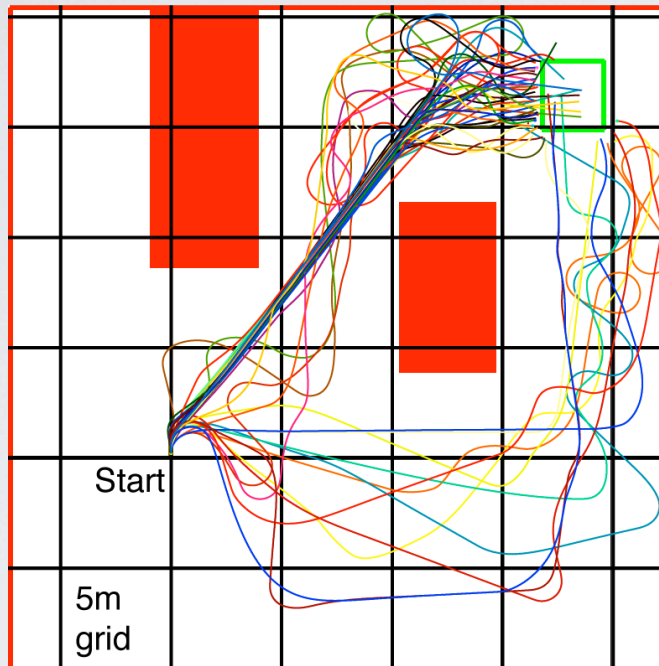
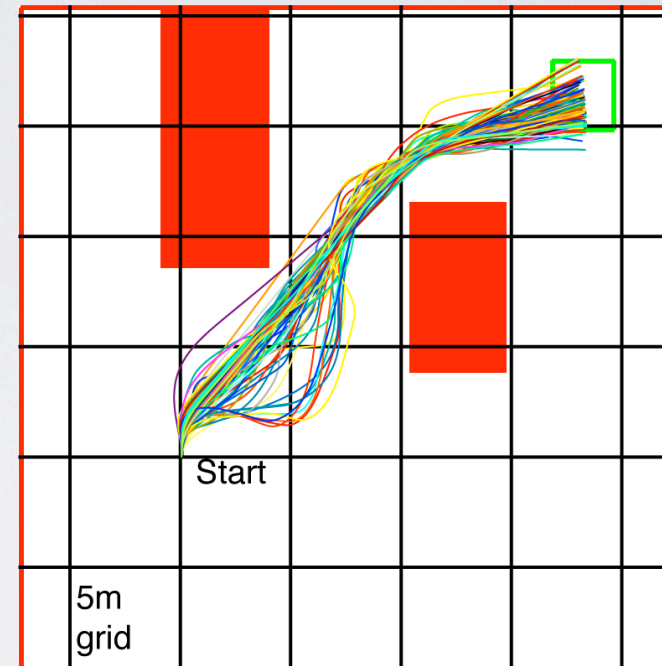


ANYTIME MOTION PLANNING USING THE RRT*



Anytime RRT



Anytime RRT*

Sertac Karaman¹, Matthew Walter², Alejandro Perez²,
Emilio Frazzoli¹, & Seth Teller²

¹MIT / LIDS

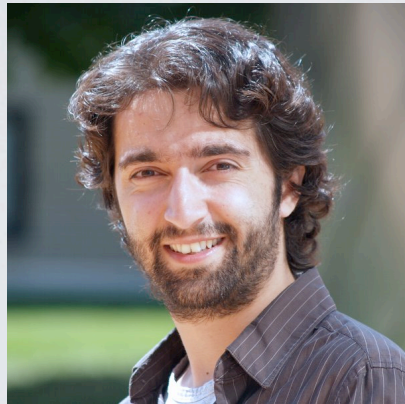
²MIT / CSAIL



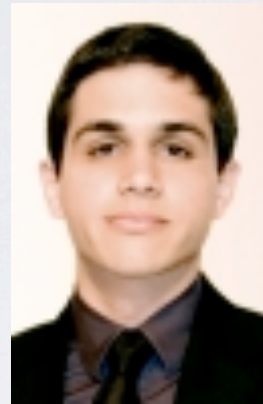
10 May 2011



JOINT WORK WITH



Sertac Karaman
(MIT/LIDS)



Alejandro Perez
(MIT/CSAIL)



Emilio Frazzoli
(MIT/LIDS)



Seth Teller
(MIT/CSAIL)

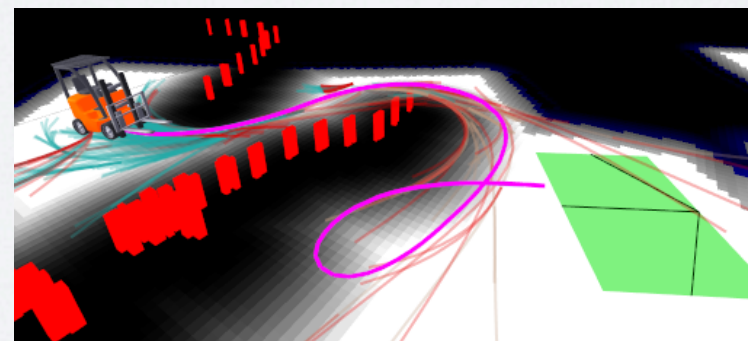


PRACTICAL MOTION PLANNING

- (Probabilistic) completeness
- Quickly find a kinodynamically feasible solution
- Computationally efficiency (limited resources)
- Plan despite incomplete, imperfect knowledge
- Accommodate dynamic environments



[Kuwata et al., GNC 2008]

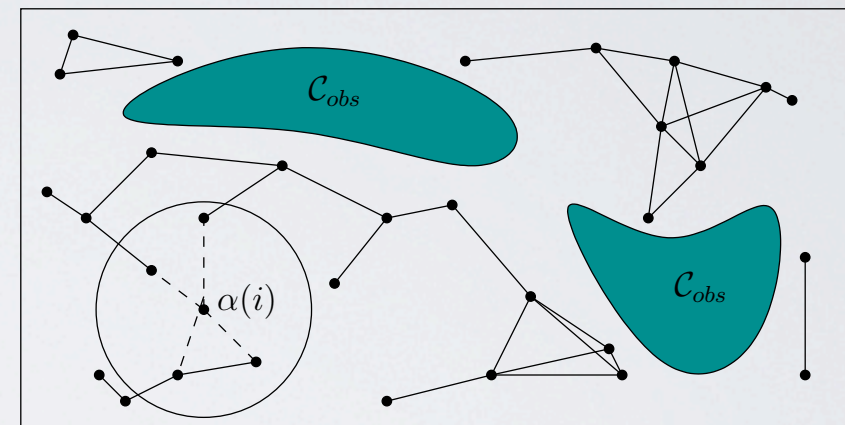


[Teller et al., ICRA 2010]

SAMPLE-BASED MOTION PLANNING

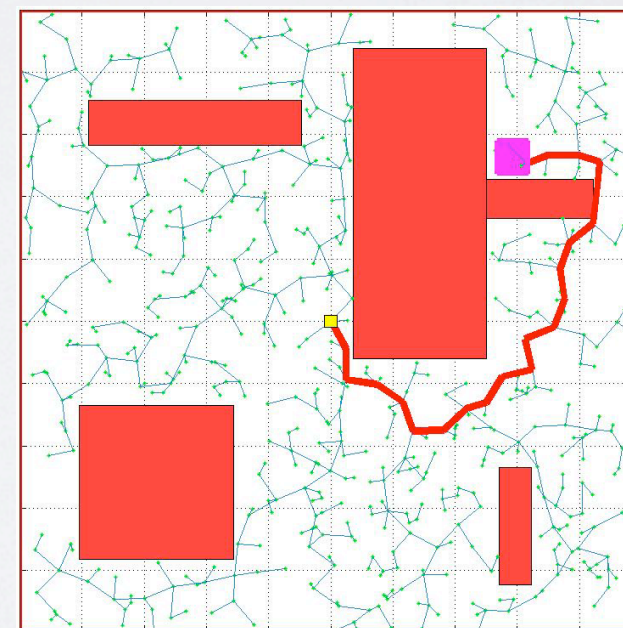
Sample-based motion planning provides an effective solution

- Probabilistic RoadMap (PRM)
[Kavraki et al., T-RA 1996]
 - Multiple query



[Credit: LaValle, Planning Algorithms 2006]

