Lower Bounds for Asymmetric Communication Channels and Distributed Source Coding

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Data Transmission

Client $\longrightarrow$ Server

Send $s \in \{0,1\}^n$

$s \leftarrow D$, $H(D) < n$

Client sends $\sim H(D)$ bits

k clients $\longrightarrow$ 1 server

Send $s_1, \ldots, s_k \in \{0,1\}^n$

$(s_1, \ldots, s_k) \leftarrow D$ (correlated!), $H(D) < nk$

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What can be done?

	1 client	k clients
D fixed	[Huffman] client sends $\lceil H(D) \rceil$	[Slepian-Wolf] clients send $\lceil H(D) \rceil$
D known by server	[Adler-Maggs] clients send $O(H(D))$ server sends $O(n)$ expected $O(1)$ rounds $\Pr[t \text{ rounds}] \geq 2^{-O(t \lg t)}$	clients send $O(H(D))$ server sends $O(kn)$ $\Omega(\frac{\lg k}{\lg \lg k})$ needed

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1. communication by server – optimal
2. rounds – quasioptimal [NEW]

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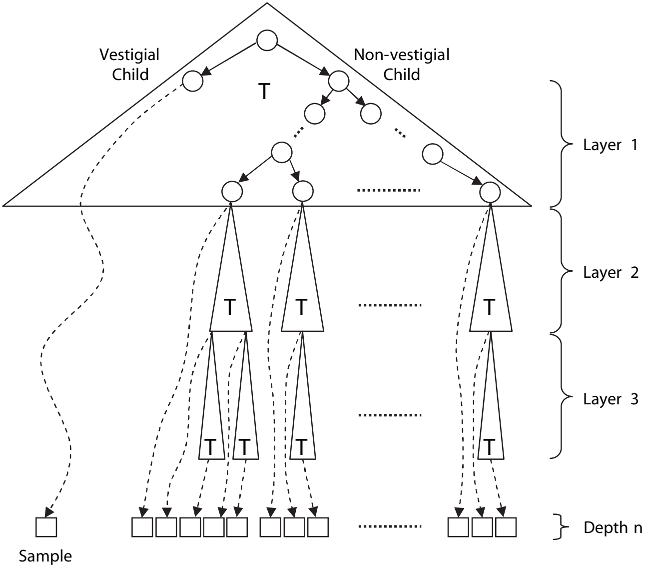
The class of hard distributions D



Intuition for hardness

Let h = height of one layer
Let $p = \Pr[\text{vestigial child}]$
$\implies H(D) = ph + (1-p)ph + (1-p)^2 ph + \ldots$

$H(D)$ is small
$\implies$ one client message cannot talk about many layers for many samples

Random choice of vestigial child (left / right)
$\implies$ don't know which samples need many layers

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Communication Complexity Tools

Message switching

Alice sends a message of $\leq a$ bits

$\Rightarrow$ eliminate, increasing Bob's message by a factor of 2^a

Round elimination lemma

$$\left.\begin{array}{l}\text{Alice gets } x_1, \ldots, x_k \\ \text{Bob gets } y, i \in [k]\end{array}\right\} \longrightarrow \text{they compute } f(x_i, y)$$

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Formal strategy

1. switch client's message
   NB: need hard upper bound on message size (Markov)

2. round elimination of server's message
   subproblems: what is below each T leaf
   prefix of client's sample chooses subproblem

3. repeat, in the smaller probability space where the sample is not vestigial at this level

Contradiction

Eliminated i rounds by introducing "small" error
With no rounds, cannot solve better than random guessing
Sample is at level $> i \Rightarrow$ nontrivial problem

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Trouble in paradise

- many complications and subtleties
- innovative communication complexity analysis

Example

Obtaining a hard bound for the client's messages:

- $\Pr[\text{sample is from level } \geq i] = (1-p)^i$
- error introduced must be small in this space
- hard bound (by Markov) must be huge $\approx H(D)/(1-p)^i$

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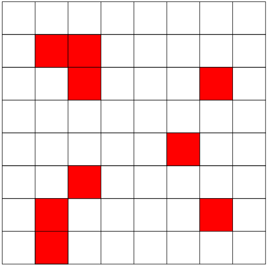
Technical insight: Unilateral error

Regular error

Unilateral error



Application

- Markov on client's message introduces unilateral error
- conditioning the sample being from level $\geq i$ does not change the marginal distribution on the client's input

$\Longrightarrow$ much better Markov bound

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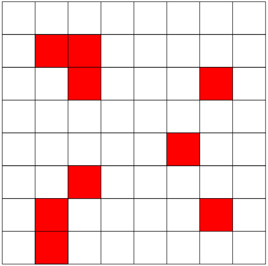

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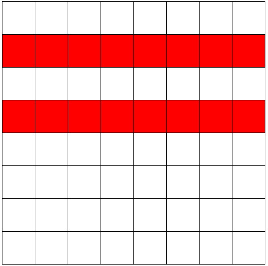

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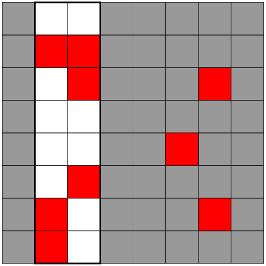

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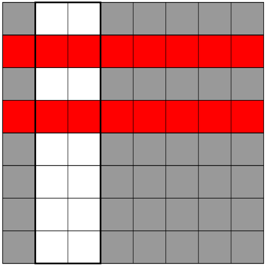

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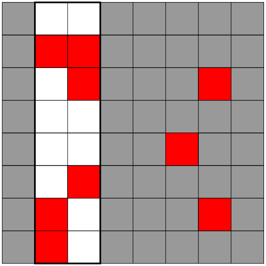

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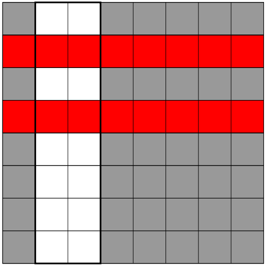

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