Higher Lower Bounds for Near-Neighbor and Further Rich Problems

Mihai Pătrașcu Mikkel Thorup





FOCS 2006

 $P_1 = [P., Thorup], P_2 = [P.], P_3 = [Andoni, Indyk, P.].$ P_3 cites P_1 ; Piotr in charge of bibliography.

 $P_1 = [P, Thorup], P_2 = [P, P_3 = [Andoni, Indyk, P, P_3], P_3 \text{ cites } P_1; Piotr in charge of bibliography. Mon APR 10, 2006$

09:00am

 P_1 , P_2 started. Piotr emailed with title for P_1 .

05:00pm

title of P_1 changed, Plotr notified

bugs discovered in P

Mikkel and Minai think and talk on the phone

TUE APR 11, 200

02:00am all bugs fixed. Mikkel goes to bed.

... Mihai writes on P_2 .

a theory student sends email: "bought donuts; 5th 1100r lounge"

00am Mihai crashes on couch in front of office

08:30am Mihai, Mikkel get up.

11:00am title of P_1 changed, Piotr notified.

first complete draft of P_1 . Authors nearing collapse.

P₂ completed; Mikkel calls: "let's spice up the abstract" of P₁.

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08:30a

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30pm

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03:45pm

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03:55pm

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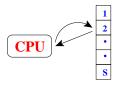
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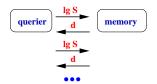
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Data Structures and Communication





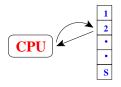
Cell-probe data structure	Communication game
space S	querier sends lg S bits
d-bit cells	memory sends d bits
query time T	T rounds

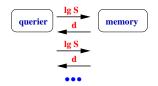
Lost in Translation

Memory can remember past communication



Data Structures and Communication





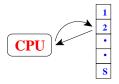
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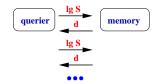
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Richness technique
$$\Rightarrow$$
 one of $\begin{cases} \text{querier sends } A \text{ bits} \\ \text{memory sends } B \text{ bits} \end{cases}$
 \Rightarrow one of $\begin{cases} T \lg S \ge A \\ Td > B \end{cases} \Rightarrow T \ge \min\{\frac{A}{\lg S}, \frac{B}{d}\}$

Best lower bound possible

$$A \sim d, B \sim n \Rightarrow T = \Omega\left(\min\{\frac{d}{\lg S}, \frac{n}{d}\}\right)$$
 ...typically, $T = \Omega(\frac{d}{\lg S})$



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	$S = n(d \lg n)^{O(1)}$	$T = \Omega(\frac{d}{\lg n})$	
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$d = O(\lg n)$	$S=n^{O(1)}$	T = O(1)	
$d = O(\lg n)$	$S = n \lg^{O(1)} n$	_	$T = \Omega(\frac{\lg n}{\lg \lg n})$



Known Richness Results

- partial match curse?
- near neighbor search:

	Deterministic	Randomized
Exact	curse?	curse?
Approximate	curse?	no curse

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MSNW STOC'95: p.m. \Omega(\sqrt{\lg d}) rand Borodin,Ostrovsky,Rabani STOC'99: p.m., ENN \Omega(\lg d) rand Barkol,Rabani STOC'00: ENN \Omega(d) rand syram,Khot,Kumar,Rabani STOC'03: p.m. \Omega(\frac{d}{\lg n}) rand Liu IPL'04: ANN \Omega(d) det
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Borodin, Ostrovsky, Rabani STOC'99: p.m., ENN $\Omega(\lg d)$ rand

Barkol, Rabani STOC'00: ENN $\Omega(d)$ rand

Jayram, Khot, Kumar, Rabani STOC'03: p.m. $\Omega(\frac{d}{\lg n})$ rand

Liu IPL'04: ANN $\Omega(d)$ det

Andoni,Indyk,P. FOCS'06: ANN $\Omega(\frac{1}{\varepsilon^2} \lg n)$ rand



• break input into k subproblems e.g.: for NN, k far clusters, $\frac{n}{k}$ points each



simmulate k queries in parallel:

T rounds
$$\left\{\begin{array}{l} \text{queriers send lg}\left(\frac{S}{k}\right) = \Theta(k \lg \frac{S}{k}) \text{ bits} \\ \text{memory sends } k \cdot d \text{ bits} \end{array}\right.$$

NB: $k \lg \frac{S}{k} < k \lg S$. This step outside communication paradigm.

1 prove direct-sum law for richness: one problem has [A, B] lower bound by richness $\Rightarrow k$ problems have $[\Omega(kA), \Omega(kB)]$ lower bound

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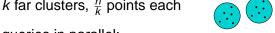
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Proof Sketch

deterministic richness: simple combinatorics randomized richness: complicated combinator

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$$\begin{cases} \text{ queriers send lg } \binom{S}{k} = \Theta(k \lg \frac{S}{k}) \text{ bits} \\ \text{ memory sends } k \cdot d \text{ bits} \end{cases}$$

NB: $k \lg \frac{S}{k} < k \lg S$. This step outside communication paradigm.

one problem has [A, B] lower bound by richness $\Rightarrow k$ problems have $[\Omega(kA), \Omega(kB)]$ lower bound

Proof Sketch

deterministic richness: simple combinatorics

randomized richness: complicated combinatorics

break input into k subproblems e.g.: for NN, k far clusters, $\frac{n}{k}$ points each





simmulate k queries in parallel:

T rounds
$$\left\{\begin{array}{l} \text{queriers send lg}\left(\frac{S}{k}\right) = \Theta(k \lg \frac{S}{k}) \text{ bits} \\ \text{memory sends } k \cdot d \text{ bits} \end{array}\right.$$

NB: $k \lg \frac{S}{k} < k \lg S$. This step outside communication paradigm.

prove direct-sum law for richness: one problem has [A, B] lower bound by richness \Rightarrow k problems have $[\Omega(kA), \Omega(kB)]$ lower bound

Conclusion:
$$\left\{ \begin{array}{l} \textit{Tk} \lg \frac{S}{k} = \Omega(\textit{kA}) = \Omega(\textit{kd}) \\ \textit{Tkd} = \Omega(\textit{kB}) = \Omega(\textit{k}\frac{n}{k}) \end{array} \right| \Rightarrow \textit{T} = \Omega(\textit{d}/\lg \frac{S\textit{d}}{n}).$$



Conclusions

GOOD: simmulating *k* queries at the same time

BAD: breaking into *k* subproblems works if hardness comes from dimension (not for ANN)

THE END

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 $THE\ END$