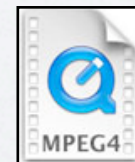
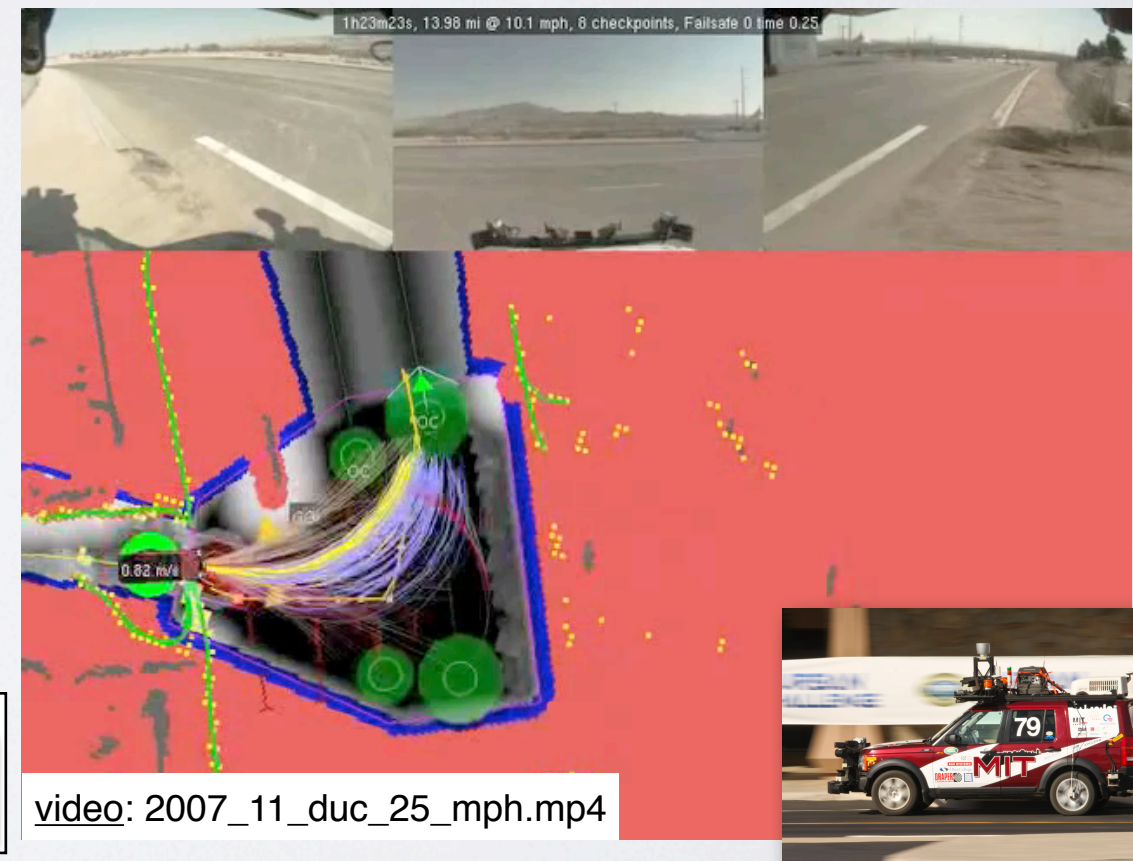
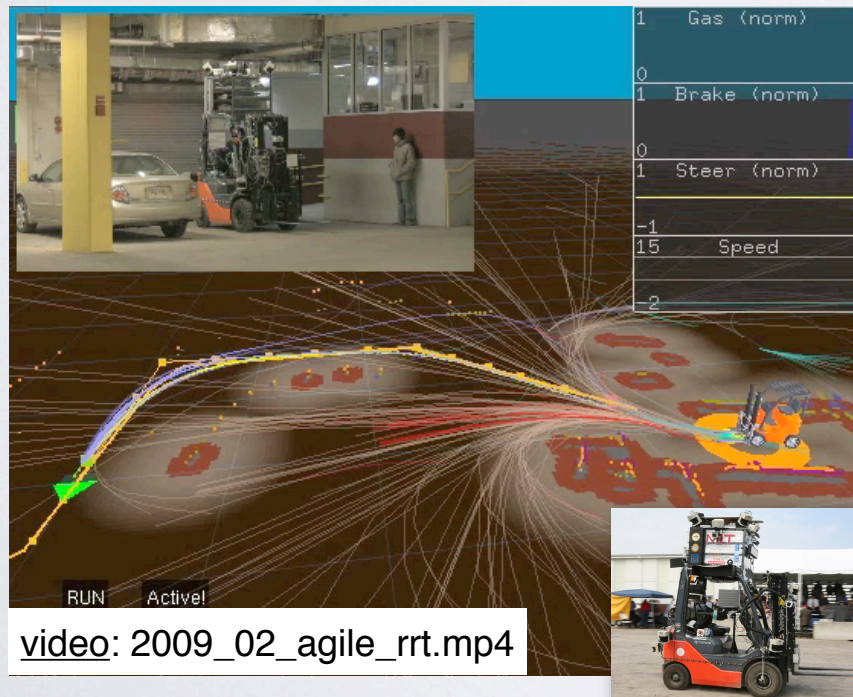
- Rapidly-exploring Random Tree (RRT)
[LaValle & Kufner, IJRR 2001]
 - Single query
 - Incremental
 - Online



INCREMENTAL SAMPLE-BASED MOTION PLANNING

- Rapidly-exploring Random Tree (RRT) [LaValle & Kufner, IJRR 2001]
 - Probabilistically complete
 - Respects kinodynamic (non-holonomic) constraints
 - Computationally efficient, scales to high dimensions
 - Relatively simple to implement

Effectively demonstrated on
state-of-the-art robotic platforms



ANYTIME MOTION PLANNING

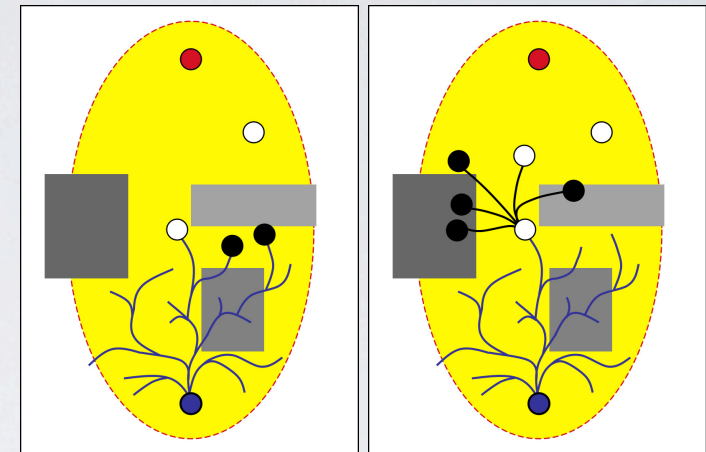
During execution, improve solution toward optimal

- Overall approach:
 1. Quickly find a solution that is feasible, but not necessarily optimal
 2. Exploit execution time to incrementally improve towards optimal solution

- Desired properties:
 1. Form of completeness guarantees
 2. Asymptotic optimality given more computation time

ANYTIME MOTION PLANNING

- Anytime RRTs [Ferguson & Stentz, IROS 2006]
 - Quickly finds an initial solution with a vanilla RRT
 - Successively generates new trees that improve solution costs via biased sampling
 - The cost of successive solutions is guaranteed to decrease, though they **do not converge to the optimum**
- CL-RRT [Kuwata et al., T-CST 2009]
 - Quickly finds an initial solution with a closed-loop RRT
 - Continues to search for other solutions (during execution)
 - Estimates an upper-bound on the cost of each solution via a cost-to-go heuristic
 - Chooses the solution with the lowest upper-bound cost
 - **No convergence guarantees**



[Credit: Ferguson & Stentz, IROS 2006]

ANYTIME MOTION PLANNING

The RRT is not asymptotically optimal

- Y_n^{RRT} denotes the cost of the best path in the RRT after n iterations
- c^* denotes the cost of an optimal path

Theorem [Karaman, Frazzoli, IJRR 2011]

The probability that the RRT converges to an optimum solution is zero

$$\mathbb{P}\left(\left\{\lim_{n \rightarrow \infty} Y_n^{\text{RRT}} = c^*\right\}\right) = 0$$

ANYTIME RRT*

Our approach: Leverage the RRT* to converge to optimal

- RRT* [Karaman & Frazzoli, RSS 2010] is both **asymptotically optimal** and **computationally efficient**
 - $Y_n^{\text{RRT}^*}$: cost of the best path in the RRT*
 - c^* : cost of an optimal solution
 - M_n^{RRT} : number of steps executed by the RRT at iteration n
 - $M_n^{\text{RRT}^*}$: number of steps executed by the RRT* in iteration n

