What Does Robustness Say About Algorithms?

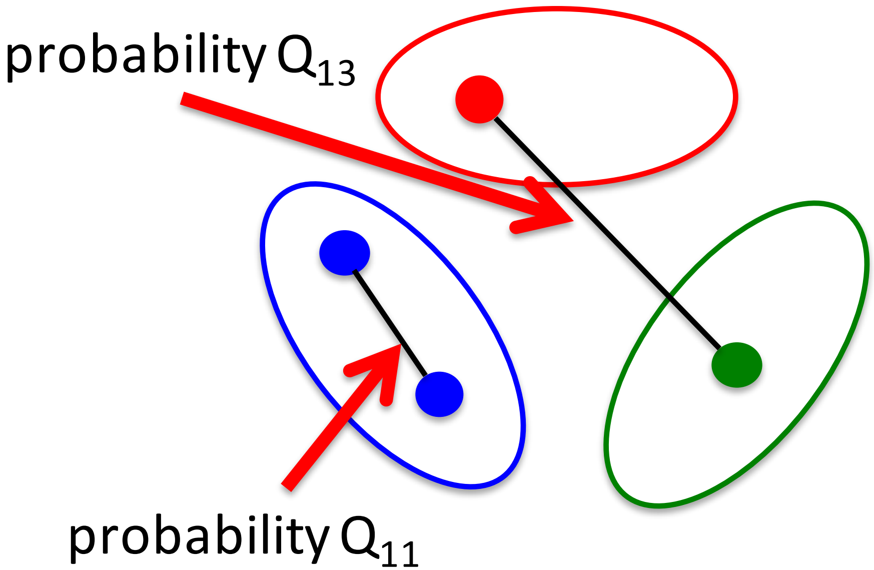
Ankur Moitra (MIT)

ICML 2017 Tutorial, August 6th

Let me tell you a story about the tension between **sharp thresholds** and **robustness**

THE STOCHASTIC BLOCK MODEL

Introduced by Holland, Laskey and Leinhardt (1983):



- k communities
- connection probabilities

Q =

Q_{11}	Q_{12}	Q_{13}
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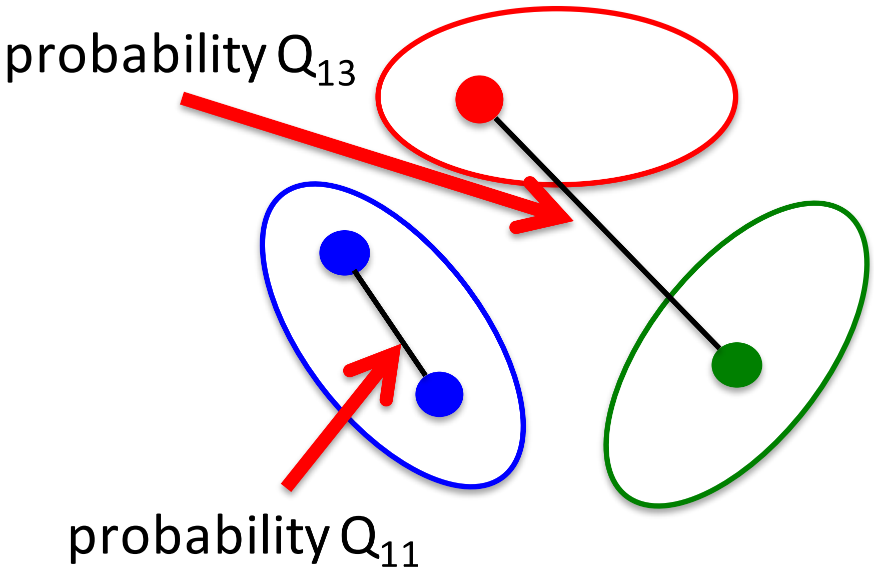

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Ubiquitous model studied in **statistics**, **computer science**, **information theory**, **statistical physics**

Testbed for diverse range of algorithms

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e.g. degree counting **[Bui, Chaudhuri, Leighton, Sipser '87]**

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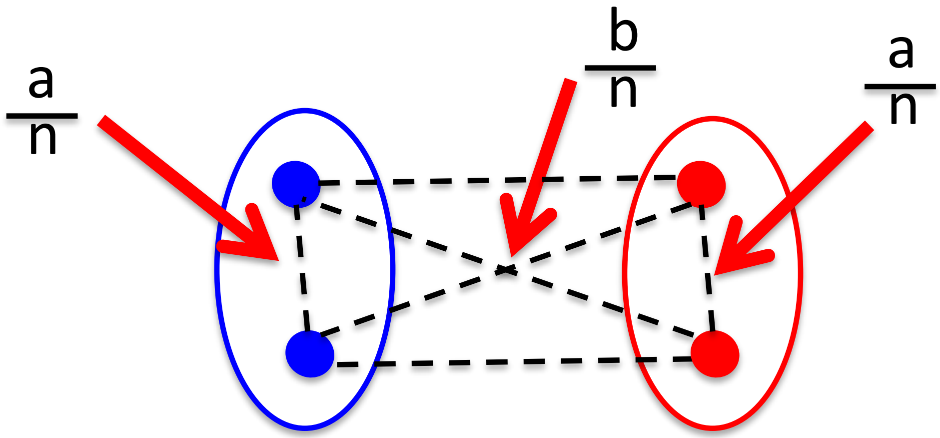
Can we reach the fundamental limits of the SBM?

Following Decelle, Krzakala, Moore and Zdeborová (2011), let's study the **sparse** regime:



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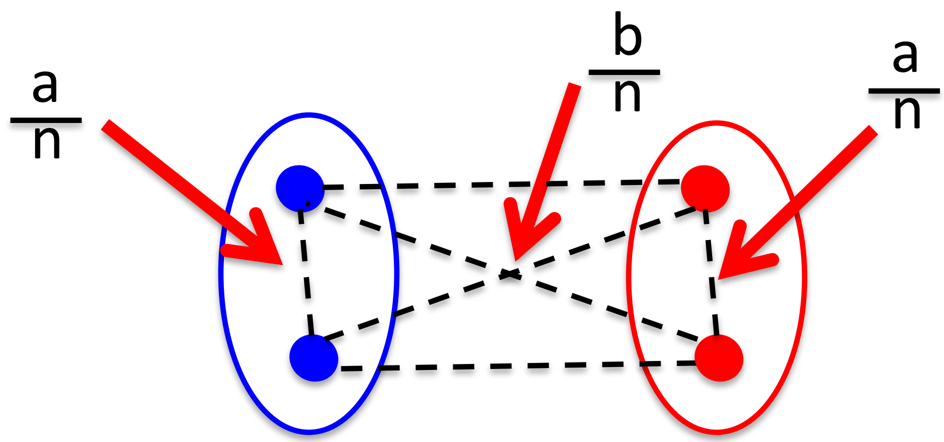

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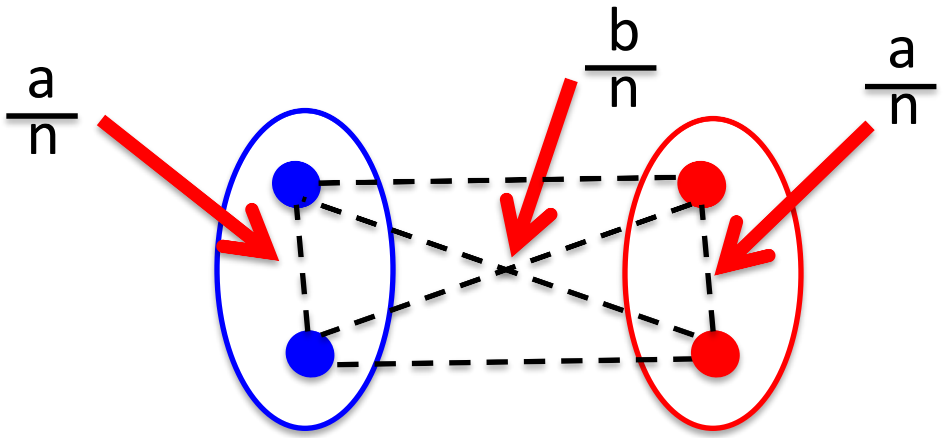

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Goal (Partial Recovery): Find a partition that has agreement better than ½ with true community structure

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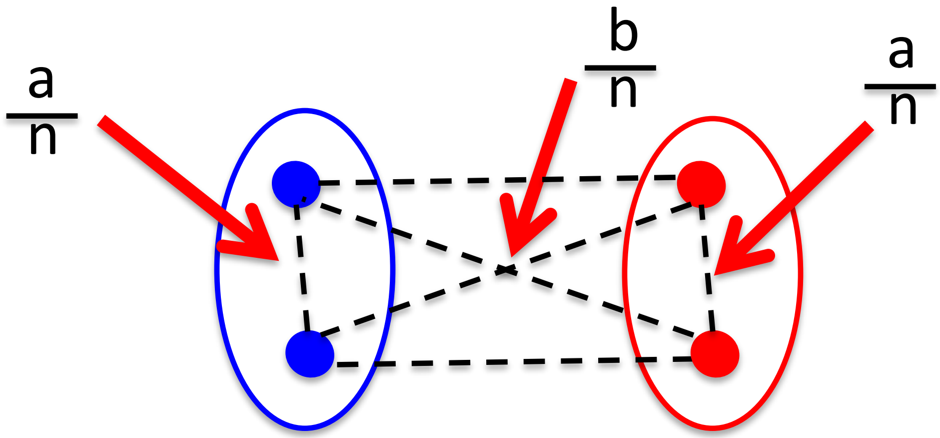

