Higher Lower Bounds for Near-Neighbor and Further Rich Problems

Mihai Pătrașcu  Mikkel Thorup

FOCS 2006
How we came up with the title

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\( P_3 \) cites \( P_1 \); Piotr in charge of bibliography.

\textbf{MON APR 10, 2006}

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\textbf{TUE APR 11, 2006}

02:00am
All bugs fixed. Mikkel goes to bed.

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A theory student sends email: “bought donuts; 5th floor lounge”

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### Communication game

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</tr>
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Lost in Translation

Memory can remember past communication
Data Structures and Communication

Cell-probe data structure

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Communication game

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**Cell-probe data structure** | **Communication game**
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Lost in Translation
Memory can remember past communication
What can this prove?

Richness technique ⇒ one of

\[
\begin{aligned}
T \lg S & \geq A \\
T d & \geq B
\end{aligned}
\]  ⇒  \( T \geq \min\{ \frac{A}{\lg S}, \frac{B}{d} \} \)

Best lower bound possible

\( A \sim d, B \sim n \Rightarrow T = \Omega \left( \min\{ \frac{d}{\lg S}, \frac{n}{d} \} \right) \) \text{ ...typically, } T = \Omega \left( \frac{d}{\lg S} \right) \)

Our result: A “black-box” improvement to \( T = \Omega \left( \frac{d}{\lg S} \right) \).

| \( S = n^{O(1)} \) | \( T = \Omega(\frac{d}{\lg n}) \) |
| \( S = n(d \lg n)^{O(1)} \) | \( T = \Omega(\frac{d}{\lg n}) \) |
| \( T = \Omega(\frac{d}{\lg d}) \) |
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| \( T = \Omega(\frac{\lg n}{\lg \lg n}) \) |
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Td \geq B
\end{array} \right. \\
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- near neighbor search:

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- Barkol, Rabani STOC’00: ENN $\Omega(d)$ rand
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1. break input into \( k \) subproblems
e.g.: for NN, \( k \) far clusters, \( \frac{n}{k} \) points each

2. simulate \( k \) queries in parallel:
   \[ T \text{ rounds} \]
   \[ \left\{ \begin{array}{l}
   \text{queriers send } \lg \left( \frac{S}{k} \right) = \Theta(k \lg \frac{S}{k}) \text{ bits} \\
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   \end{array} \right. \]

NB: \( k \lg \frac{S}{k} < k \lg S \). This step outside communication paradigm.

3. prove direct-sum law for richness:
one problem has \([A, B]\) lower bound by richness
   \[ \Rightarrow k \text{ problems have } [\Omega(kA), \Omega(kB)] \text{ lower bound} \]
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Proof Sketch

deterministic richness: simple combinatorics
randomized richness: complicated combinatorics
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3. Prove direct-sum law for richness:
   - One problem has $[A, B]$ lower bound by richness
   - $\Rightarrow k$ problems have $[\Omega(kA), \Omega(kB)]$ lower bound

Conclusion:

\[
\begin{align*}
Tk \lg \frac{S}{k} &= \Omega(kA) = \Omega(kd) \\
Tk \cdot d &= \Omega(kB) = \Omega(k \frac{n}{k})
\end{align*}
\Rightarrow T = \Omega(d/ \lg \frac{S \cdot d}{n}).
\]
GOOD: simulating $k$ queries at the same time
BAD: breaking into $k$ subproblems
    works if hardness comes from dimension (not for ANN)

THE END
Conclusions

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