Theorem [Karaman & Frazzoli, IJRR 2011]

(i) The RRT* algorithm is asymptotically optimal

$$\mathbb{P}\left(\left\{\lim_{n \rightarrow \infty} Y_n^{\text{RRT}^*} = c^*\right\}\right) = 1$$

(ii) RRT* algorithm has no substantial computational overhead when compared to the RRT:

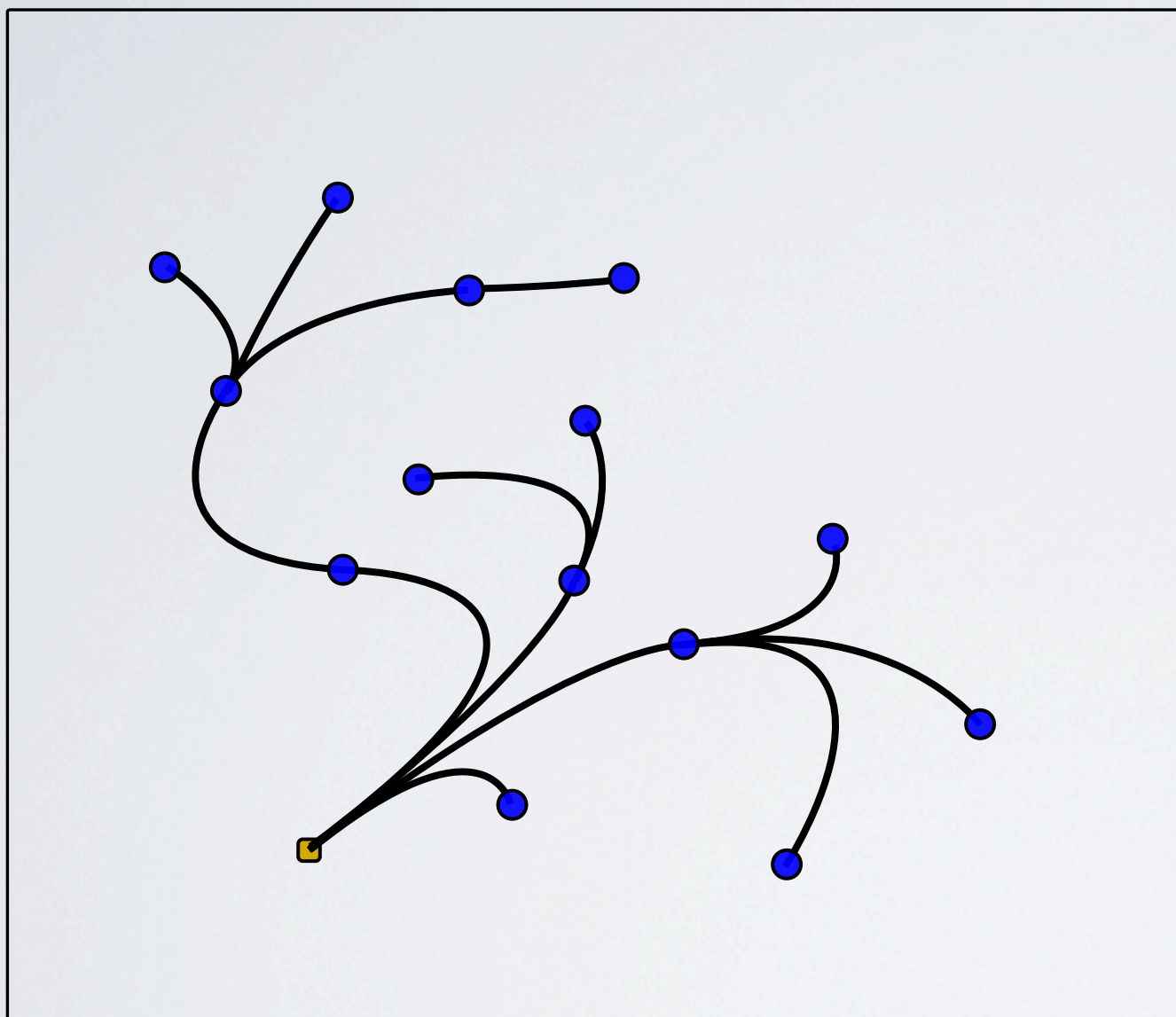
$$\lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{M_n^{\text{RRT}^*}}{M_n^{\text{RRT}}} \right] = \text{constant}$$

ANYTIME RRT*

Our approach: Leverage the RRT* to converge to optimal

- Closed-loop formulation of the RRT*
 - Samples from the space of control inputs utilizing a prediction model to grow the tree
- Quickly find a feasible, possibly sub-optimal solution
- Exploit available computation time during execution to rewire the tree
- Introduce heuristics for online implementation and efficiency
 - Committed trajectory
 - Branch-and-bound