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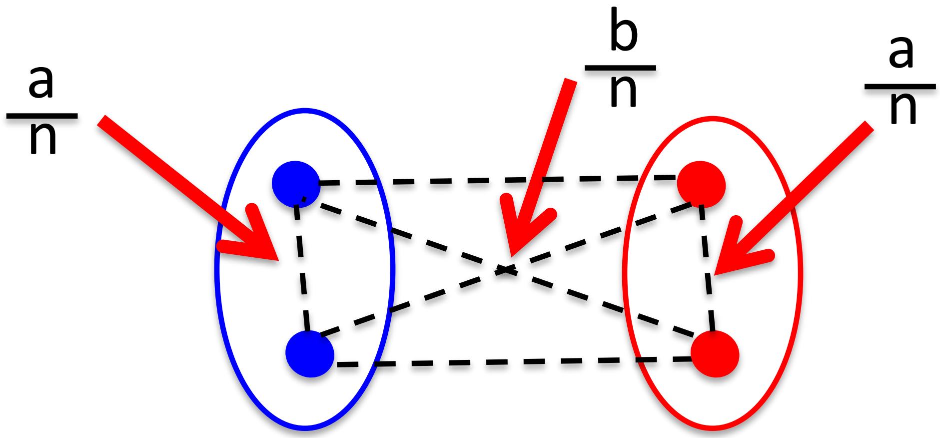

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Conjecture is based on fixed points of **belief propagation**…

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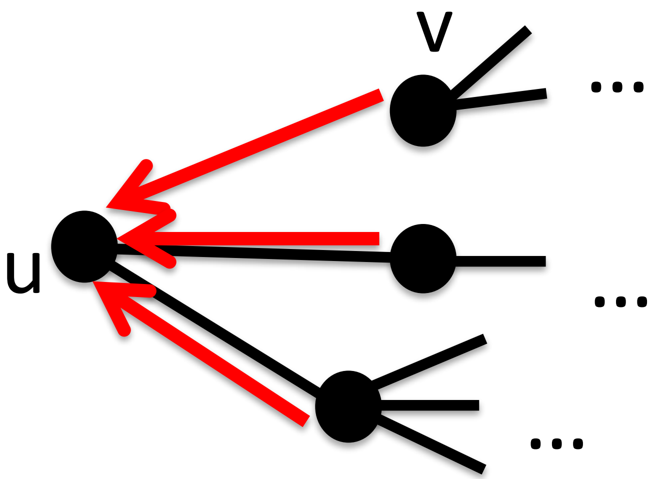
BELIEF PROPAGATION

Introduced by Judea Pearl (1982):

"For fundamental contributions ... to probabilistic and causal reasoning"

Adapted to community detection:



Message v→u

Probability I think
I am community #1,
community #2, ...

Do same for all nodes

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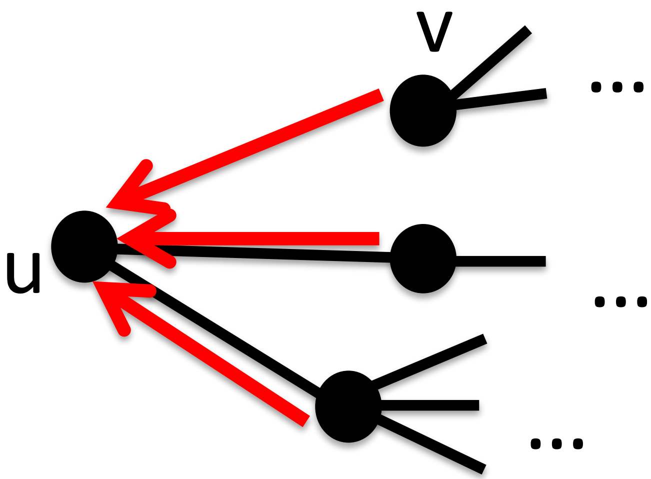

update beliefs

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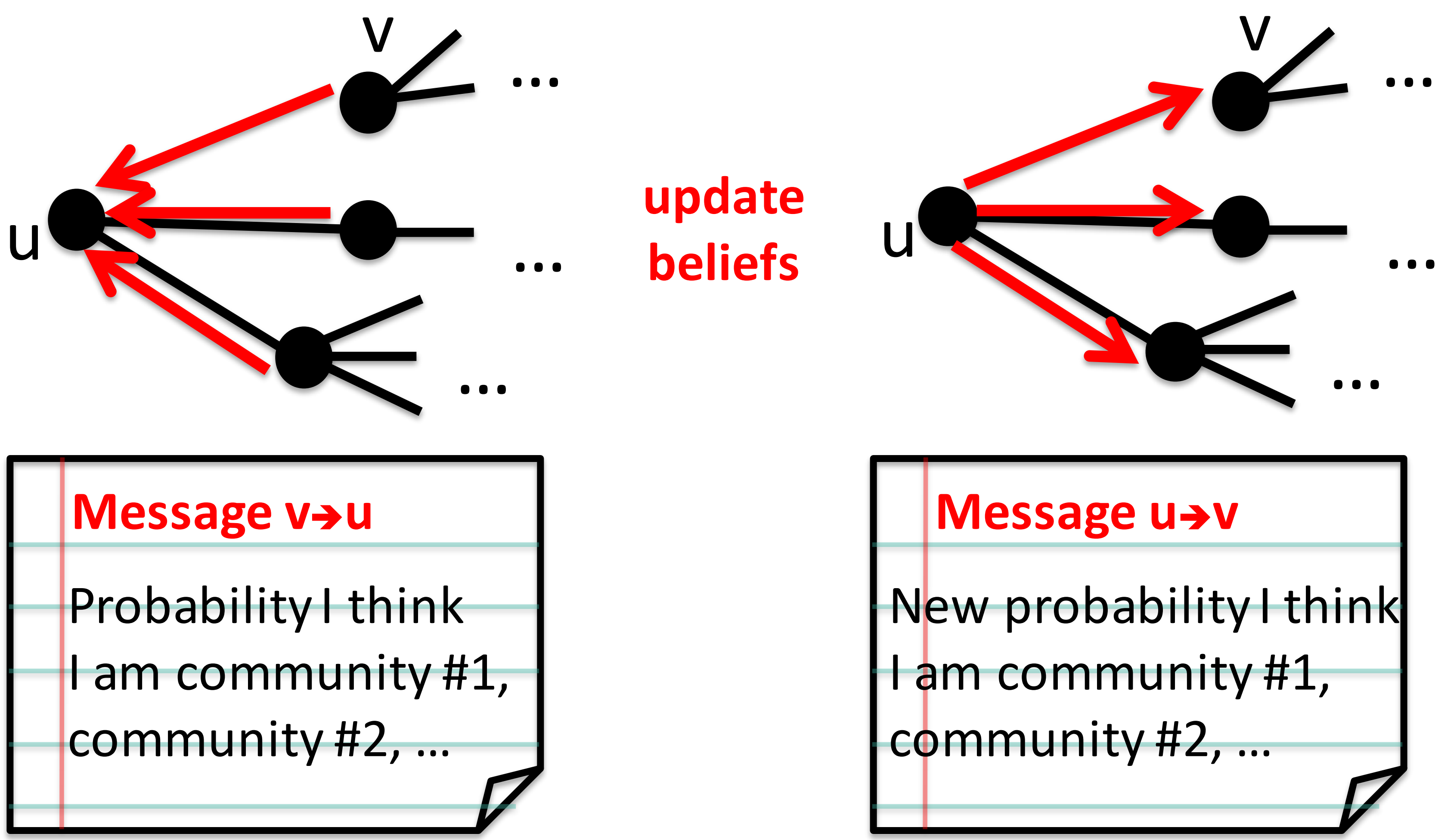

