Non-Adaptive Fault Diagnosis for All-Optical Networks via Combinatorial Group Testing on Graphs

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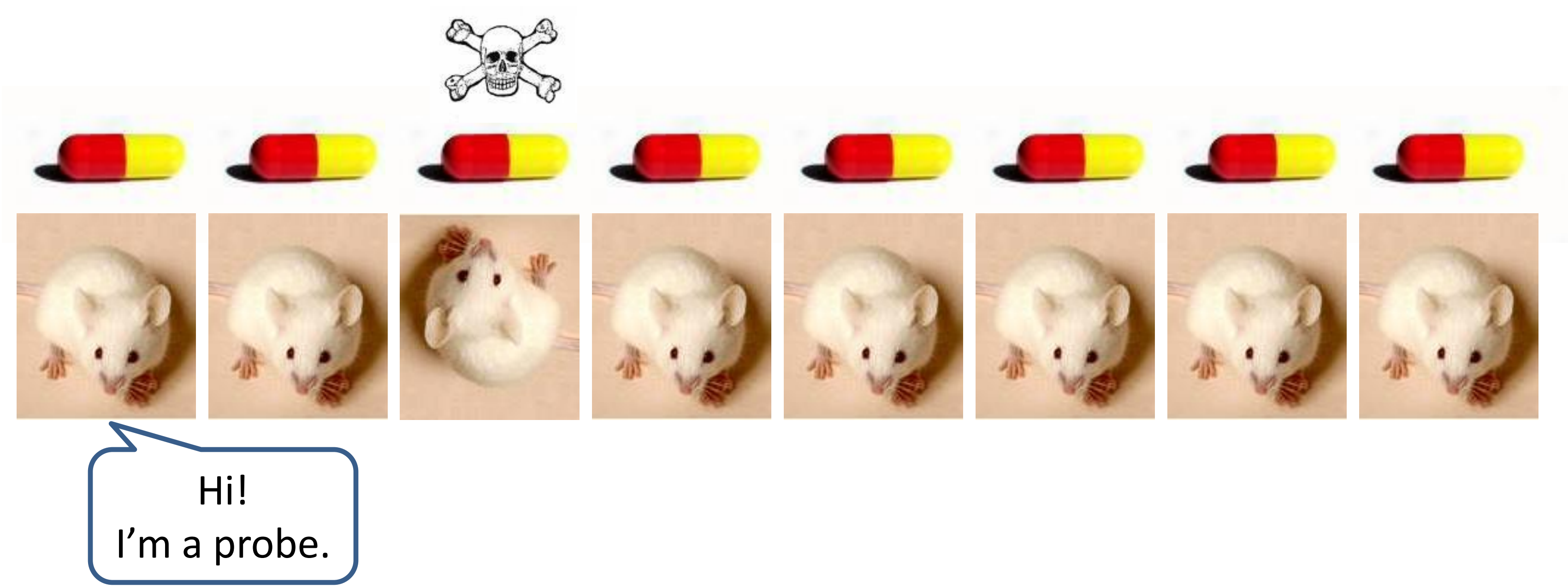
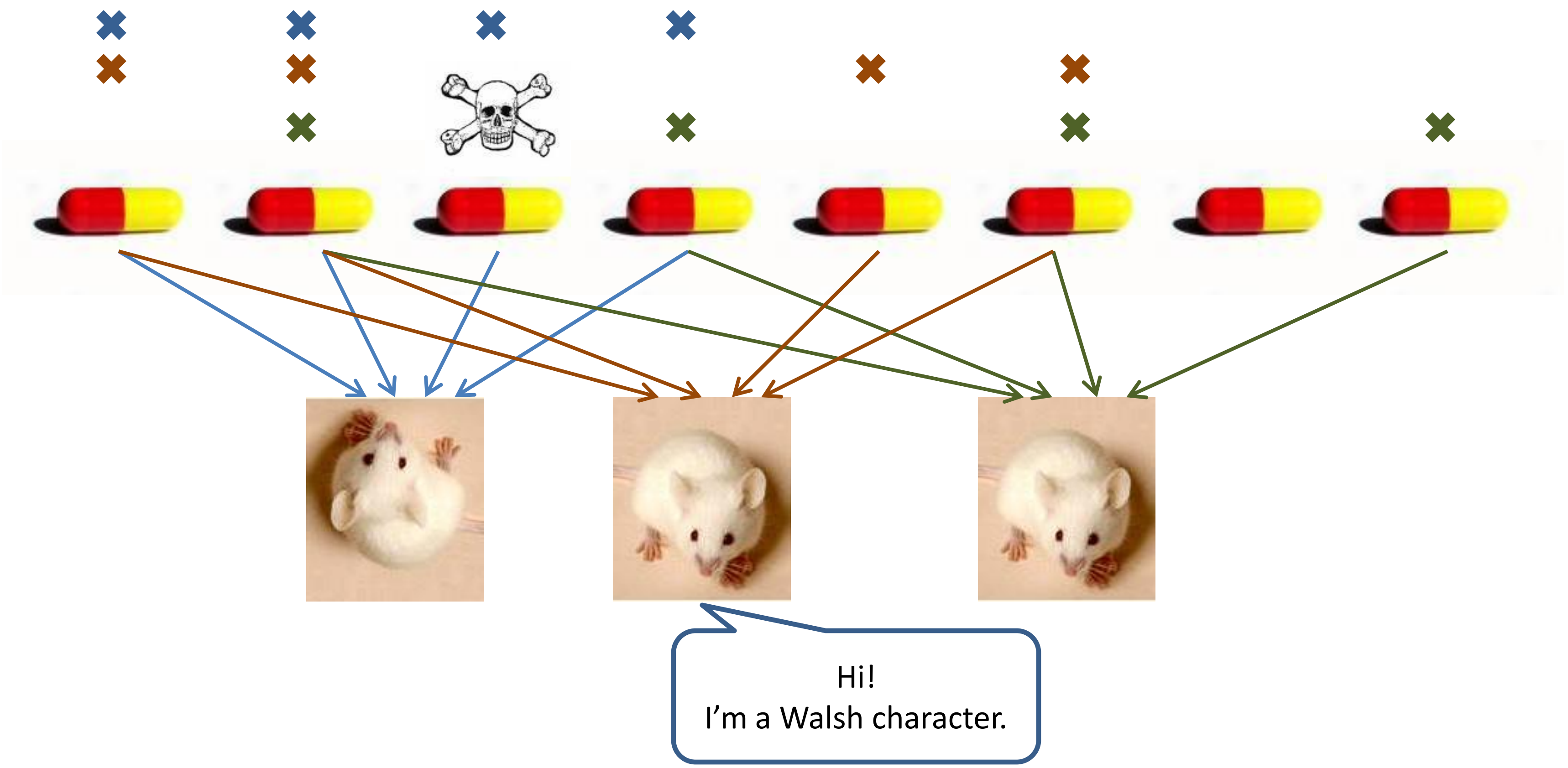
Group Testing