THE RRT*



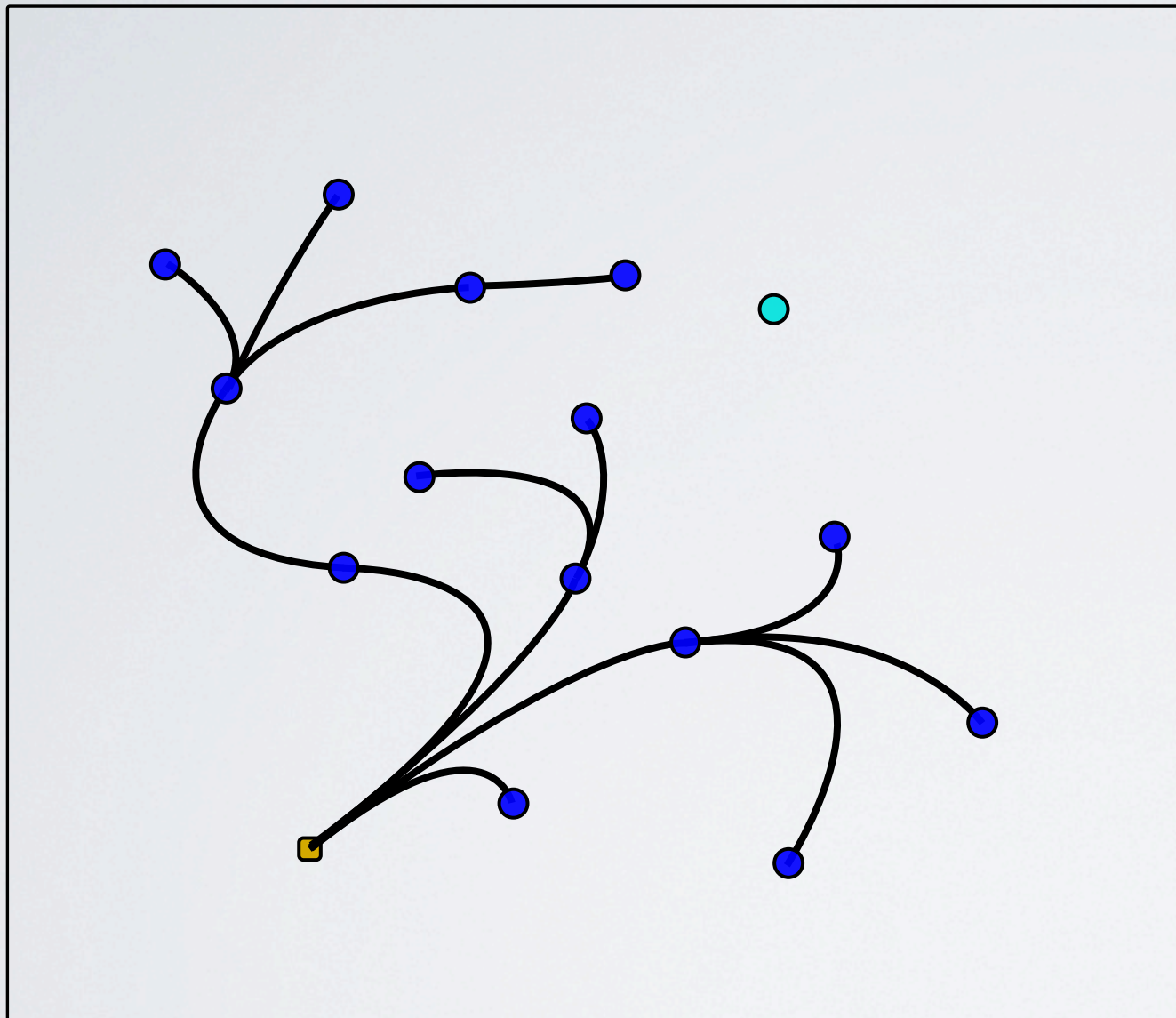
```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

So far, a standard RRT

THE RRT*



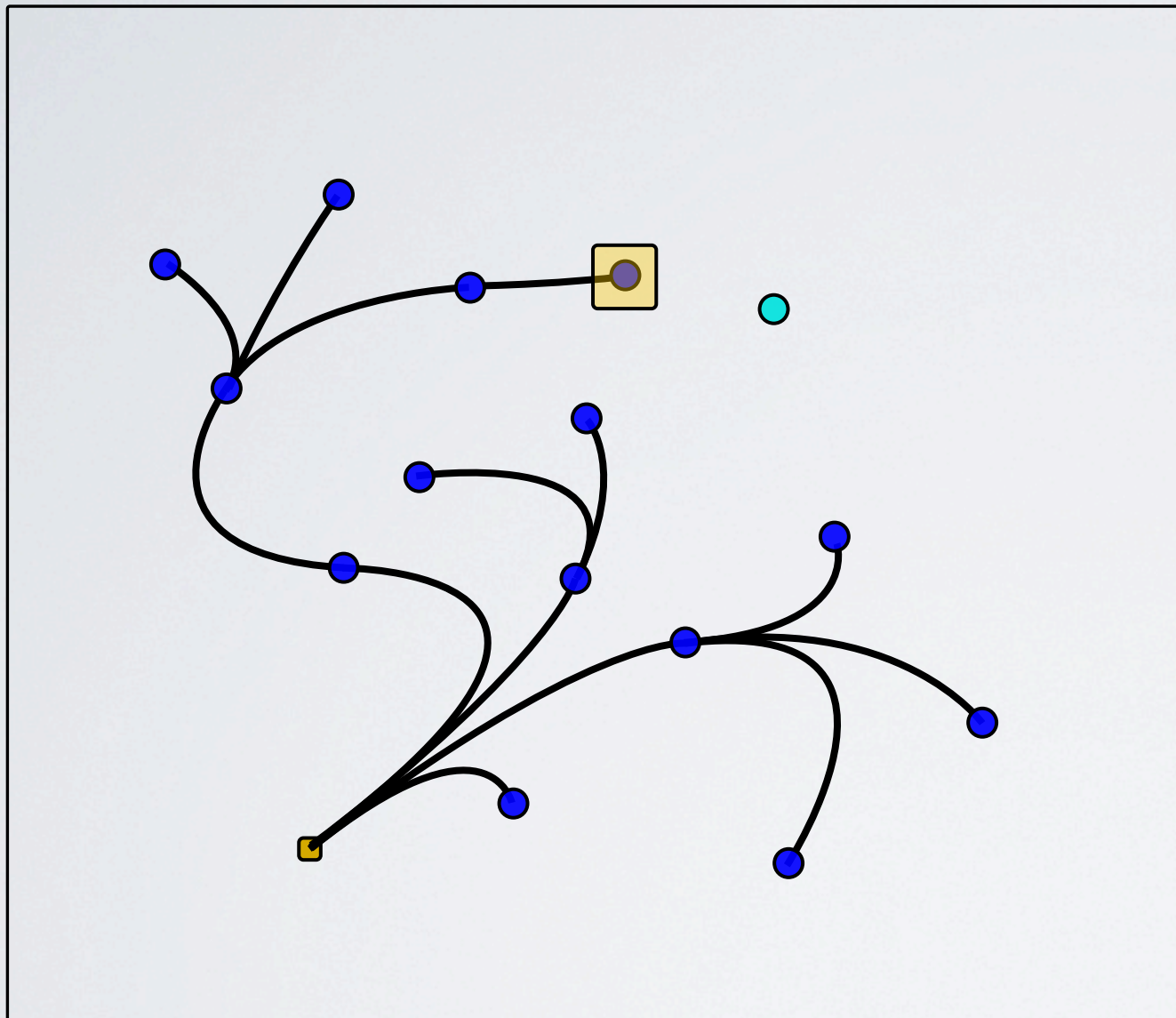
```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

