# Reinforcement Learning without Intractable Oracles?

# Ankur Moitra (MIT)

April 7<sup>th</sup>, ONR Program Meeting

# OUTLINE

#### **Part I: Introduction**

- Models and Problems
- Hardness and Beyond Worst-Case Analysis
- Our Results

#### **Part II: Planning**

#### Part III: Learning

• Approximate MDPs via Barycentric Spanners

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# **REINFORCEMENT LEARNING (RL)**





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Standard model is a Markov Decision Process







- State Space  ${\cal S}$  , start at  $s_0$
- Action Space  ${\cal A}$
- Rewards  $R_h(s,a)$
- Transition Probabilities

 $\mathbb{T}_{h}(s'|s,a)$ 



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 $\bullet \operatorname{Horizon} H$ 





**Goal:** Find a policy  $\pi: \mathcal{S} \to \mathcal{A}$  that maximizes expected reward

Main problems:

(1) **Planning:** Given a full description of the MDP, **compute** an optimal policy

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  - e.g. value iteration, policy iteration, linear programming
- (2) Learning: Given budget of iterations with the environment (e.g. simulator, episodic), learn an optimal policy
  - e.g. model based, q-learning, actor-critic, policy gradient

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function approximation, block MDPs, etc



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#### Partially observable MDPs (POMDPs)

There is a rich understanding of how to augment the model, and still be able to bound sample complexity

# WHAT ABOUT COMPUTATIONAL COMPLEXITY?

Returning to our earlier picture



Modern RL is generally built on computationally intractable oracles

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Modern RL is generally built on computationally intractable oracles

Are there computationally efficient algorithms with strong end-to-end provable guarantees?

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• Horizon H



# PARTIALLY OBSERVABLE MDPS (POMDPS)



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# **PLANNING IS HARD**

Classic lower bound:

# **Theorem [Papadimitriou, Tsitsiklis]:** Optimal planning in a POMDP is PSPACE hard

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Optimal action only depends on current state $\pi: \mathcal{S} \to \mathcal{A}$	

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Can you succinctly represent an optimal policy?

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Natural approaches use exponential space  $(|\mathcal{A}||\mathcal{O}|)^H$  or  $C^{|\mathcal{S}|}$ 

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Why should real-world POMDPs have succinct descriptions of good policies?
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Could this enable tractable planning/learning?

**Definition:** We say the POMDP is  $\gamma$ -observable if for all h and all distributions b,b' on states we have

$$\|\mathbb{O}_h b - \mathbb{O}_h b'\|_1 \ge \gamma \|b - b'\|_1$$

i.e. well-separated distributions on states lead to well-separated distributions on observations

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Key Point: No assumption on transition dynamics like e.g. deterministic transitions or mixing (under every possible policy)

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# MAIN RESULTS (PLANNING)

There is a quasi-polynomial time algorithm for planning under observability:

**Theorem [Golowich, Moitra, Rohatgi]:** Given the description of a  $\gamma$ -observable POMDP there is an algorithm running in time  $H(|\mathcal{O}||\mathcal{A}|)^{C\log(|\mathcal{S}|H/\epsilon)/\gamma^4}$ 

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Key Idea: The Bayes filter is exponentially stable compute posterior on states, given actions/observations

# MAIN RESULTS (PLANNING), CONTINUED

Moreover these results are tight

**Theorem [Golowich, Moitra, Rohatgi]:** Under the Exponential Time Hypothesis, there is no algorithm running in time  $(|\mathcal{S}||\mathcal{A}|H|\mathcal{O}|)^{o(\log(|\mathcal{S}||\mathcal{A}|H|\mathcal{O}|/\epsilon)/\gamma)}$ 

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It's hard even in the lossy case, where you observe the state with probability  $\gamma$  independently at each step

## WHAT ABOUT LEARNING?



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#### Can we build a learning algorithm on top of this primitive?

Assumption 1: The POMDP is undercomplete, i.e.  $|\mathcal{S}| \leq |\mathcal{O}|$ And moreover  $\sigma_{min}(\mathbb{O}_h) \geq \alpha$  for all h

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But optimism is very hard!

# MAIN RESULTS (LEARNING)

We show how to solve learning by using barycentric spanners to construct a policy cover. As a result:

**Theorem [Golowich, Moitra, Rohatgi]:** There is an algorithm with running time and sample complexity

$$(|\mathcal{O}||\mathcal{A}|)^{C\log(H|\mathcal{S}||\mathcal{O}|/\epsilon\gamma)/\gamma^4}$$

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These are the first end-to-end algorithmic guarantees for learning POMDPs, without oracles or strong assumptions about the model dynamics

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**Theorem:** Fix any  $\gamma$  -observable POMDP and policy  $\pi$  . Then



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Parallels well-known stability results for Kalman filtering

## **BELLMAN UPDATES FOR POMDPS**

Can find an optimal policy through:



## TRUNCATED BELLMAN UPDATES

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We only need a quasi-polynomial number of belief states

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(3) States in M can mapped to beliefs (using a uniform prior).

By belief contraction, M and P approximate each other
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How can we find a mixture of policies that visits all latent states?

### **BARYCENTRIC SPANNERS**

**Definition:** Given a set  $\mathcal{X} \subseteq \mathbb{R}^d$ , a  $\lambda$ -approximate barycentric spanner is a set  $\mathcal{C} \subseteq \mathcal{X}$  of size d such that every point in  $\mathcal{X}$  can be expressed as a linear combination of points in  $\mathcal{C}$  with coefficients in the range  $[-\lambda, \lambda]$ 

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**Theorem [Awerbuch, Kleinberg '04]:** Given an oracle for optimizing linear functions over  $\mathcal{X}$ , there is a polynomial time algorithm for constructing a  $\lambda$ -approximate barycentric spanner with

$$O(d^2 \log_\lambda d)$$

calls to the optimization oracle (assuming  ${\mathcal X}$  is compact)

### **POLICY COVERS**

Now let

### $\mathcal{X}=rac{\mathrm{set}\ \mathrm{of}\ \mathrm{all}\ \mathrm{distributions}\ \mathrm{on}\ \mathrm{observations}}{\mathrm{at}\ \mathrm{step}\ \mathrm{h}\ \mathrm{that}\ \mathrm{can}\ \mathrm{be}\ \mathrm{obtained}\ \mathrm{by}\ \mathrm{a}\ \mathrm{policy}}$

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**Claim:** By observability, if we can construct policies

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whose induced distributions on observations at step h are an approximate barycentric spanner, we must visit each latent state with nonnegligible probability

Our approach is:

M<sub>h</sub> MDP that approximates P up to step h









Our approach is:



Without explorability, need more complex measure of progress

### LOOKING FORWARD

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Takeaway: Improved regret bounds, where you only need to explore state-action pairs with substantially inaccurate predictions, even without knowing which ones are accurate in advance

#### Summary:

- Modern RL is built on computationally intractable oracles. Are there end-to-end guarantees?
- Quasi-polynomial time algorithm for planning in observable POMDPs, no assumption on dynamics
- New framework for learning without optimism

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# Thanks! Any Questions?