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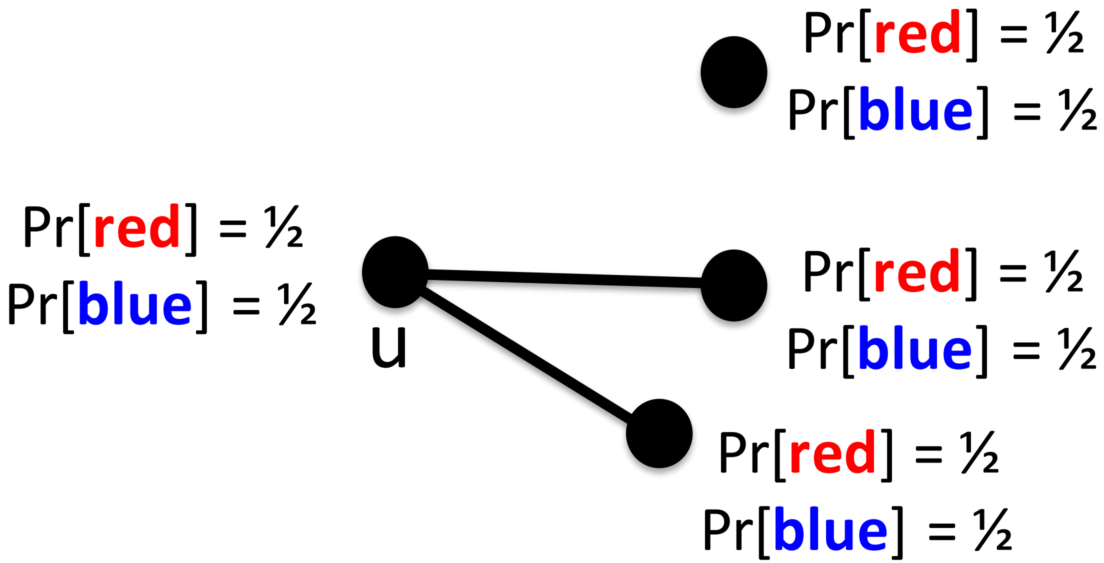

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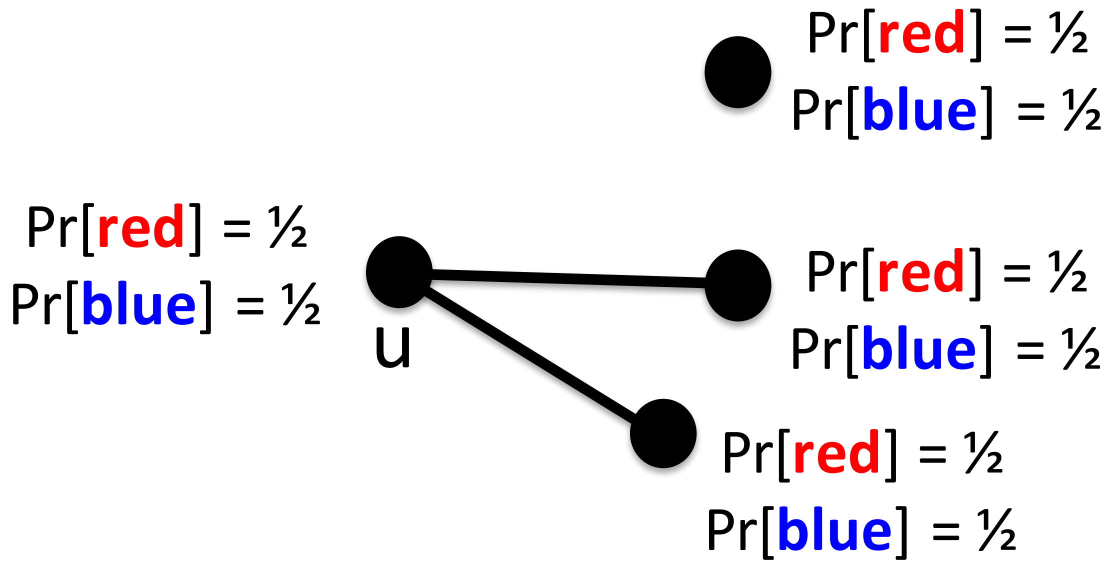

Claim: No one knows anything, **so you never have to update your beliefs**

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And if $(a-b)^2 \leq 2(a+b)$ and it does get stuck, then maybe partial recovery is **information theoretically impossible**?

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Theorem: It is possible to find a partition that is correlated with true communities iff $(a-b)^2 > 2(a+b)$

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Robustness will be a key player in the answers

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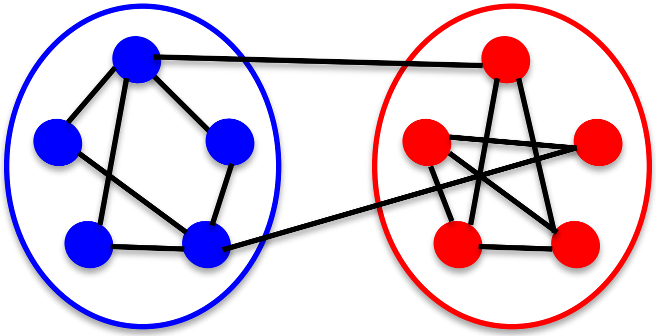

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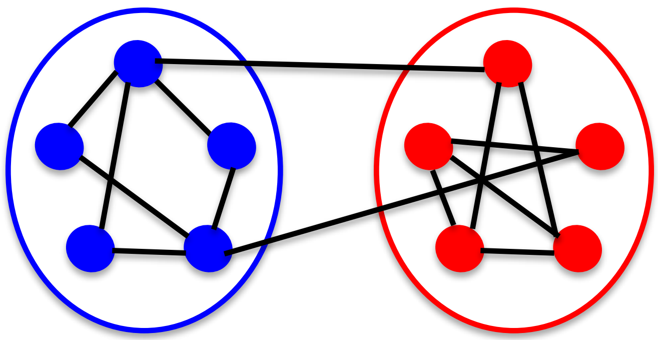
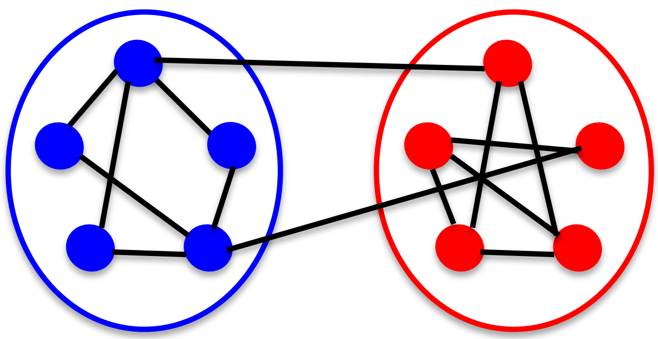


(2) Adversary can add edges within community and delete edges crossing



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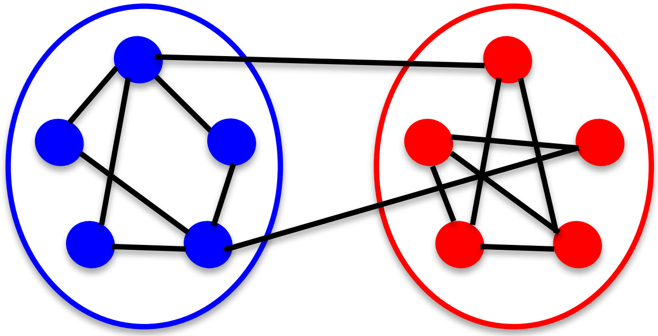

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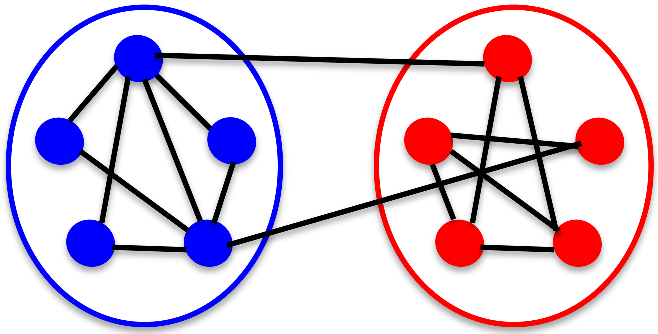

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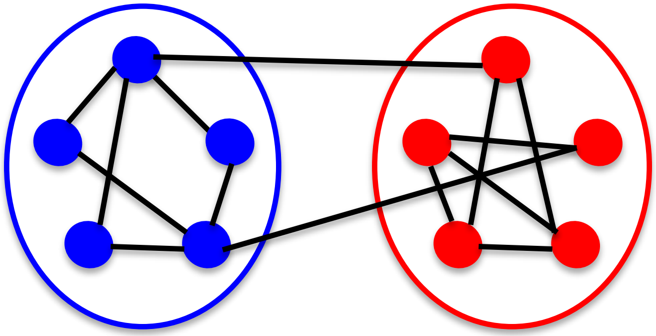

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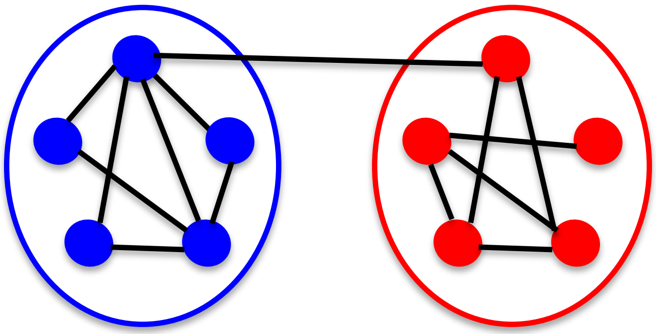

