Higher Lower Bounds for Near-Neighbor and Further *Rich* Problems

Mihai Pătraşcu Mikkel Thorup

FOCS 2006

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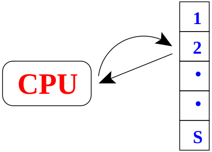

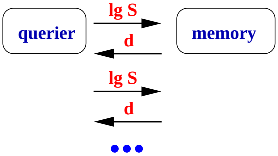

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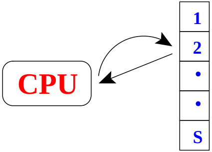
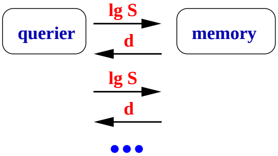
Lost in Translation

Memory can remember past communication

Data Structures and Communication





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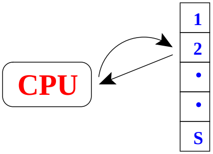
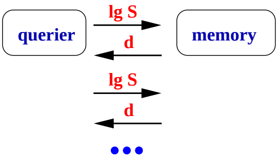
Data Structures and Communication





Cell-probe data structure	*Communication game*
space S	querier sends $\lg S$ bits
d-bit cells	memory sends d bits
query time T	T rounds

Lost in Translation

Memory can remember past communication

What can this prove?

Richness technique $\Rightarrow$ one of $\begin{cases} \text{querier sends } A \text{ bits} \\ \text{memory sends } B \text{ bits} \end{cases}$

$\Rightarrow$ one of $\begin{cases} T \lg S \geq A \\ Td \geq B \end{cases}$ $\Rightarrow T \geq \min\{\frac{A}{\lg S}, \frac{B}{d}\}$

Best lower bound possible

$A \sim d, B \sim n \Rightarrow T = \Omega\left(\min\{\frac{d}{\lg S}, \frac{n}{d}\}\right)$ ...typically, $T = \Omega(\frac{d}{\lg S})$

Our result: A "black-box" improvement to $T = \Omega(d / \lg \frac{Sd}{n})$.

	$S = n^{O(1)}$	$T = \Omega(\frac{d}{\lg n})$	
	$S = n(d \lg n)^{O(1)}$	$T = \Omega(\frac{d}{\lg n})$	$T = \Omega(\frac{d}{\lg d})$
$d = O(\lg n)$	$S = n^{O(1)}$	$T = O(1)$	
$d = O(\lg n)$	$S = n \lg^{O(1)} n$	—	$T = \Omega(\frac{\lg n}{\lg \lg n})$

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Known Richness Results

- partial match — *curse?*
- near neighbor search:

	Deterministic	Randomized
Exact	*curse?*	*curse?*
Approximate	*curse?*	*no curse*

MSNW STOC'95: p.m. $\Omega(\sqrt{\lg d})$ rand
Borodin,Ostrovsky,Rabani STOC'99: p.m., ENN $\Omega(\lg d)$ rand
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Liu IPL'04: ANN $\Omega(d)$ det
Andoni,Indyk,P. FOCS'06: ANN $\Omega(\frac{1}{\varepsilon^2} \lg n)$ rand

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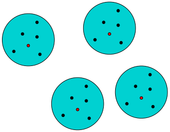
Proof Technique

1. break input into k subproblems
   e.g.: for NN, k far clusters, $\frac{n}{k}$ points each



2. simmulate k queries in parallel:

   T rounds $\begin{cases} \text{queriers send } \lg \binom{S}{k} = \Theta(k \lg \frac{S}{k}) \text{ bits} \\ \text{memory sends } k \cdot d \text{ bits} \end{cases}$

NB: $k \lg \frac{S}{k} < k \lg S$. This step outside communication paradigm.

3. prove direct-sum law for richness:
   one problem has $[A, B]$ lower bound by richness
   $\Rightarrow$ k problems have $[\Omega(kA), \Omega(kB)]$ lower bound

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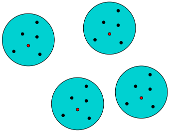

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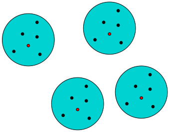

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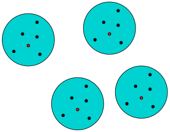

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deterministic richness: simple combinatorics

randomized richness: complicated combinatorics

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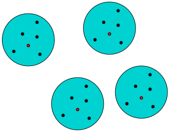

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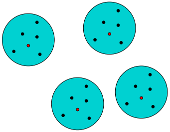

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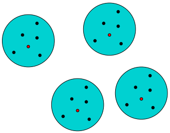

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Conclusion: $\begin{cases} Tk \lg \frac{S}{k} = \Omega(kA) = \Omega(kd) \\ Tkd = \Omega(kB) = \Omega(k \frac{n}{k}) \end{cases} \Rightarrow T = \Omega(d / \lg \frac{Sd}{n})$.

Conclusions

GOOD: simmulating k queries at the same time

BAD: breaking into k subproblems
works if hardness comes from dimension (not for ANN)

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