Dynamic Connectivity: Connecting to Networks and Geometry

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Part I:

Graph Theory

Dynamic Subgraph Connectivity

computer network

Maintain undirected graph **G** under:

add new cable,
cable cuts

- insert/delete edges

router reboot,
misconfiguration, etc

- turn nodes ON/OFF

through ON nodes

- query: “are nodes **u** and **v** connected ?”

Results

[Chan STOC'02]

- $t_u = O(m^{0.94})$, $t_q = \tilde{O}(\sqrt[3]{m})$ using <u>F</u>ast <u>M</u>atrix <u>M</u>ult.
- $t_u = O(m^{0.89})$ in the ideal case of FMM in $O(n^2)$
- "moral $\Omega(\sqrt{m})$" from triangle-finding, etc
- conjecture: no $O(m^{1-\varepsilon})$ without FMM

[CPR FOCS'08]

- $t_u = \tilde{O}(m^{2/3})$, $t_q = \tilde{O}(\sqrt[3]{m})$
- cute, simple$^{(?)}$, no FMM

Dynamic Graph Problems

edge updates

	Amortized	Worst-case
• dynamic connectivity	$O(\lg n \cdot (\lg\lg n)^3)$	$O(\sqrt{n})$
• dynamic MST	$O(\lg^2 n)$	$O(\sqrt{n})$
• dyn. reachability (directed)	$O(n^2)$	$O(n^2)$
• dynamic APSP	$\tilde{O}(n^2)$	$\tilde{O}(n^{2.75})$

✔ node updates

Dynamic Graph Problems

edge updates

	Amortized	Worst-case
• dynamic connectivity	$O(\lg n \cdot (\lg\lg n)^3)$	$O(\sqrt{n})$
• dynamic MST	$O(\lg^2 n)$	$O(\sqrt{n})$
• subgraph connectivity	$\tilde{O}(m^{2/3})$	$O(m)$
• dyn. reachability (directed)	$O(n^2)$	$O(n^2)$
• dynamic APSP	$\tilde{O}(n^2)$	$\tilde{O}(n^{2.75})$

✔ node updates

The Algorithm: Idea 1

- can always do t_u= degree * Õ(lg n)

  … be smart about large degree nodes!

- **H** = graph of nodes with degree ≥ $m^{1/3}$

  * $O(m^{2/3})$ nodes

  * edges = contract components of low-degree nodes

How fast?

[Chan] subquadratic via FMM

✓ update high-degree node: O([illegible])

✓ query high-degree node: O(lg n)

The Algorithm: Idea 2

- H = high-degree nodes + components with $\geq m^{1/3}$ edges

  * $O(m^{2/3})$ nodes

  * edges = contract small components

construction time: $O(m^{4/3})$

$O(m^{2/3})$ amortized

turn ON a small degree node:

  * add to **H**, rebuild after $m^{2/3}$ insertions

The Algorithm: Idea 3

Turn OFF a node from a small component

* recompute adjanced edges of H in $(m^{1/3})^2$ time

Turn OFF a node from a large component

* use dynamic connectivity to find subcomponents

* leave largest subcomponent in place, move other(s)

$\Rightarrow$ $O(\lg m)$ work per edge in total (halving trick)

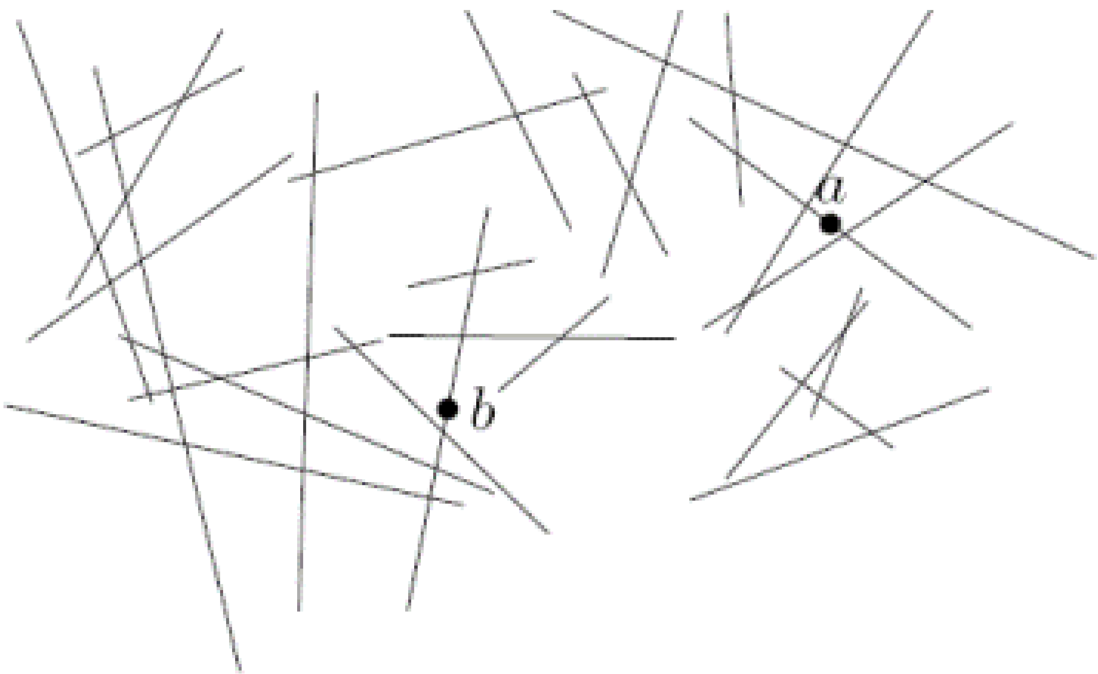
Part II:

Computational Geometry

Dynamic Geometric Connectivity

Maintain collection **S** of objects:

- update: insert/delete objects
- query: “are objects **u** and **v** connected thru intersections?”

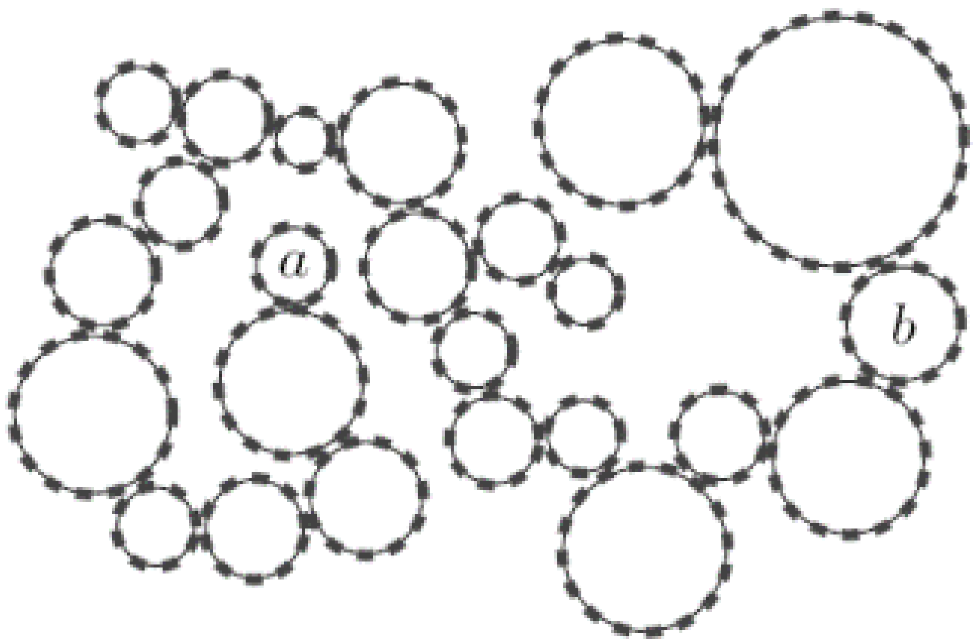


Is b reachable from a staying on the roads?

Dynamic Geometric Connectivity

Maintain collection **S** of objects:

- update: insert/delete objects
- query: “are objects **u** and **v** connected thru intersections?”



Do the gears transmit rotation from a to b?

Connecting to Subgraph Connectivity

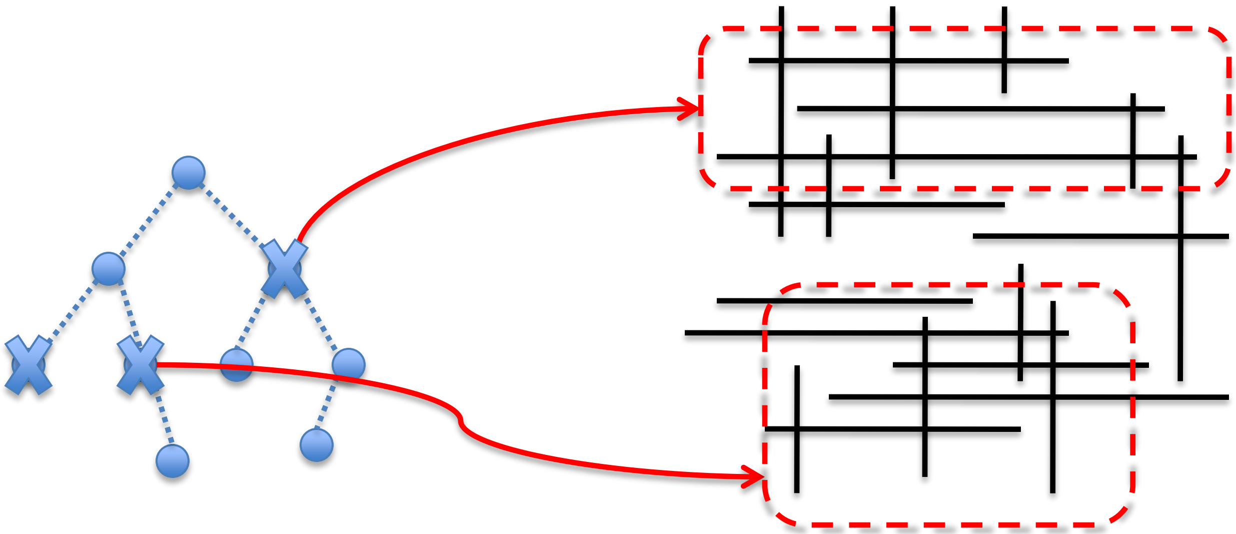
[Chan STOC'02]