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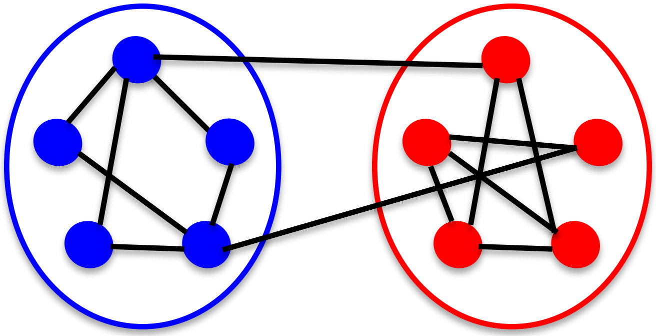

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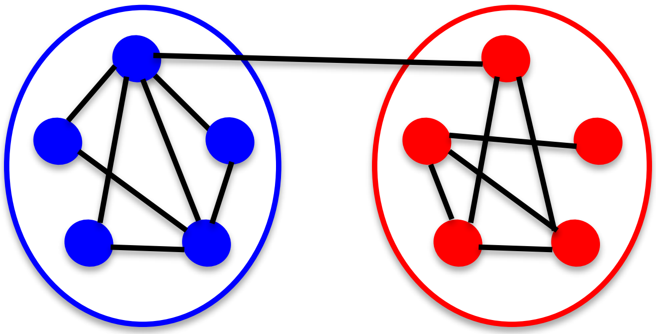

Algorithms can no longer over tune to distribution

A NON-ROBUST ALGORITHM

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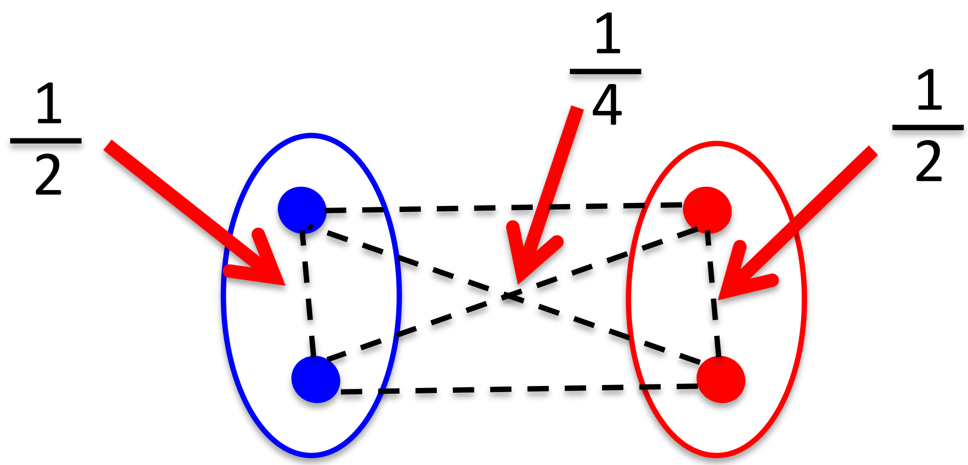

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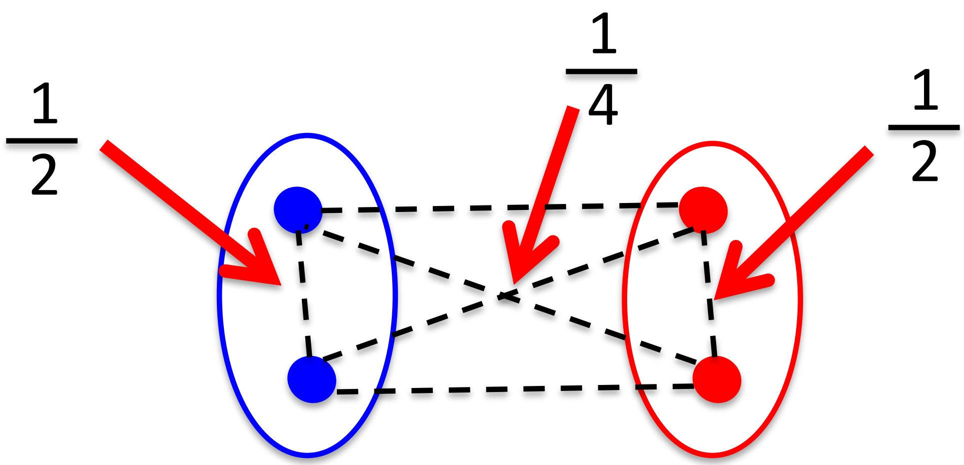

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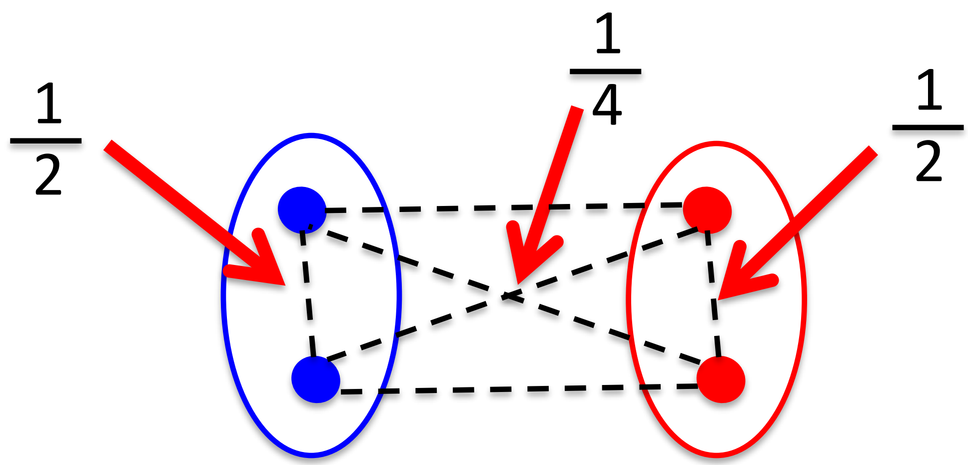

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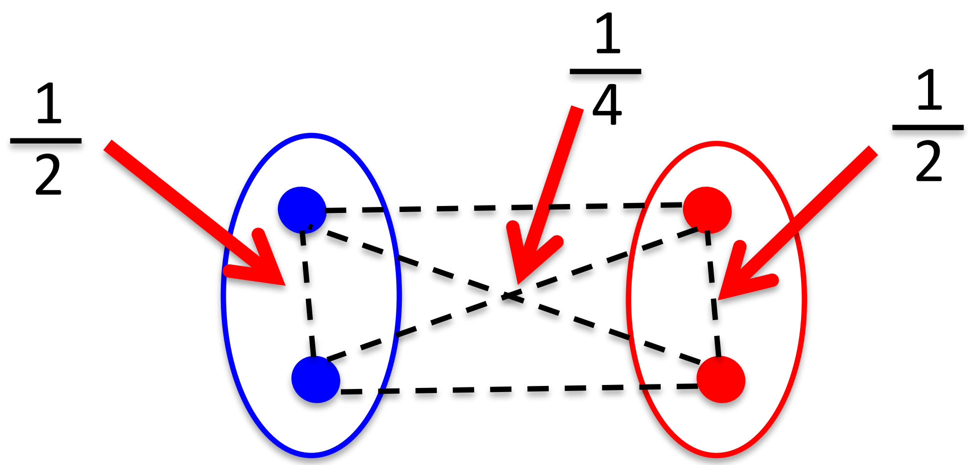

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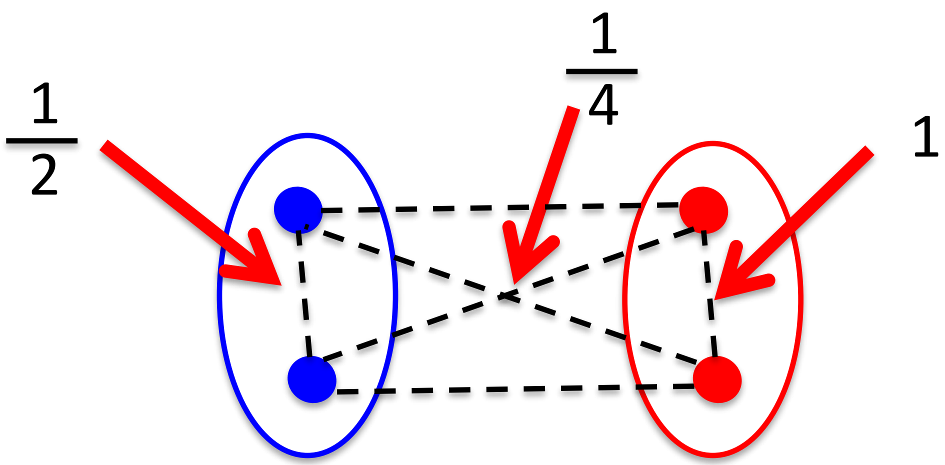