- n drugs: n-1 good, 1 bad

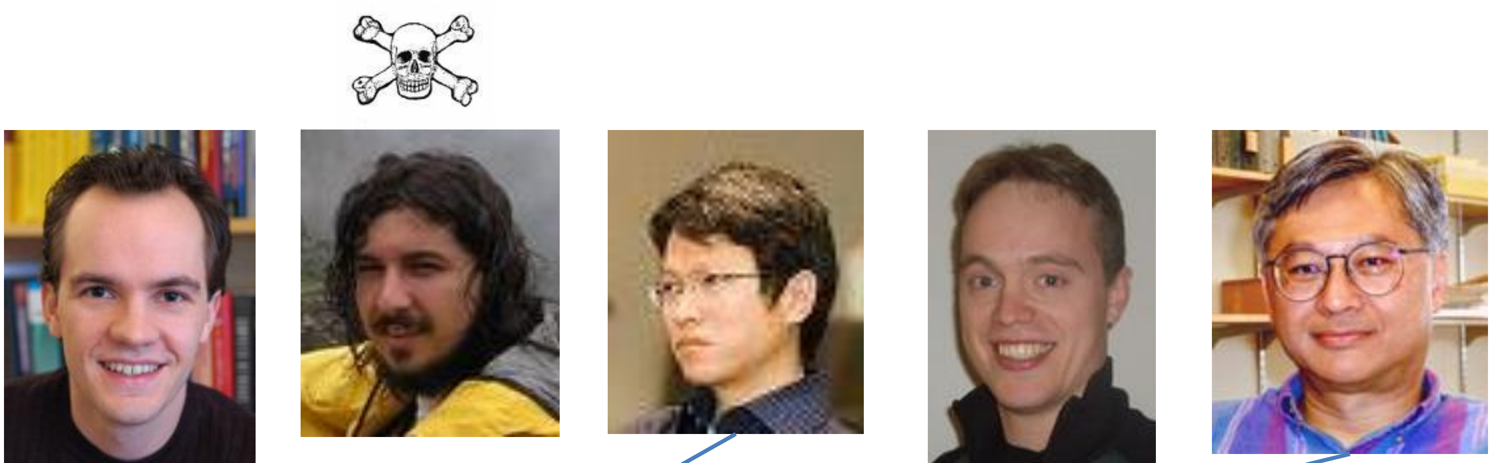


Group Testing

- n drugs: n-1 good, 1 bad
- better idea:



Very Useful in Practice



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To: nickh, eewyg, chan
Subject: is any of you willing to give the talk?
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Combinatorial Group Testing