So far, a standard RRT

THE RRT*



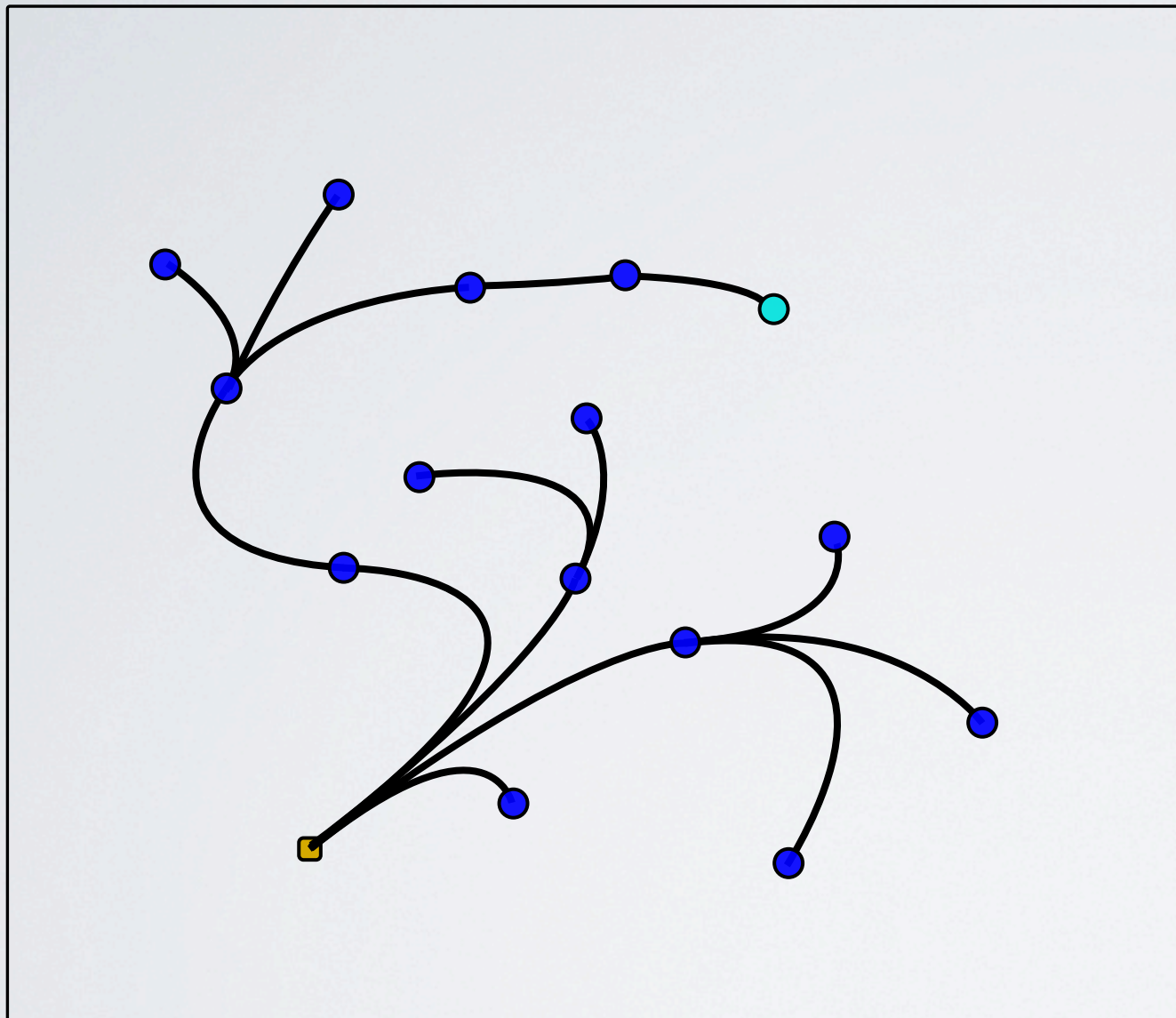
```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

So far, a standard RRT

THE RRT*



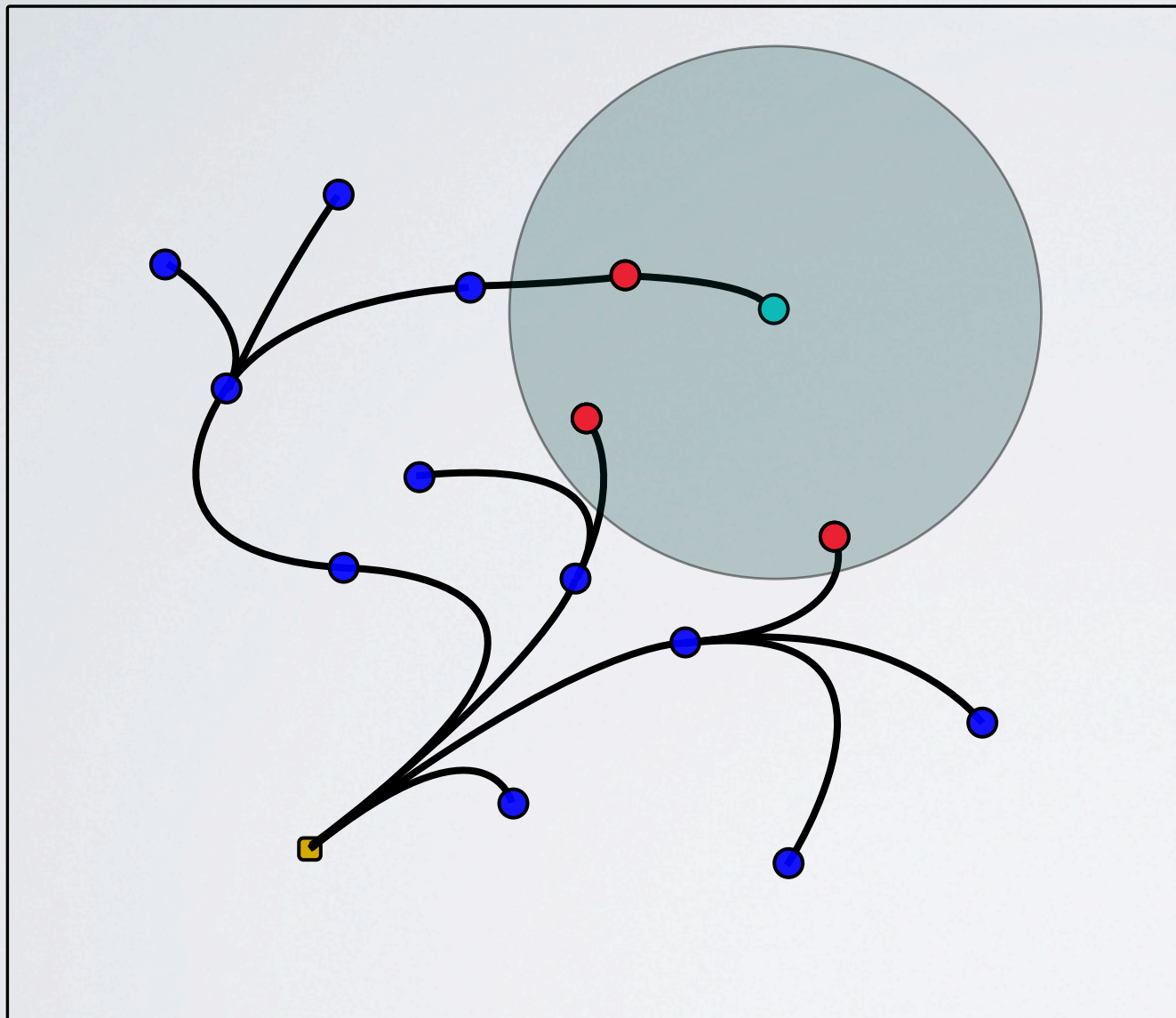
```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