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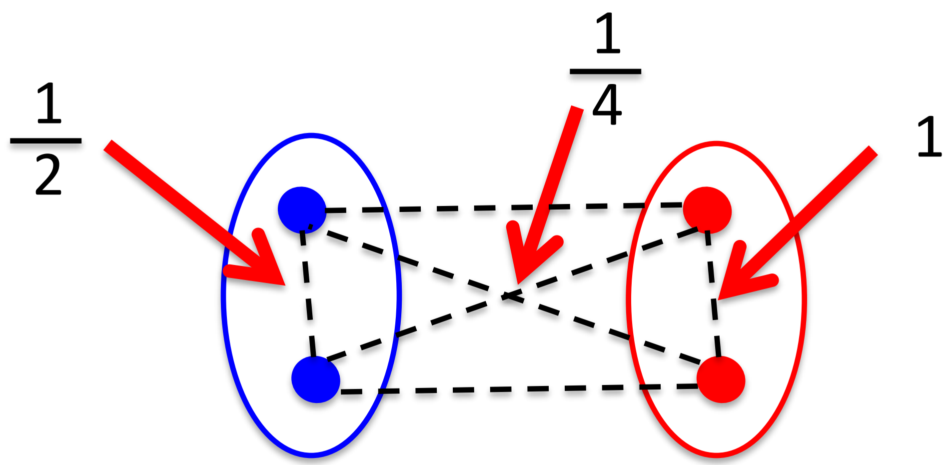

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Nodes from **blue** community: $\left(\frac{1}{2}\right)^2 \frac{n}{2} + \left(\frac{1}{4}\right)^2 \frac{n}{2}$

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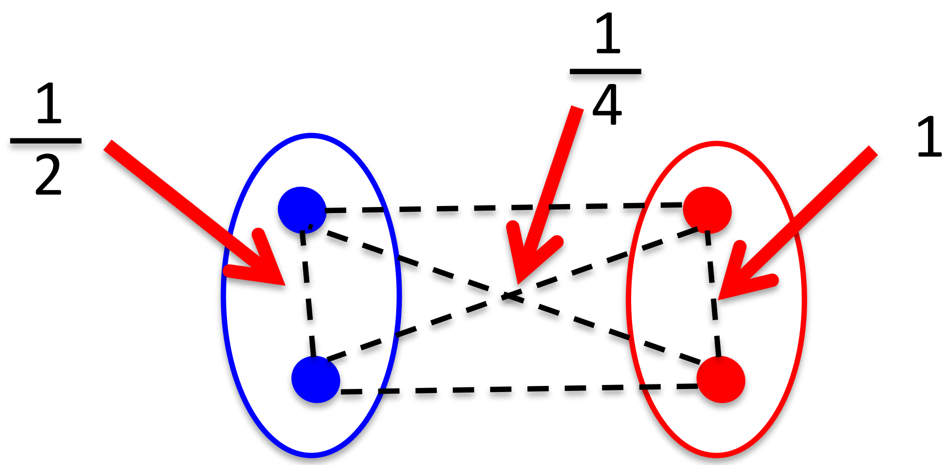

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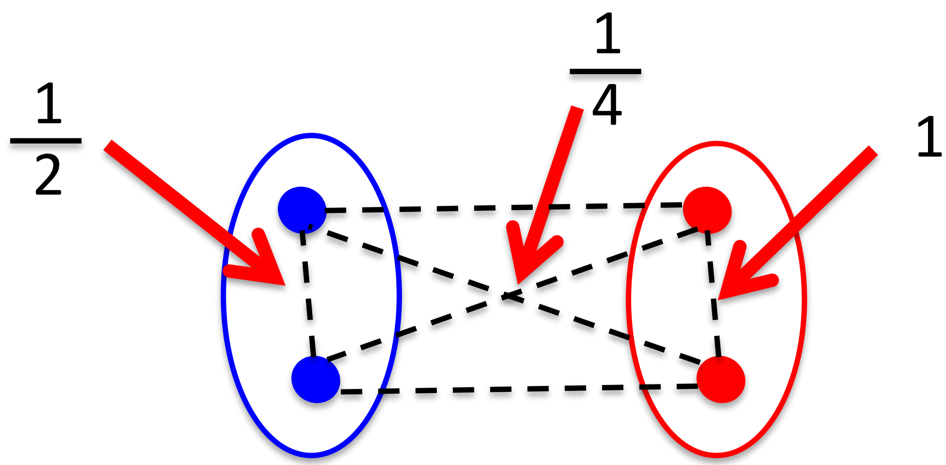

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Theorem [Moitra, Perry, Wein '16]: It is **information theoretically impossible** to recover a partition correlated with true communities for $(a-b)^2 \leq C_{a,b}(a+b)$ for some $C_{a,b} > 2$ in the semirandom model

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Reaching the sharp threshold for community detection requires exploiting the structure of the noise

Let’s explore robustness vs. sharpness in a simpler model

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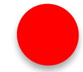

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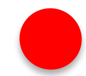

(2) Each node gives birth to **Poi(a/2)** nodes of same color and **Poi(b/2)** nodes of opposite color

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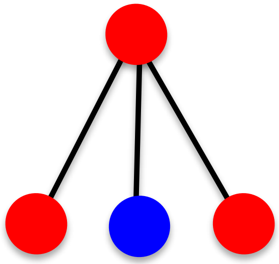

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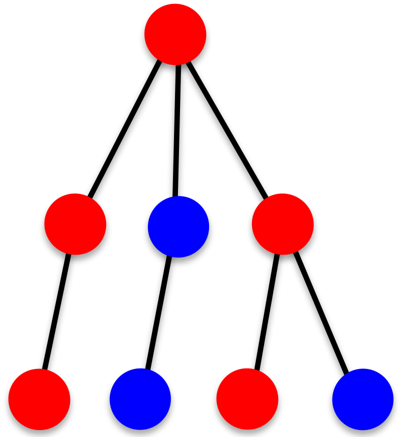

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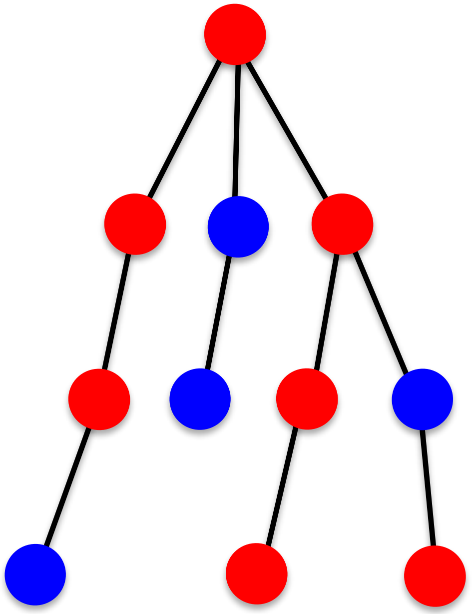

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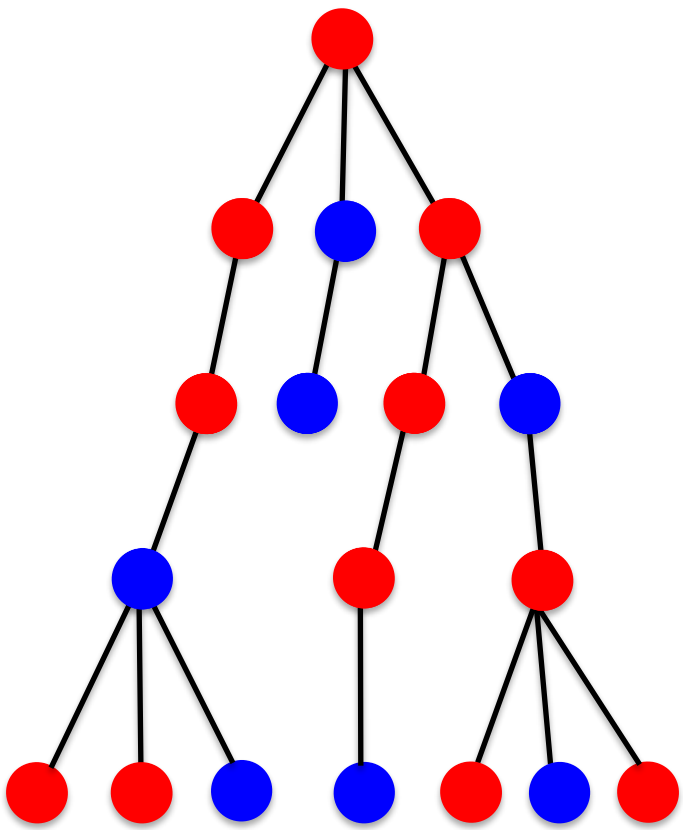