- adaptive / nonadaptive
- m = # elements
- s = # bad elements
- T* = # necessary probes

$$\Omega\left(\frac{s^2}{\log s}\log m\right) \le T^*(m,s) \le O(s^2 \log m)$$

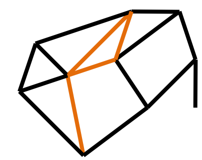
adaptive: $O(s \log m)$

Group Testing on Graphs

- elements = edges


- bad elements = failed edges


- probes = connected subgraphs

Assumptions

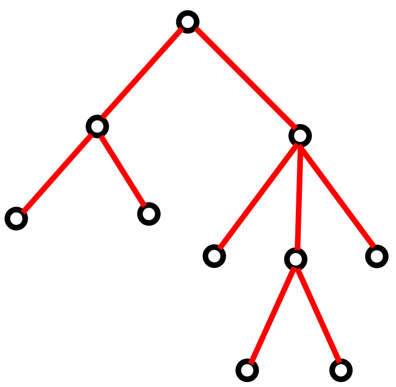
☹	1 element = $1
Graph Testing:	1 connected subset = $1
CGT:	1 subset = $1

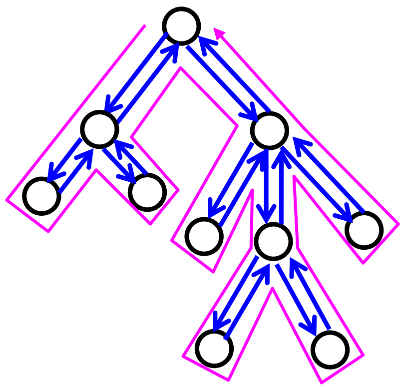
Cost Model: All-Optical Networks

Practical assumption: undirected graph

an edge = 2 parallel optical fibers

=> testing entire connected subgraph = 1 lightpath



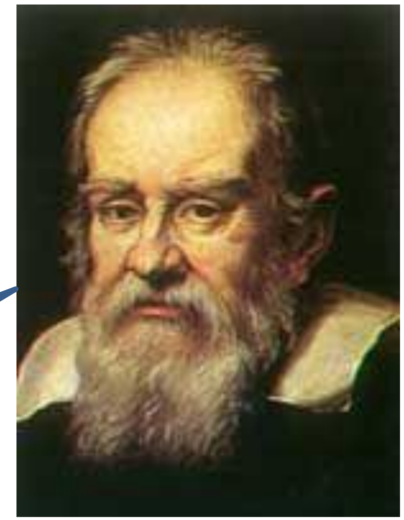


Cost Model: All-Optical Networks

"1 lightpath = $1"

routing with
negligible attenuation

speed of light = fast



Optical routing

=> real cost: transmission / reception

=> real delay: adaptivity

Assumptions

Failure model:

- links fail (cable cuts…)
- permanent failure model (link = up/down)
- adversarial failures (up to s)

Network model:

- undirected
- separate control network

  [alternative: want to find the connected component of a central node]

Some Special Cases

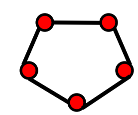
adaptive: $O\left[s \log m\right]$

Nonadaptive group testing on graphs: $L^*(G,s)$

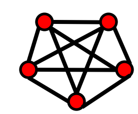
$$L^*(G,s) \geq T^*(m,s)$$

$$T^*(m,s) \approx s^2 \log m$$

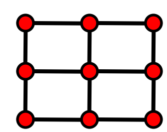
- line, ring $1 \cdots n$ $\qquad L^*(s=1) \;=\; \lceil n/2 \rceil$



- complete graph K_n $\qquad L^*(s=1) \;=\; O(\log n)$

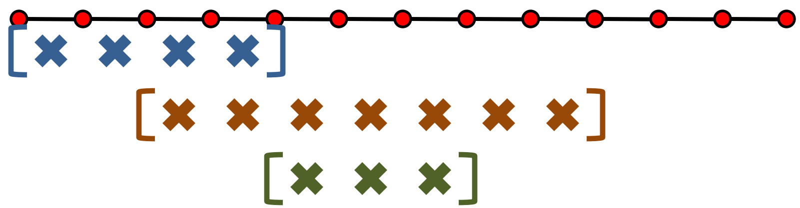


- grid, torus $\sqrt{n} \times \sqrt{n}$ $\qquad L^*(s=1) \;=\; O(\log n)$, $\quad L^*(s \geq 7) \;=\; \Omega(n)$



Lesson 1: Small Connectivity Hurts

Lower bound for line:

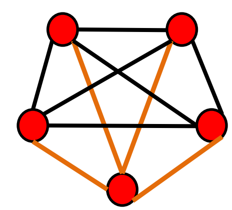


- probes = interval
- if one vertex has no [or]