So far, a standard RRT

THE RRT*

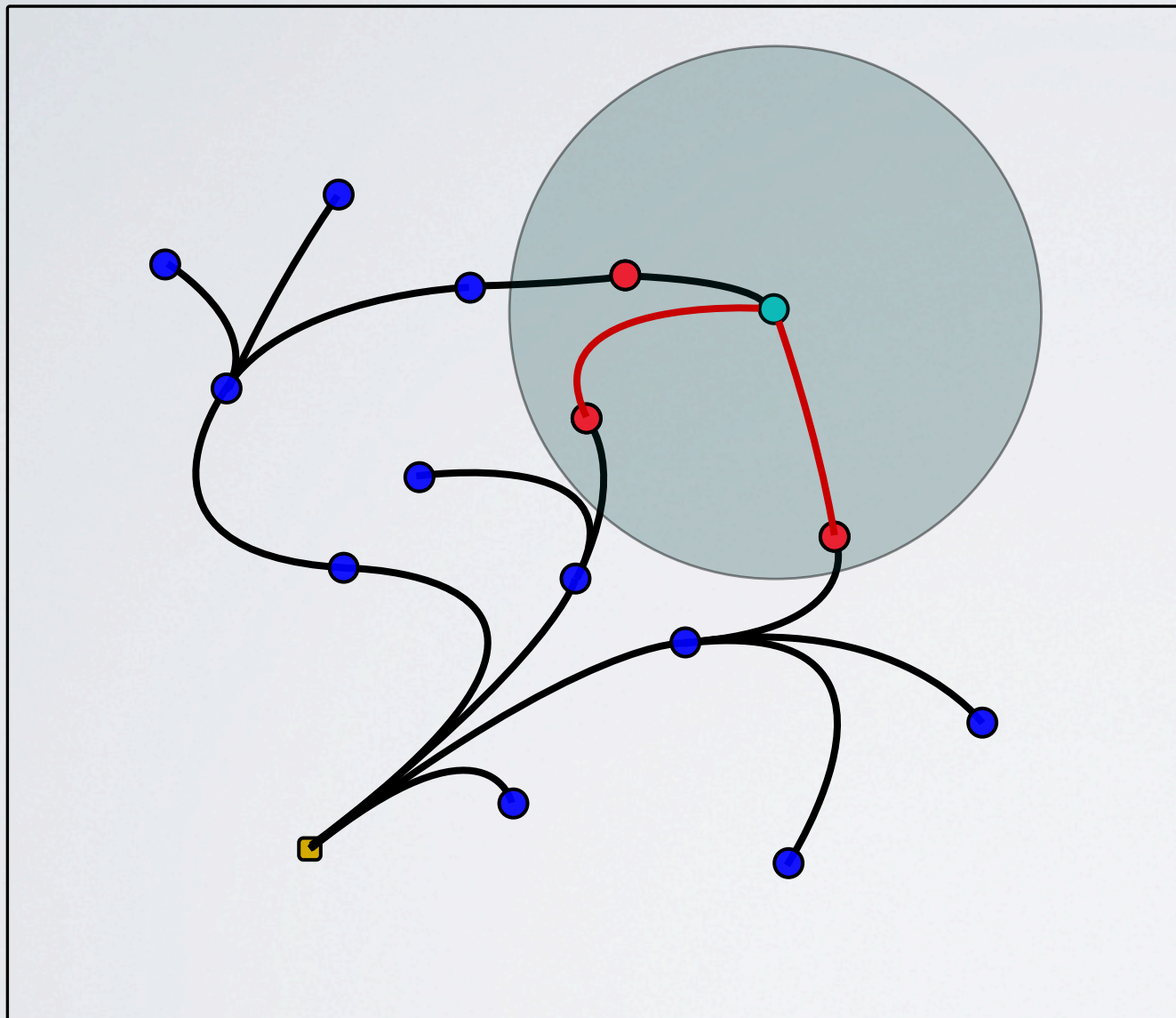


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

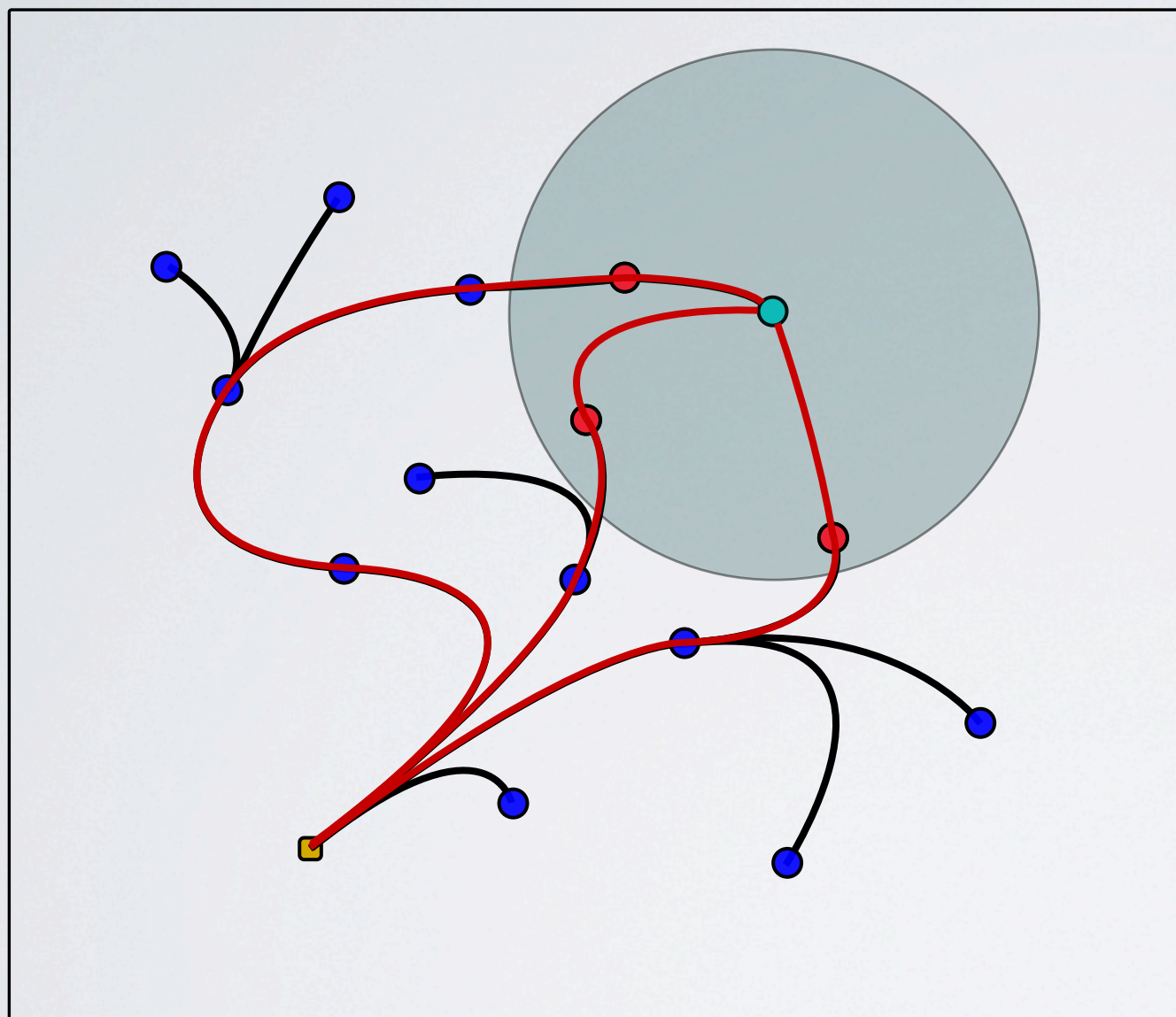


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

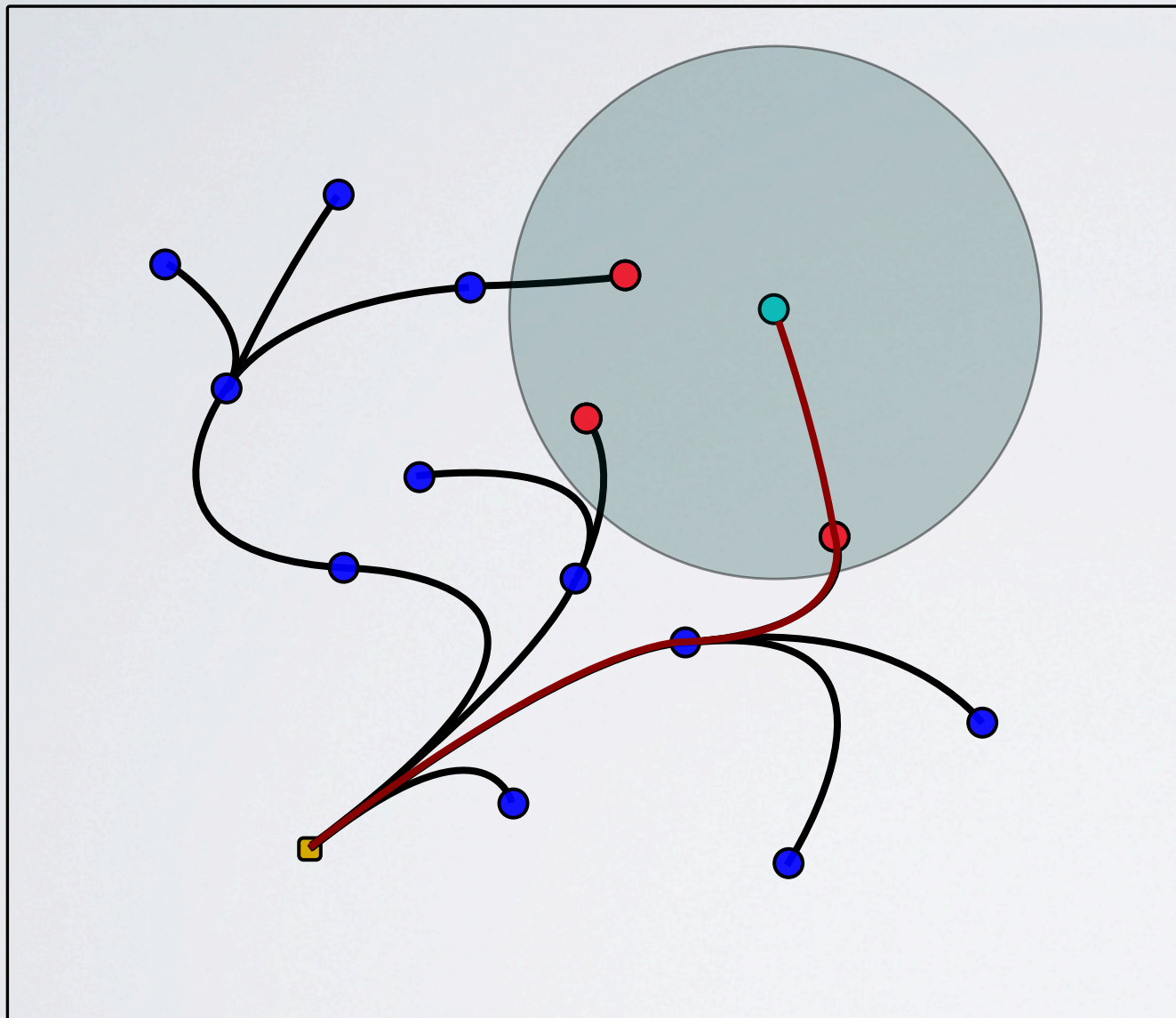


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T};$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