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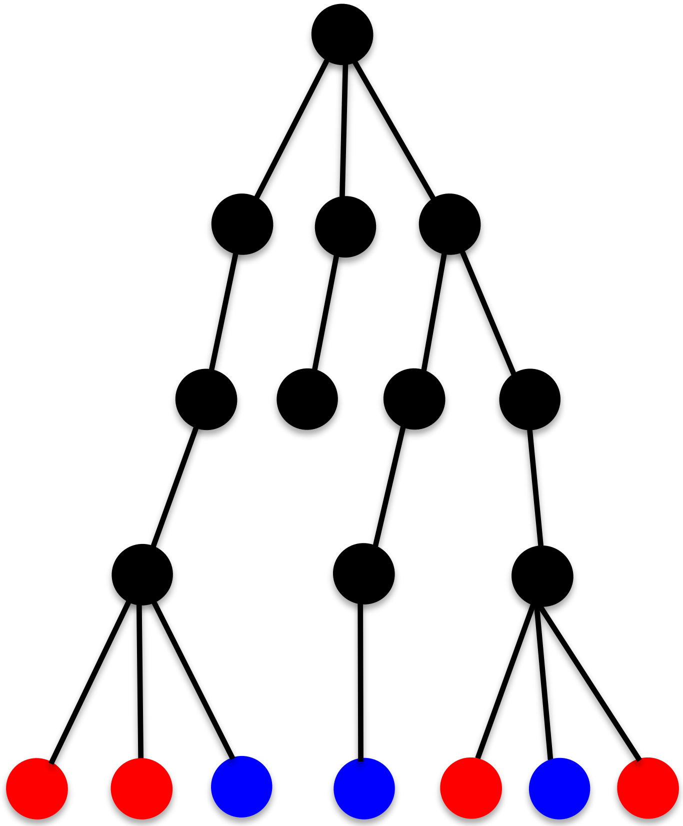

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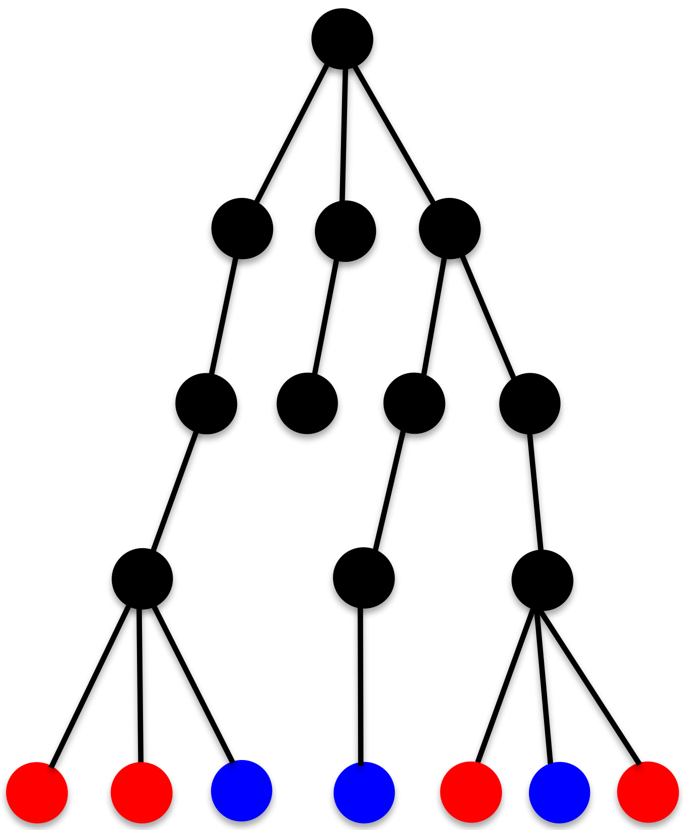

This is the natural analogue for partial recovery

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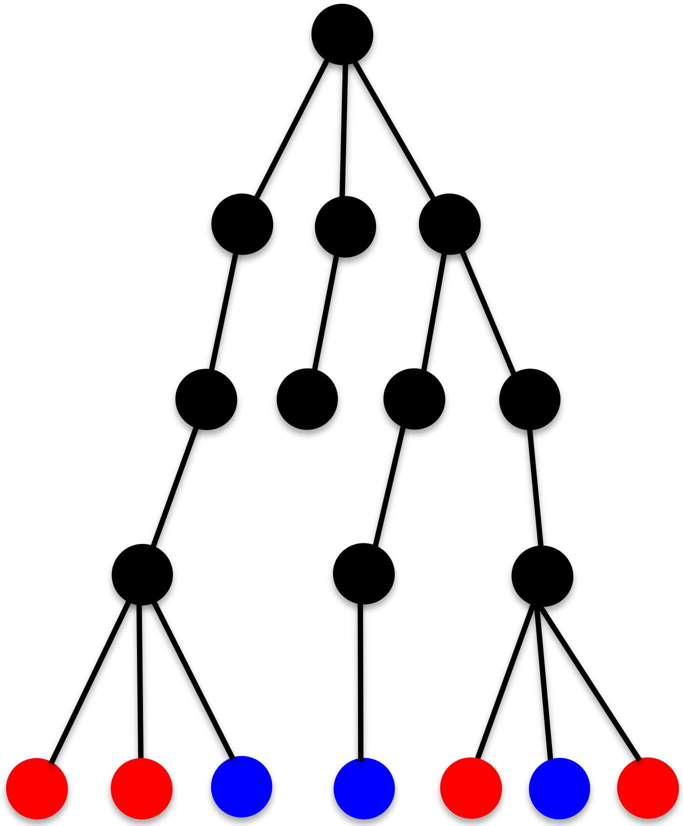

For what values of a and b can we guess the root?

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Local view in SBM = Broadcast Tree

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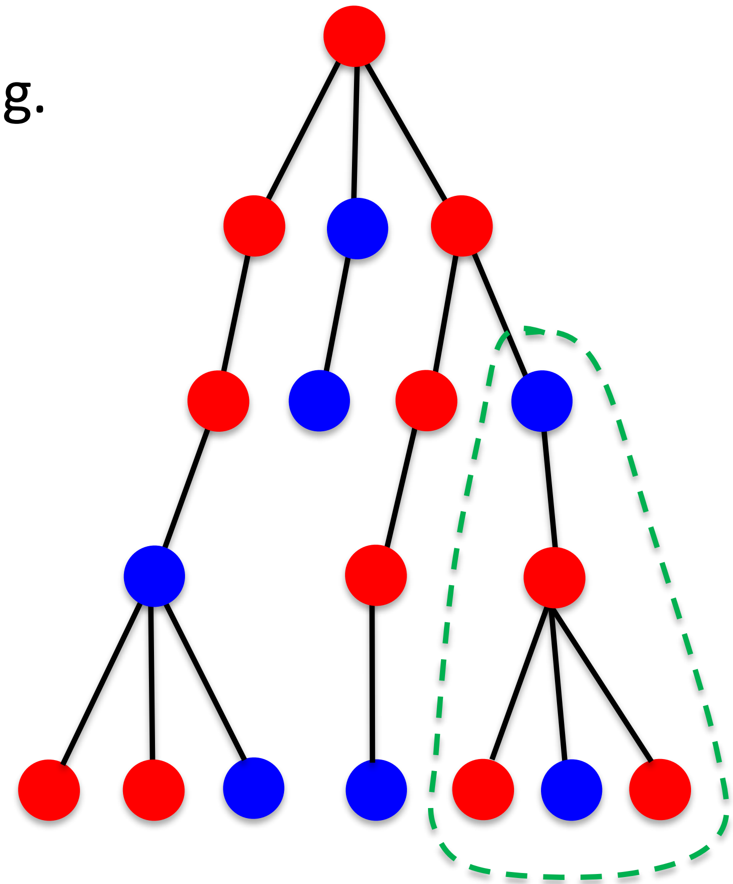

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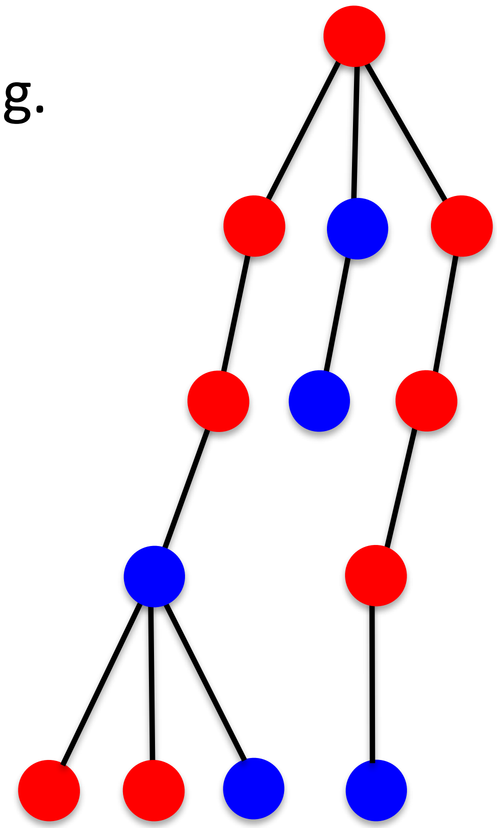

