\zeta_2 \ \dots \ \phi'_m/\zeta_m]$
if $\exists \ n_{min}, \ n \geq n_{min}$ **then**
  $P_{\mathsf{HM}} := P$
else
  if $n\zeta_i\phi_i > \lambda\alpha_i, \ \forall i$ **then**
    $n_{min} := n$
  else
    $n := n+1$
  end if
  restart 1
end if
return P_{HM}

A. Accuracy and Complexity

The extended network is parametrized by the travel time discretization τ. This presents a trade-off between two degrees of accuracy in our urban mobility model. For a base network of size m, this determines the number of auxiliary nodes that will be added to the extended network. Assuming an average travel time of t_0 between two clusters, each base node inherits $2(m-1)$ auxiliary connections with $\tau_0 = \max\{\text{round}(t_0/\tau), 1\}$ auxiliary nodes connecting it to other base nodes and one self-loop with τ_0 auxiliary nodes. This yields a total of $m[2(m-1)+1]\tau_0 = O(m^2)\tau_0$ auxiliary nodes. Thus a smaller discretization τ means a larger ratio τ_0, which increases the size of the extended network as $O(m^2)$.

A smaller τ means a more accurate representation of the road network. However, the increase in accuracy comes at the cost of generality. Since customers arrive only at base nodes, a greater increase in the number of nodes in the extended network means a coarser granularity of customer origins and destinations. Conversely, for a given extended network size m', a larger base network size m and smaller travel time discretization ratio means a larger fraction of the m' nodes are considered as origins and destinations.

Finally, as discussed in Section IV-A, restricting arrivals to base nodes allows us to formulate the search problem for the base network and transform the result to the extended network. Thus the complexity of the linear program depends only on m and there is no penalty incurred with finer travel time discretization. This means that for a given base network size, we can achieve arbitrary travel time granularity for a fixed computational cost.



(a) Experimental Stability (b) Quality of Service (c) Fuel Consumption

Fig. 2: Simulation Results. Fig. 2a shows the percentage of requests serviced for each policy with increasing n. Fig. 2b shows the decrease in service time for each policy with increasing n (QOS metric). Fig. 2c shows the improvement in fuel consumption for each policy with increasing n (FC metric).

VI. Policy Criteria

We are interested in examining feasible solutions from three independent objectives: urban planning (UP), quality of service (QOS), and fuel consumption (FC). In order to meaningfully evaluate a proposed solution, we establish metrics that correspond to the three criteria under consideration.

1) Urban Planning: We assume that the main goal of the municipal authority is to reduce congestion in the city by minimizing the number of vehicles on the streets. From the UP perspective, the redistribution policy is irrelevant: the objective is simply to employ the minimum number of taxis that yields a valid solution. We define the UP metric as

$$L_{\mathsf{UP}} = n/n_{min} \tag{16}$$

i.e. the metric expresses the degree by which more taxis are employed in a given solution than is strictly necessary for stability. Note that $L_{\mathsf{UP}} \in [1, \infty)$, and $L_{\mathsf{UP}} = 1$ when $n = n_{min}$, i.e. the ideal UP policy is any solution that uses n_{min} taxis.

2) Quality of Service: Since the stability condition is satisfied for feasible solutions, and all customers in the system are being serviced, we reason that the net revenue from all customers is constant, regardless of the redistribution policy P. Thus we assume that the main incentive of the taxi company is to provide the best possible service quality. QOS is measured as the average service time (the queueing time plus the time waiting for a taxi) for a customer in the system. We define the QOS metric as

$$L_{\mathsf{QOS}} = \sum_i \alpha_i R_i \tag{17}$$

where R_i is the average service time for a customer waiting at node i. Note that $L_{\mathsf{QOS}} \in [0, \infty)$, and $L_{\mathsf{QOS}} = 0$ when $R_i = 0$, $\forall i$, i.e. the ideal QOS policy is a solution where the service time at all nodes is zero.

3) Fuel Consumption: We assume that it is in the interests of the taxi driver to minimize the fuel costs associated with the operation of their vehicle, and that the fuel consumption of a vehicle is reasonably characterized by its total daily mileage. We define the FC metric as

$$L_{\mathsf{FC}} = \sum_i \phi_i (1 - \beta_i) \sum_{j \neq i} p_{i,j}. \tag{18}$$

i.e. fuel consumption is minimized by maximizing the amount of time that the redistribution policy dictates the vehicle to remain on standby and wait for a customer at its current location. Note that $L_{\mathsf{FC}} \in [0, 1]$, and $L_{\mathsf{FC}} = 0$ when $S = I_m$, i.e. the ideal FC policy is the solution where all vehicles remain at their current locations and do not redistribute to other nodes.

VII. Experiments

A simulation framework was implemented in MATLAB. A base cluster network of 27 clusters was created from the Singapore taxi dataset according to the methodology described in Section IV. With a discretization of $\tau = 60s$, this yields an extended network of size $m' = 8615$. A one hour epoch was chosen for parameter measurements. The customer arrival rate λ was learned by recording the number of trips made by a taxi with a customer on board; this was calculated to be approximately 48 arrivals per minute. The customer transition matrix D was learned by recording the distribution of destination nodes of taxis leaving each node with a customer on board. In order to evaluate our proposed policy within the context of the actual taxi system in Singapore, simulations were conduction in comparison with the following two test policies.