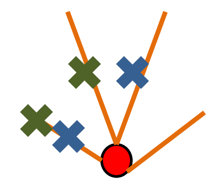
  => can't distinguish adjacent edges

Lesson 2: Just Need a Trusted Subnet

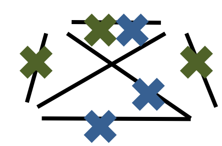
Solution for complete graph:



1. test the star $(1,2),(1,3),\cdots,(1,n)$



2. assuming 1. says "fail":
   apply CGT on star edges



3. assuming 1. says "not fail":
   apply CGT on all other edges

Cost: $1 + T^*(n, s=1) + T^*\{(n-1)(n-2)/2, s=1\}$

$$= O(\log n)$$

General Results: Well Connected

min-cut 2(s+1)

If G containts s+1 edge-disjoint spanning trees:

$$T^*(m,s) \le L^*(G,s) \le O(s \cdot T^*(m,s))$$

$O\left[s^3 \log m\right]$

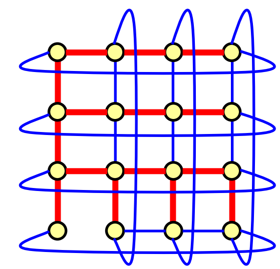
Proof:

- test each subtree => at least one is ok
- use this subtree to do CGT on the other edges

Corollaries:

- 2D torus has min-cut 4 => $L^*(s=1) = O(\log n)$

  2D grid similar, more complicated



- complete graph has min-cut n-1 => can handle $s \le \left[n-3\right]/2$

General Results: Trees

Tree T of depth D: $\Omega(D + \log n) \leq L^*(T, s=1) \leq O(D + \log^2 n)$

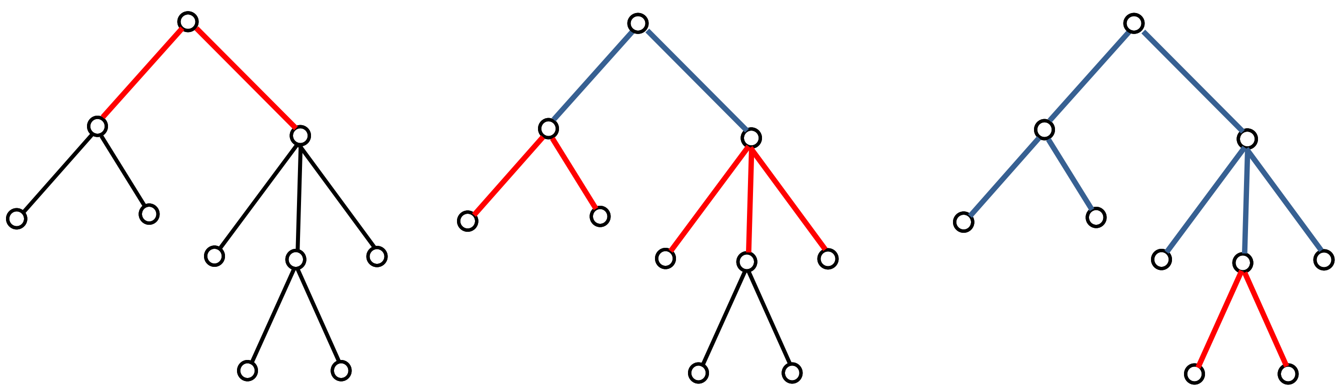
Proof:

- for all $d \leq D$, assume failure is at depth d
  use tree to depth $d-1$ to do CGT at depth d

$O(D \cdot \log n)$

centroid decomposition

$O(D + \log^2 n)$



General Results: Low Diameter

G connected, diameter D: $L^*(G,1) \leq \mathrm{O}(D + \log^2 n)$

Proof:

	Cost
• let T = shortest path tree (depth D)	
• test T	1
• assuming T is ok, do CGT on G\T	O(log *n*)
• assuming T in not ok, apply tree algorithm on T	
	O(*D*+log^2*n*)

Summary

- trees $\Omega(D + \log n) \le L^*(T, s=1) \le O(D + \log^2 n)$

  * line, ring: $L^*(s=1) = \Theta(D) = \Theta(n)$

- diameter D: $L^*(G, s=1) \le O(D + \log^2 n)$

- s+1 edge-disjoint spanning trees

$$\Omega\left(\frac{s^2}{\log s}\log m\right) \le L^*(G,s) \le O(s^3 \log m)$$

  * complete graph $\forall s \le \lfloor n-3 \rfloor / 2$
  * torus, grid

$$\begin{aligned} L^*(s=1) &= O(\log n) \\ L^*(s \ge 7) &= \Omega(n) \end{aligned}$$

The End

I have a question about slides {1,2,5,6,9}