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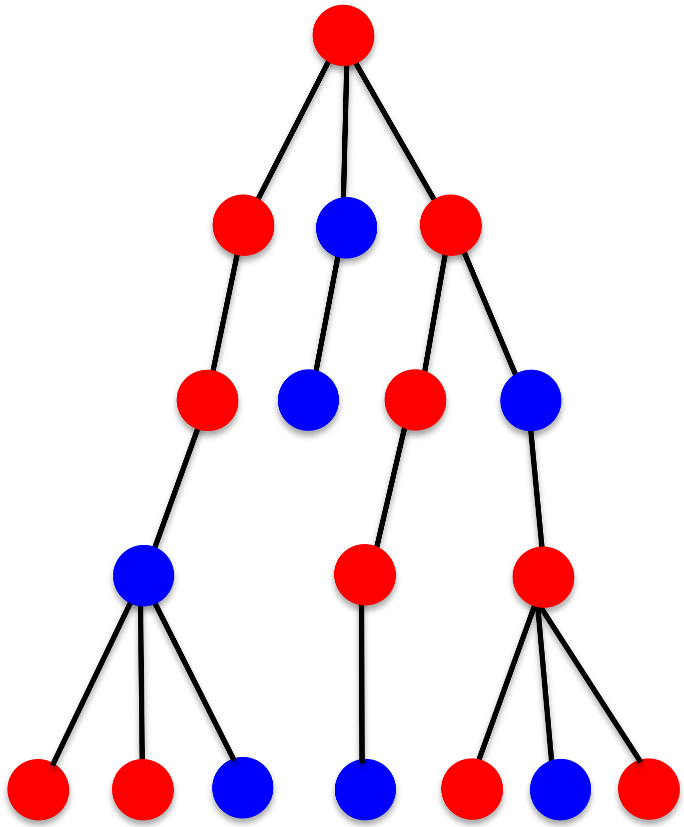
Can the adversary usually flip the majority vote?

Key Observation: Some node's descendants vote **opposite** way



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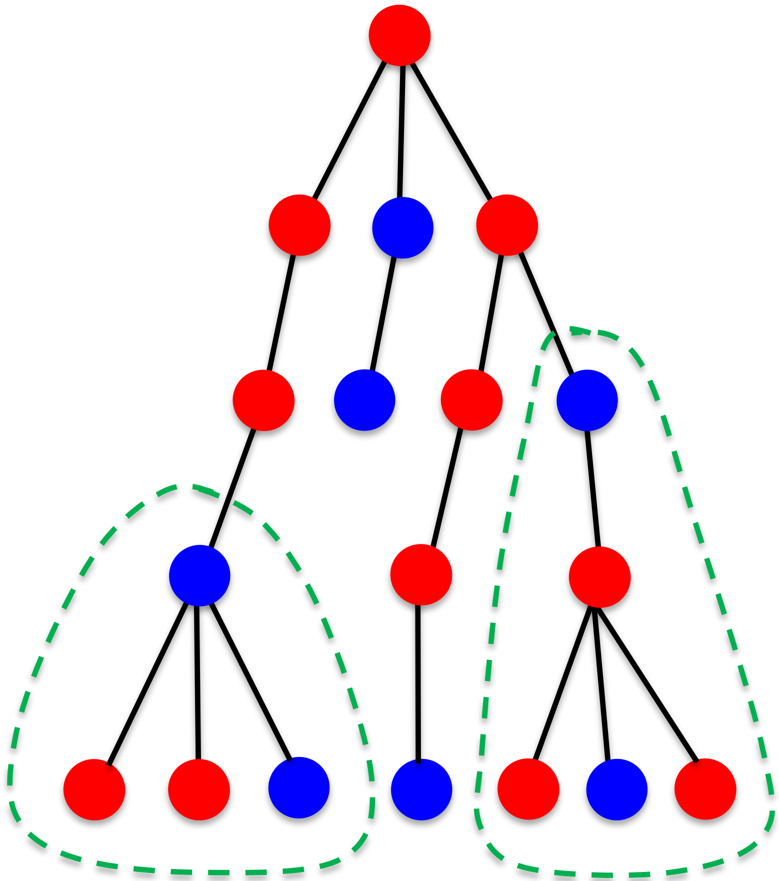

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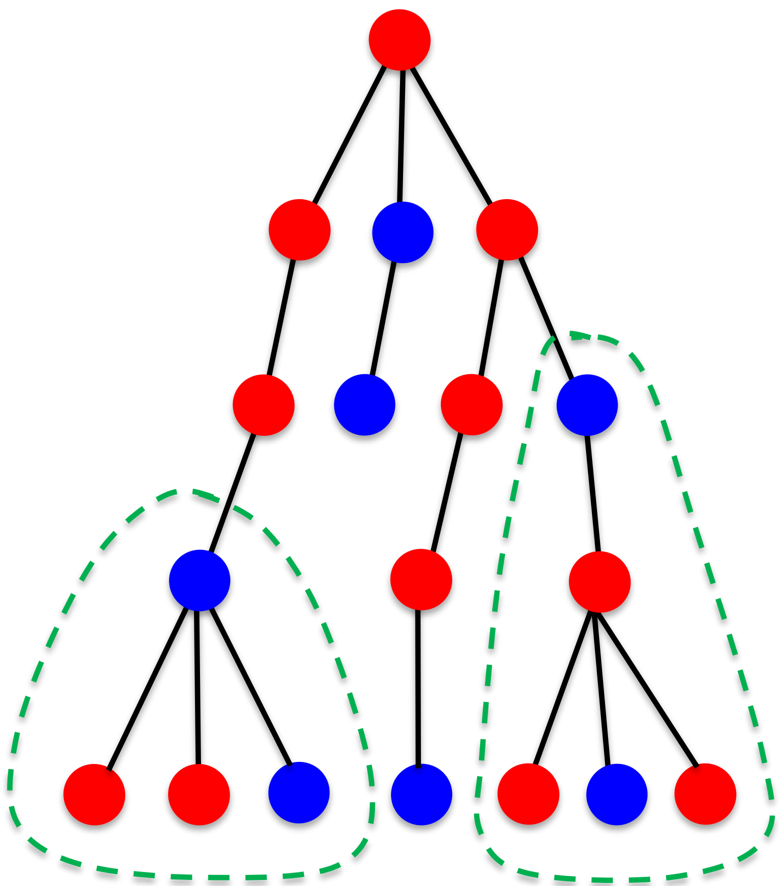

Near the Kesten-Stigum bound, this happens **everywhere**

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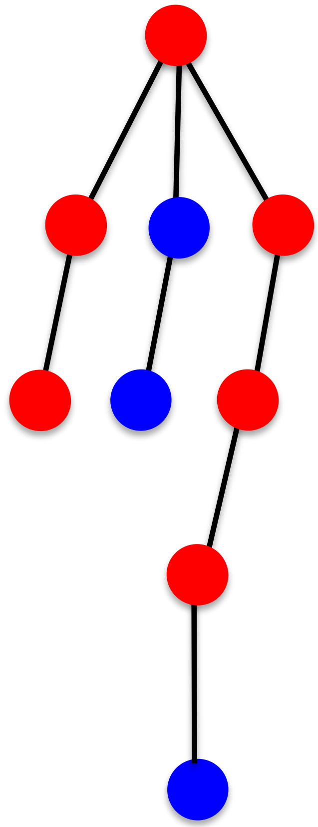

By cutting these edges, adversary can usually flip majority vote

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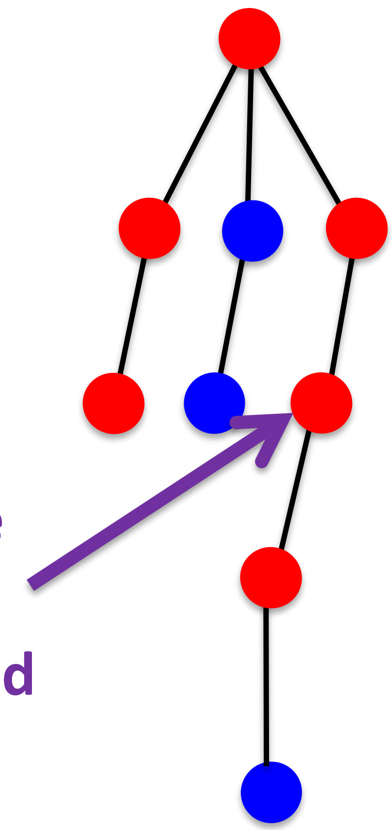
e.g. If we cut every subtree where this happens, would mess up independence properties



More likely to have red children, given his parent is red and he was not cut

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Need to design adversary that puts us back into *nice* model

e.g. a model on a tree where a sharp threshold is known

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Following **[Mossel, Neeman, Sly]** we can embed the lower bound for semi-random BTM in semi-random SBM

e.g. Usual complication: once I reveal colors at boundary of neighborhood, need to show there's little information you can get from rest of graph