If objects allow range reporting in space S, query τ…

Geometric connectivity reduces to subgraph connectivity
* graph has $m=S$ edges
* update slow down by a factor of τ

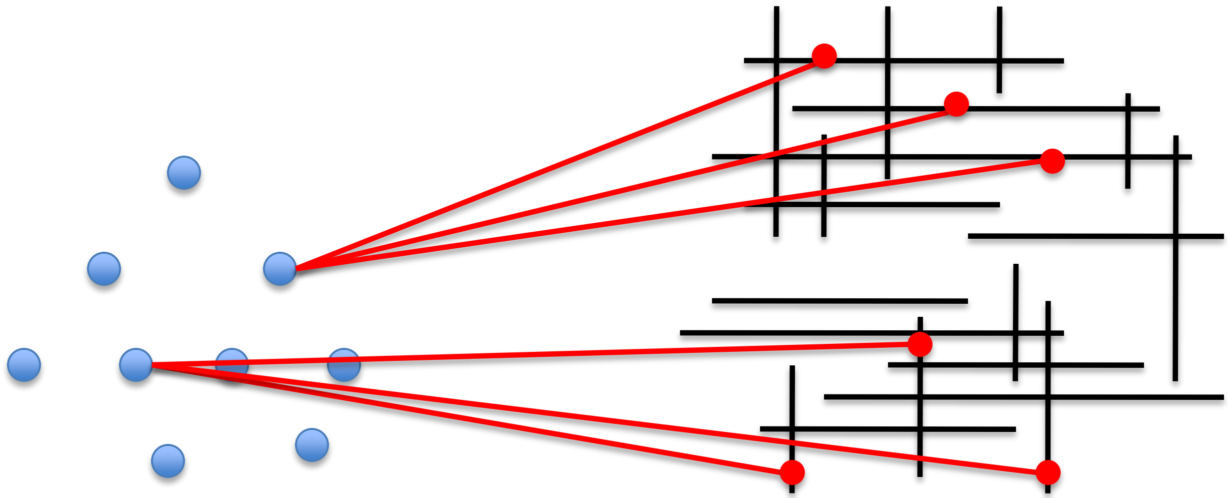
Example: axis parallel boxes (reduction loses polylogs)

Connecting to Subgraph Connectivity



Query answers = union of objects in O(τ) nodes

Connecting to Subgraph Connectivity



Query answers = union of objects in O(τ) nodes

=> turn those nodes ON

Results

[Chan STOC'02]

- axis-parallel boxes ⇒ subgraph connectivity
- in $3D^+$, subgraph connectivity ⇒ axis-parallel boxes

[Afshani, Chan ESA'06]

- axis-parallel, 2D: $t_u=\tilde{O}(n^{10/11})$, $t_q=O(1)$

[Eppstein'95]

- equal-radius balls ⇒ MST ⇒ range searching
- in 2D: $t_u=O(\lg^{10} n)$ via [Chan SODA'06]

Our Result

[Agarwal, Matousek]
anything under the sun

If objects allow range reporting in space $\tilde{O}(n)$, query $O(n^{1-\alpha})$

$\Rightarrow$ geometric connectivity can be solved in $O(n^{1-\beta})$

Examples:

- 2D line segments $\tilde{O}(n^{9/10})$
- 3D disks $\tilde{O}(n^{35/36})$

Open Problems

Graph Theory:

- beat $t_u = \tilde{O}(m^{2/3})$? …ideally $O(\sqrt{m})$
- what query time is possible? spanning tree?
- $o(n)$, for any m? … likely impossible
- worst-case $o(m)$? … “batched dynamic connectivity”

Geometry:

- “real algorithms” for interesting special cases

The End

Questions?