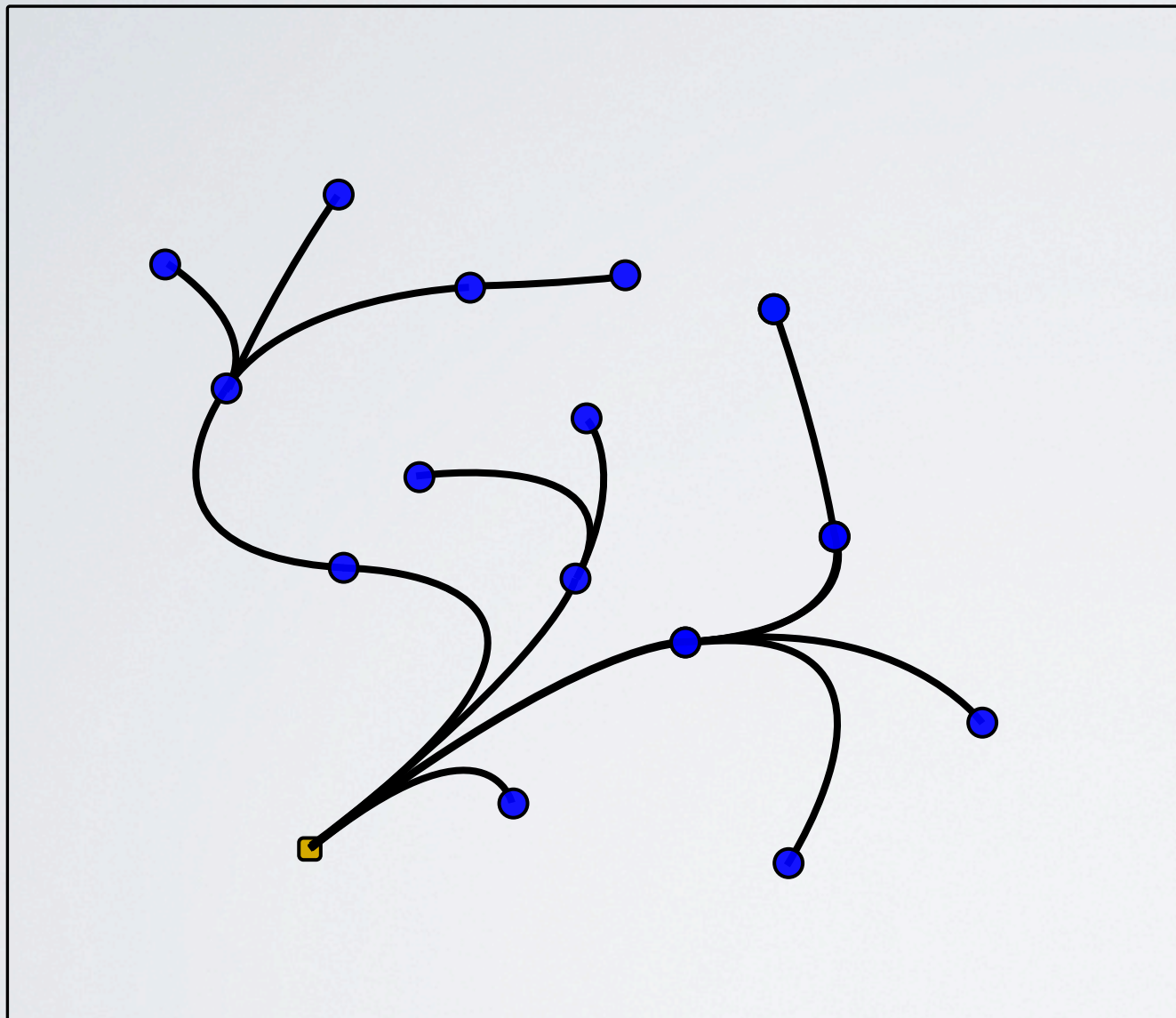


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

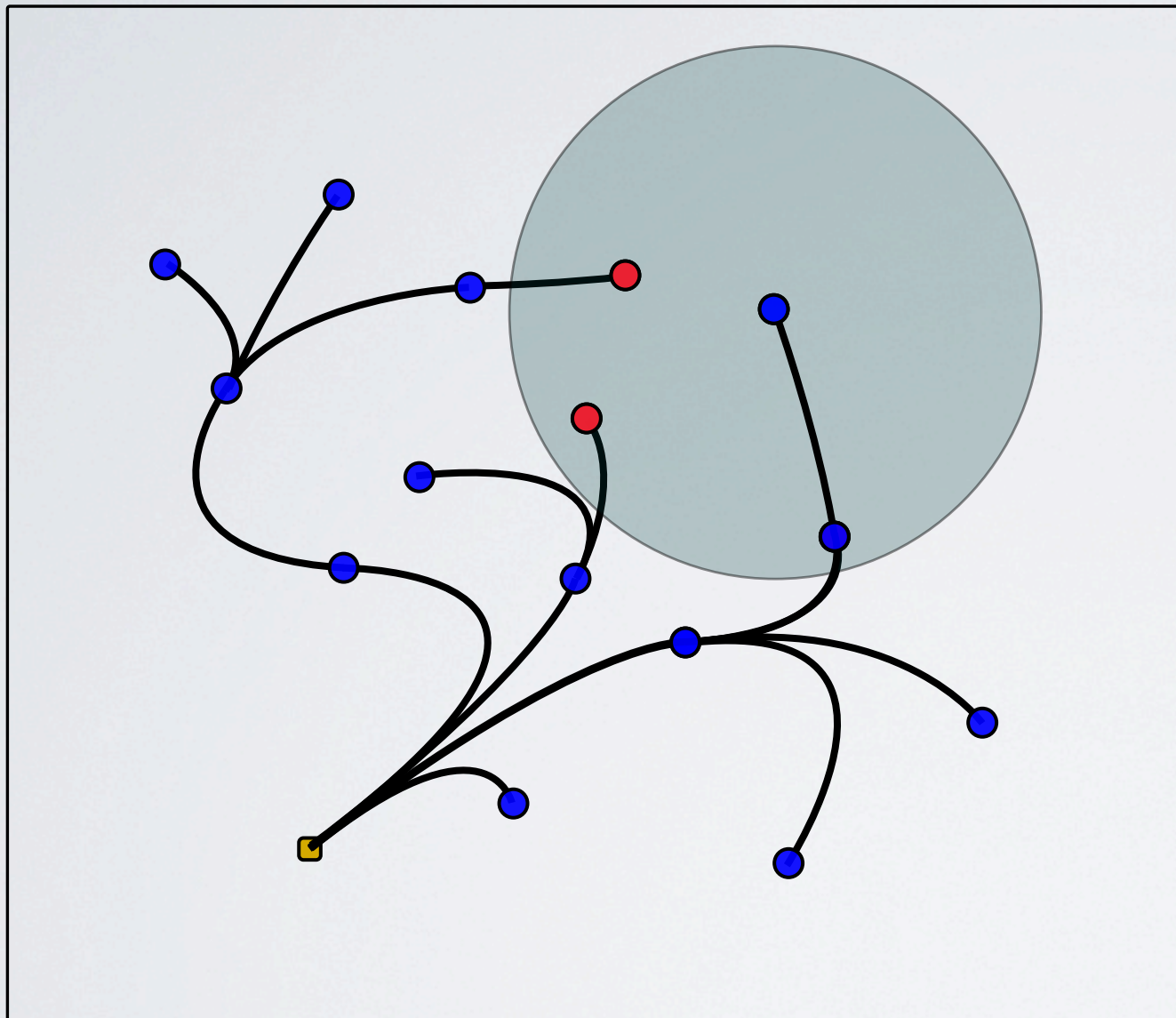


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

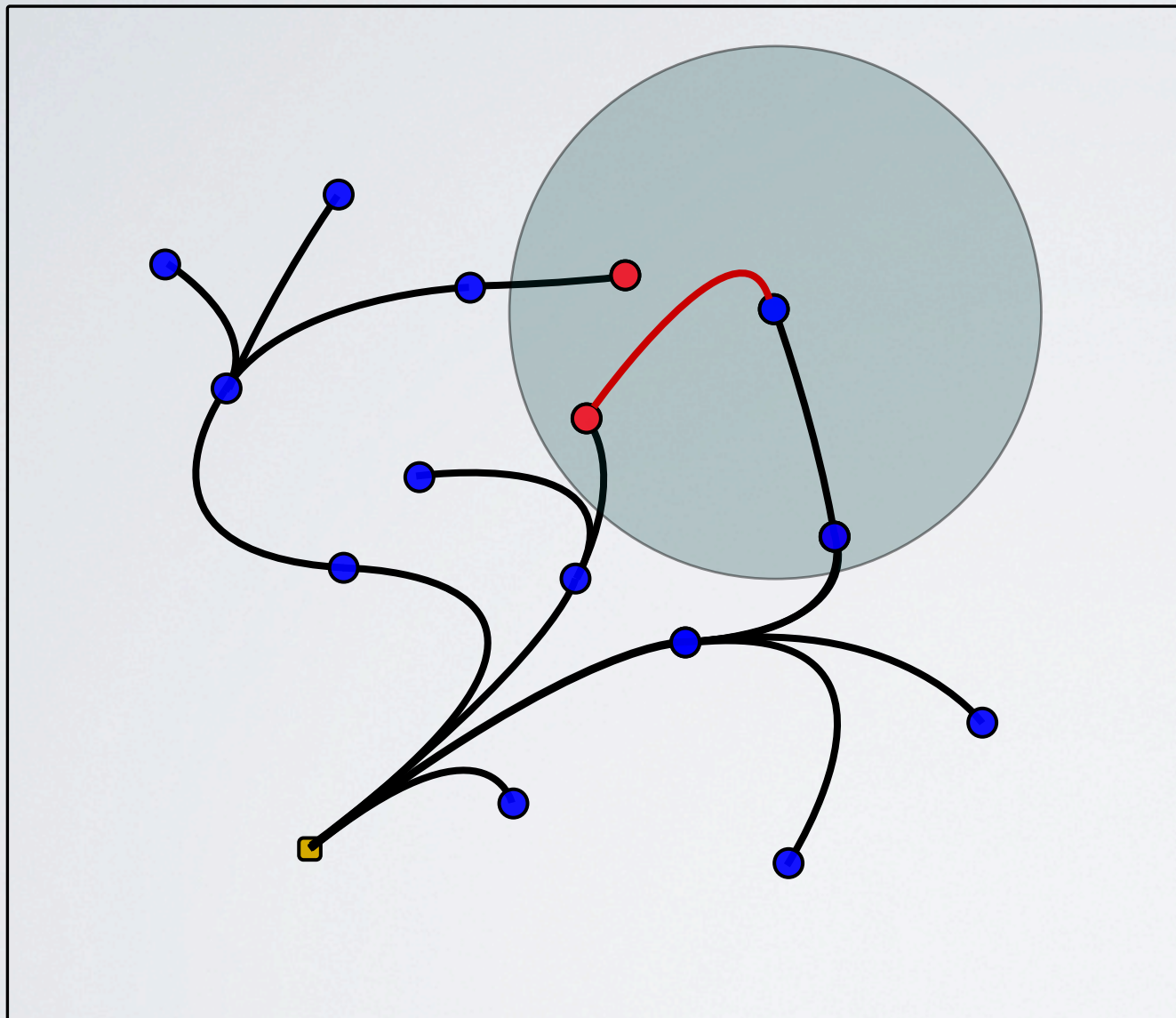