SHARPNESS VS. ROBUSTNESS, CONTINUED

"Helpful" changes can hurt:

Theorem [Moitra, Perry, Wein '16]: Reconstruction in semi-random broadcast tree model is information theoretically impossible for $(a-b)^2 \leq C_{a,b}(a+b)$ for some $C_{a,b} > 2$

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Theorem [Moitra, Perry, Wein '16]: **Recursive majority** succeeds in semi-random broadcasttree model if

$$(a-b)^2 > (2 + o(1))(a+b) \log \frac{a+b}{2}$$

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This is an axis on which recursive majority is superior

BETWEEN WORST-CASE AND AVERAGE-CASE

Spielman and Teng (2001):

"Explain why algorithms work well in practice, despite bad worst-case behavior"

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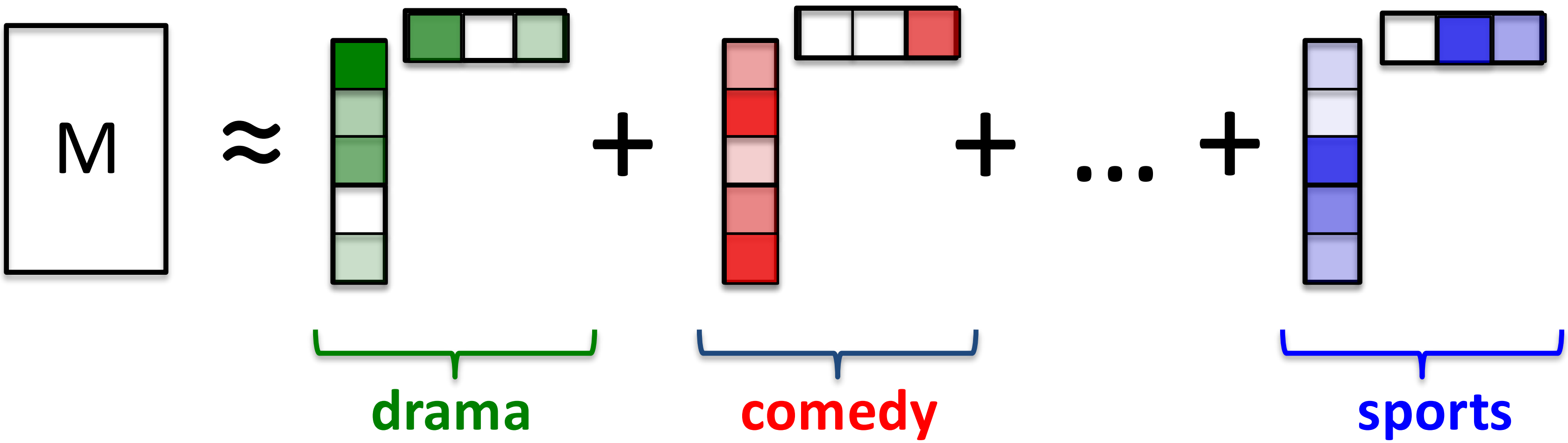
What else are we missing, if we only study problems in the average-case?

THE NETFLIX PROBLEM

Let M be an unknown, low-rank matrix



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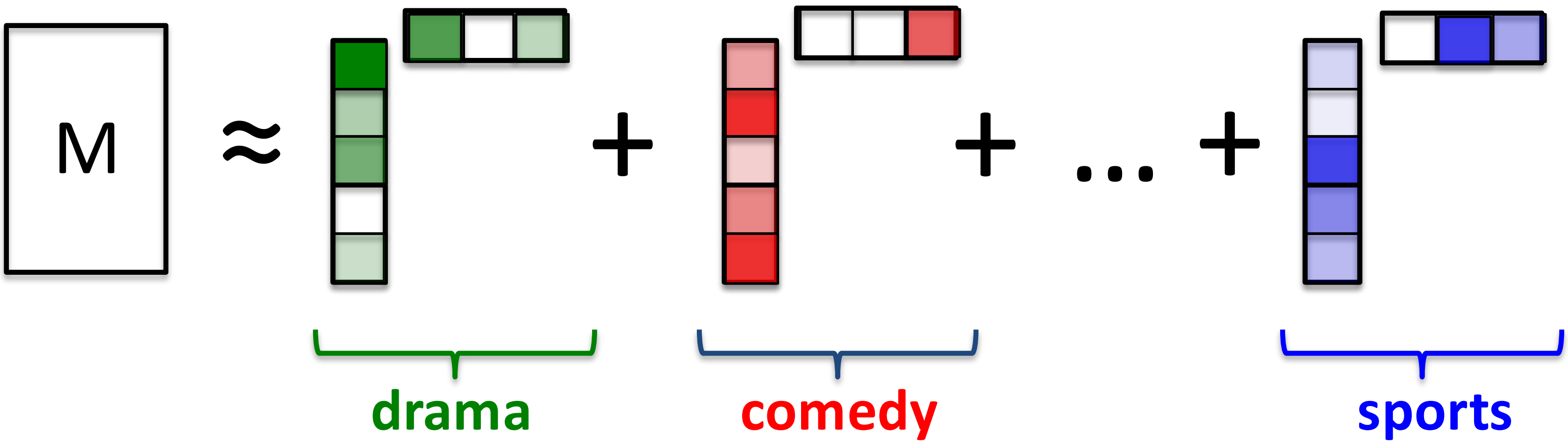

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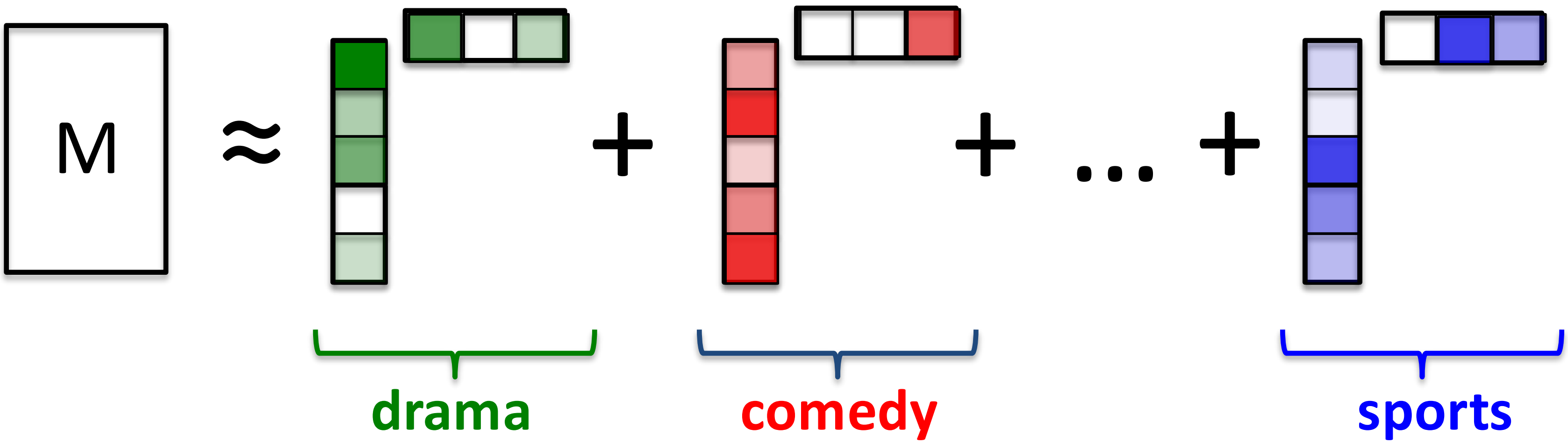

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Is there an efficient algorithm to recover M?

CONVEX PROGRAMMING APPROACH

$$\min \|X\|_* \text{ s.t. } \sum_{(i,j)\in\Omega} |X_{i,j} - M_{i,j}| \leq \eta \qquad \textbf{(P)}$$

Here $\|X\|_*$ is the **nuclear norm**, i.e. sum of the singular values of X

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Theorem: If M is n x n and has rank r, and is C-incoherent then **(P)** recovers M exactly from $C^6 nr\log^2 n$ observations

ALTERNATING MINIMIZATION

Repeat: $U \leftarrow \operatorname{argmin}_U \sum_{(i,j)\in\Omega} |(UV^T)_{i,j} - M_{i,j}|^2$

$$V \leftarrow \operatorname{argmin}_V \sum_{(i,j)\in\Omega} |(UV^T)_{i,j} - M_{i,j}|^2$$

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Running time and space complexity are better

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Convex program:

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still works, it's just more constraints

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Alternating minimization:

Analysis completely breaks down

observed matrix is no longer good spectral approx. to M

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Alternating minimization:

Are there variants that work in semi-random models?

LOOKING FORWARD

Are there nonconvex methods that match the robustness guarantees of convex relaxations?

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Thanks! Any Questions?