1) Observed Policy: P_{obs} is the actual redistribution policy derived from the Singapore taxi data. This was learned by analyzing the distribution of taxis leaving each node without a customer on board. This is the "ground truth" policy and represents the actual redistribution behavior of an unmanaged taxi fleet. This is the policy that is of most interest to us because it provides an insight into the effectiveness of unmanaged redistribution. Further, the number of taxis n_{obs} that was actually observed to be in operation on Singapore roads was estimated by recording the number of individual taxi IDs that registered journeys within the given epoch.

2) Arrival Policy: P_{arr} is a "smart" but naive policy that provides a reasonable model for individual taxi driver behavior. The arrival policy is defined by $p_{i,j} = \alpha_j$, i.e. the taxi driver will choose his next location based on the chances that a customer will arrive there.

Policy	n	L_{UP}	L_{QOS}	L_{FC}	% serviced
P_{HM}	n_{min}	1	**0.40**	**0.94**	99.96
P_{obs}		1	0.67	0.99	99.91
P_{arr}		1	0.55	0.99	99.94
P_{HM}	$n_{min} \times 1.2$	1.2	**0.17**	**0.92**	99.99
P_{obs}		1.2	0.40	0.99	99.95
P_{arr}		1.2	0.32	0.98	99.98
P_{HM}	$n_{min} \times 1.4$	1.4	**0.11**	**0.91**	99.99
P_{obs}		1.4	0.26	0.99	99.97
P_{arr}		1.4	0.18	0.99	99.99
P_{HM}	$n_{min} \times 1.6$	1.6	**0.05**	**0.89**	99.99
P_{obs}		1.6	0.19	0.99	99.98
P_{arr}		1.6	0.12	0.98	99.99
P_{HM}	$n_{min} \times 1.8$	1.8	**0.03**	**0.88**	99.99
P_{obs}		1.8	0.15	0.99	99.99
P_{arr}		1.8	0.09	0.98	99.99
P_{HM}	$n_{min} \times 2$	2	**0.03**	**0.87**	99.99
P_{obs}		2	0.12	0.99	99.99
P_{arr}		2	0.07	0.99	99.99
P_{HM}	$n_{min} \times 4$	4	**0.01**	**0.83**	99.99
P_{obs}		4	0.05	0.99	99.99
P_{arr}		4	0.02	0.97	99.99

TABLE I: Simulation Results. A lower metric indicates better performance. The best policy is highlighted for each n.

Simulations were carried out for 8 simulation hours each (at $\tau = 60s$). Each simulation was carried out 5 times for each policy and for each value of n, and the results aggregated. First, n_{min} was determined by means of bootstrap simulations as described in Section V. Then the main test simulations were carried out (105 in total) employing P_{obs}, P_{arr} and P_{HM} policies with n set to increasing multiples of n_{min}. Results were evaluated using the metrics described in Section VI.

A. Results

Table I summarizes the simulation results. The optimization algorithm yielded $n_{min} = 1000$ (rounded to the nearest 100 vehicles) for $\lambda = 48$. The number of taxis that was actually observed to be in operation on Singapore roads was recorded from the taxi data as $n_{obs} = 10,088$. This clearly suggests that there are many more taxis in operation in Singapore than strictly necessary to service all customers without a buildup of queues. Since simulations were carried out for fixed n, the UP metric is the same for all policies for a given n. Fig. 2a shows a plot of the average percentage of customers serviced for different values of n. Observe that for $n = n_{min}$ the policies marginally satisfy stability constraints, and for $n > n_{min}$ all policies achieve almost full service.

The rightmost columns of Table I summarize the results for the QOS and FC metrics. Fig. 2b shows the calculated average waiting time of customers for increasing n. We see the HM policy improves substantially over the both test policies for the same number of taxis. Fig. 2c shows the average fuel consumption of taxis for increasing n. Again, the HM policy shows an improvement over both test policies.

These results confirm that the unmanaged redistribution behavior is indeed suboptimal. The fact that the HM policy shows a significant improvement for both QOS and FC metrics is intriguing as it indicates that the interests of the taxi driver and the customer may in fact be aligned, and suggests the potential for a natural incentive-based redistribution model for drivers that also improves quality of service.

VIII. Conclusions

In this paper we presented a Markov-based model for an urban transportation network. We considered three evaluation criteria: urban planning, fuel consumption, and quality of service. We presented a scalable optimization framework and proposed a solution that can be computed efficiently. We compared the computed policy to the activity of a fleet of 16,000 taxis in Singapore. The simulation experiments show that there is potential to improve the efficiency of the physical transportation system. We believe that this study is an important step toward understanding the basic trade-offs between the number of transportation vehicles in the system, vehicle fuel consumption, and the overall quality of service provided by the transportation system. Our next steps are to characterize these trade-offs, do extensive case studies in simulation and in the context of taxi systems in real urban settings, and to derive specific policies that optimize each of these individual criteria.

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