Rethinking Robustness: From Classification to Contextual Bandits

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UAI 2021 Keynote

Based on work with Sitan Chen, Frederic Koehler, Morris Yau

In this talk, we will explore models for corruption that **blend worst-case and average-case analysis**, in hopes of designing **more robust algorithms** for classic problems in learning

OUTLINE

Part I: Supervised Learning

- PAC Learning and Robustness
- Our Results and Framework
- Applications to Fairness

Part II: Online Learning

- Regression and Clean MSE
- Dynamic Range vs. Variance
- Our Results and Extensions to Contextual Bandits

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- (2) Assume Y = h(X) for some unknown hypothesis h that is in a known class H

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Probably Approximately Correct

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Random Classification Noise: Each label is flipped with some fixed probability



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[Blum et al.]: Efficient algorithm for halfspaces under RCN

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[Kalai et al.], [Awasthi, Balcan, Long], [Daniely]

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Are there distribution-independent algorithms for learning with Massart noise?

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Theorem [Diakonikolas, Goulekakis, Tzamos '19]: There is a polynomial time algorithm for **improperly** learning halfspaces under Massart noise with error $\eta + \epsilon$

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Is there a proper learning algorithm?

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Theorem: There is a polynomial time algorithm for learning **generalized linear models** under Massart noise

i.e
$$\mathbb{E}[Y|X] = \sigma(\langle w^*, X \rangle + b)$$

link function: monotone, Lipschitz

In particular, this includes noisy logistic regression as a special case

For RCN, a natural approach is to use the Leaky ReLU...



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How can we tolerate varying noise rates?

A GENERAL FRAMEWORK

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$$\begin{array}{c} \min \max \mathbb{E}[\mathbf{c}(\mathbf{X})\ell_{\lambda}(-Y\langle w,X\rangle)]\\ \|w\| \leq 1 \quad \mathbf{c} \quad \mathbf{Leaky ReLU} \quad \mathbf{c} \quad \mathbf{v} \quad \mathbf{$$

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Claim: The optimal solution for the min-player is w^{*}

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Claim: The optimal solution for the min-player is w^{*}

Unfortunately, optimizing over the max-players strategies is both statistically and computationally hard

A GENERAL FRAMEWORK, CONTINUED

Instead we work with a relaxation where the max-player can only restrict the distribution to **slabs along the current w**

$$\min_{\|w\| \leq 1} \max_{\mathbf{r} > \mathbf{0}} \mathbb{E}[\ell_{\lambda}(-Y\langle w, X \rangle)| - r \leq \langle w, X \rangle \leq r]$$

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We show that any approximate equilibrium in this game necessarily corresponds to a hypothesis with low error

THE ALGORITHM

How do we find an approximate equilibrium?



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Main Theorem: Converges in a polynomial number of iterations and provably solves the Massart learning problem

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We measure overall accuracy and accuracy on the part of the target group that is above \$50k

Target group: African Americans



Target group: Female



Target group: Female



Many natural algorithms (e.g. logistic) amplify bias in the data

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Is ours more fair because it can tolerate heterogenous noise?

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In each time step, we

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(2) Predict the response
$$y_t = \langle w^*, x_t \rangle + \sigma_t$$

true regressor additive noise

and incur loss based on the squared error

Classic Solution: Online Gradient Descent, see e.g. [Hazan, '19]

MODELS FOR NOISE

What if some of the responses are corrupted?

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Definition: In the Huber Contamination Model, a random η fraction of the responses are arbitrarily corrupted



ROBUSTNESS GUARANTEES

Proposition [folklore]: Online gradient descent achieves



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Lower Bound: CAMSE must be at least $\Omega(\eta^2 \sigma^2)$

variance of stochastic noise

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The cone of lines achieving nearly optimal CAMSE is wide, and corruptions cannot mess things up too much!

Hard Case: The range² is much larger than the variance



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The cone of lines achieving nearly optimal CAMSE is narrow

Hard Case: The range² is much larger than the variance



The cone of lines achieving nearly optimal CAMSE is narrow, and corruptions can mess things up badly
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Our algorithm is a simple twist on least trimmed squares

NOTES

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Many works in stronger contamination models, but work in offline setting and make distributional assumptions

[Klivans, Kothari, Meka], [Prasad et al.], [Diakonikolas et al.], [Bakshi, Prasad], [Zhu et al.], [Cherapnamjeri et al.], ...

LEAST TRIMMED SQUARES

In 1984, Rousseeuw introduced a powerful methodology



LEAST TRIMMED SQUARES, REVISITED

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Our twist: Set
$$S \leftarrow \arg \min_{S \in \mathcal{F}} \sum_{i \in S} \left(y_i - \langle \widehat{w}, x_i \rangle \right)^2$$

where \mathcal{F} is the set of all subsets whose covariance is approx. the same as covariance of all points

ONLINE SETTING

Finally, we can build an online algorithm from the offline one using cutting planes methods

Set N = 10000, d = 500, R = 1, σ = 0.1 and true regressor

$$w = [1, 0, 0, \cdots, 0]$$

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And to model rare, but predictive features set

$$x_t = \begin{cases} \text{approximately 0, with probability 0.8} \\ [1, 0, 0, \cdots, 0], \text{ with probability 0.1} \\ [-1/2, 0, 0, \cdots, 0], \text{ else} \end{cases}$$

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Adversary: Zero out a random fraction of the responses



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Note: Can extend to infinite dimensional spaces, using kernels

Many applications:

E-commerce: Selecting ads to display, based on user history e.g. [Abe, Nakamura]

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Mobile Health: Just-in-time interventions to modify behavior, adapted to the user e.g. [Nahum-Shani et al.]



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Yes! Standard approach uses linear regression as a subroutine

make accurate predictions about the loss of an action [Foster, Rakhlin] good action

Thus we get new algorithms for linear contextual bandits that are **provably resistant to adversarial corruptions**

CONCLUDING REMARKS

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Are there real-world applications where provably robust estimators can replace their non-robust counterparts?

Thanks!

Any Questions?