De Dictionariis Dynamicis Pauco Spatio Utentibus

(*lat.* On Dynamic Dictionaries Using Little Space)

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Dictionaries

Maintain $S \subset [u], |S| \le n$ under:

INSERT(x) : $S \leftarrow S \cup \{x\}$

DELETE(x) : $S \leftarrow S \setminus \{x\}$

MEMBER(x) : is $x \in S$?

OPUS CLASSICUM : [FKS'82]

query: $O(1)$ worst-case

update: $O(1)$ expected $\longrightarrow$ w.h.p. [DMadH'90]

space: $O(n \lg u)$ bits $\longrightarrow$ $O(n \lg \frac{u}{n})$ bits [RR'03]

We achieve both, through new techniques.

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Basic Idea

How to get w.h.p. from expected bounds? independence

Idea: $\sqrt{n}$ independent tables

1. permute universe randomly: π
2. distribute elements by $hi(\pi(x))$
3. store $lo(\pi(x))$ in hash table

Permutation needed!

To be information-efficient, need to store just $lo(\pi(x))$.

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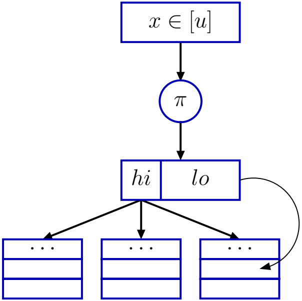

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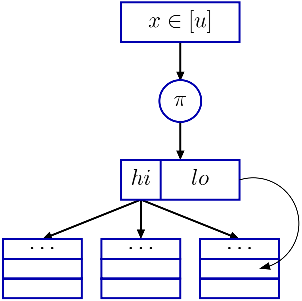

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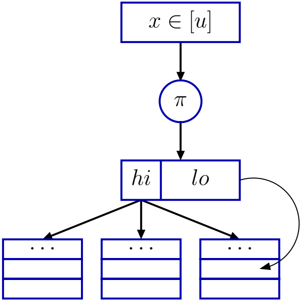

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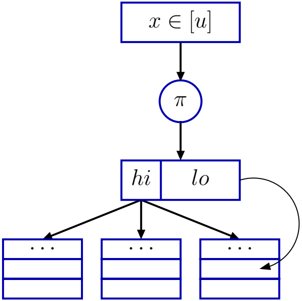

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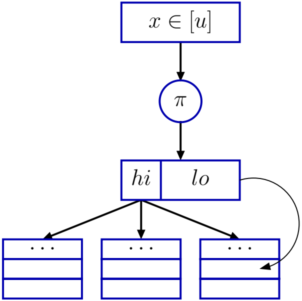

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Wait in line. . .

Idea: construct permutations with good k^{th} moment bounds.

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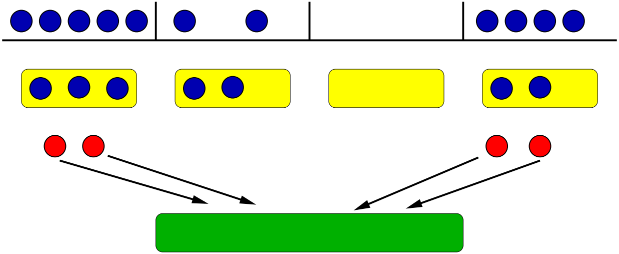
1. break universe into $n/\lg n$ segments
2. minidictionary in each segment with capacity $2\lg n$
3. fall back to high performance dictionary:



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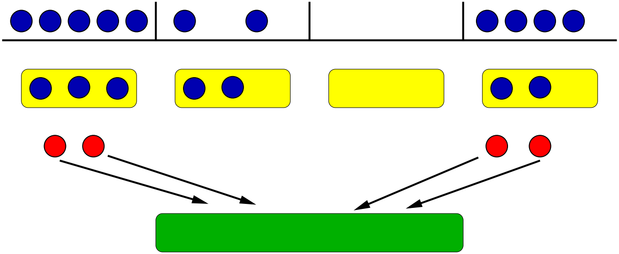

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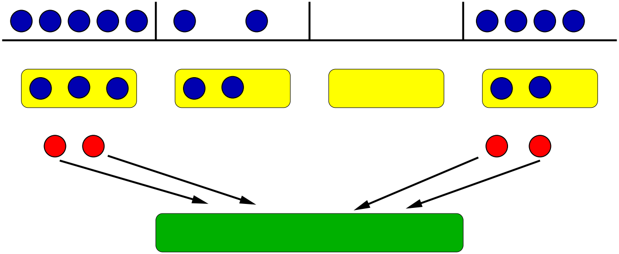

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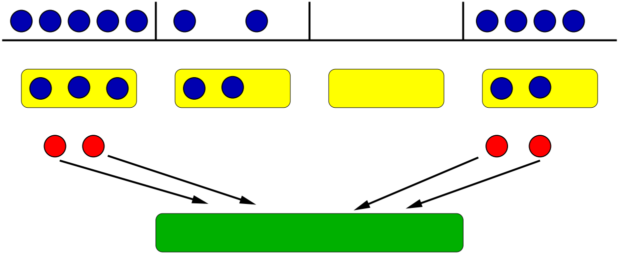
   - elements overflowing capacity of minidictionary
   - elements for which the minidictionary failed



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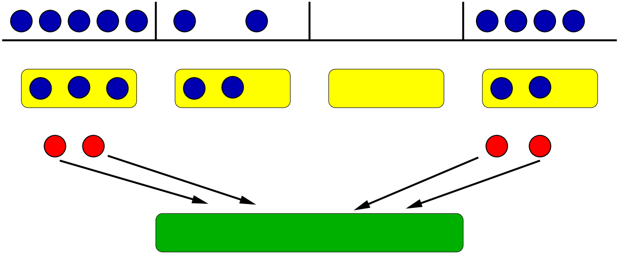
exp. $O(1)$ bad elements from each segment
$\Rightarrow O(n/\lg n)$ total w.h.p.



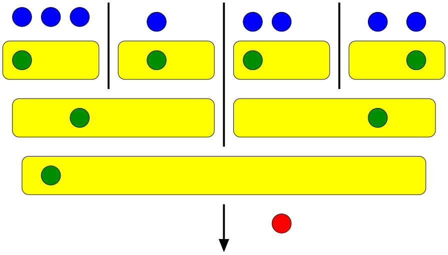
Life is tough

Minidictionary stores $O(\lg n)$ elements in arbitrary order

$\Rightarrow \Omega(\lg\lg n)$ bits per element!

For small universes, need to do something crazy...

$O(\lg\lg n)$ levels of filters
on disjoint segments of the universe } all packed in a word!



Dictionary? What dictionary?

What is the query?

membership: is $x \in S$?

requires $\geq \lg \binom{u}{n} = \Omega(n \lg \frac{u}{n})$ bits

retrieval: return $data(x)$ if $x \in S$

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perf hashing: return unique, immutable ID for any $x \in S$

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Recapitulare

1. independence through permutation hashing
2. multilevel word-packed dictionaries
3. dictionaries without membership

FINIS

GRATIAS AGO VOBIS