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

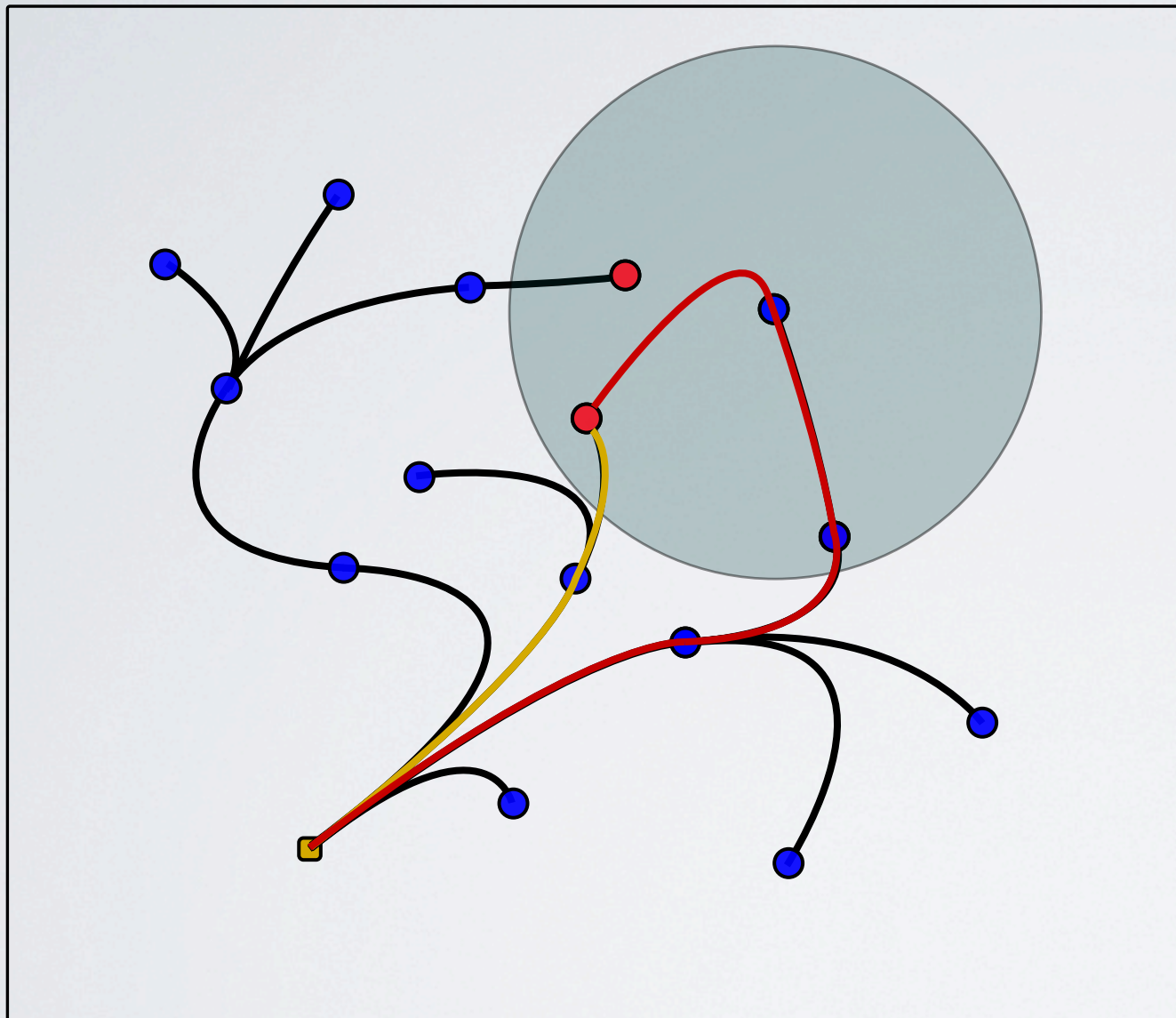


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

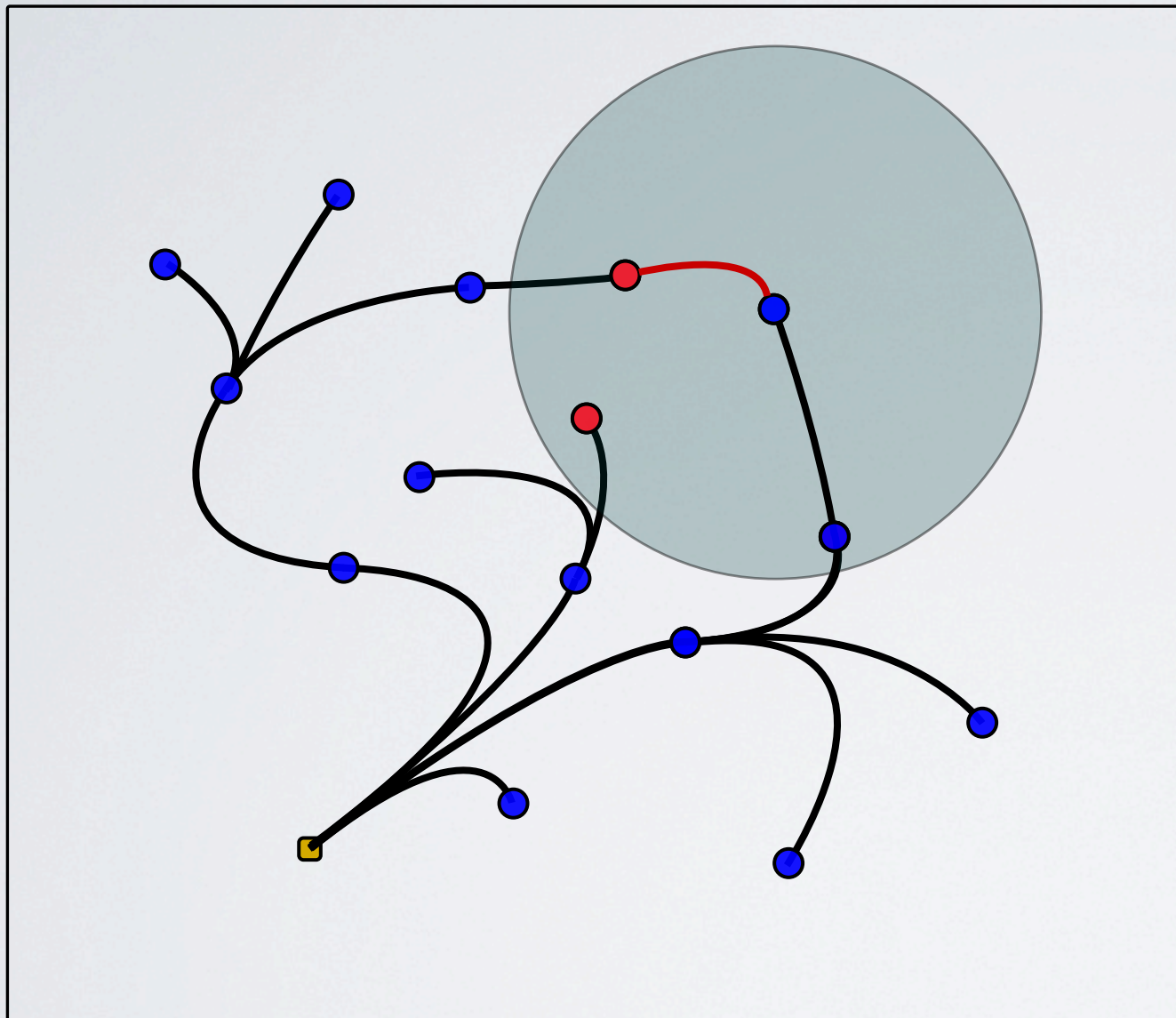


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

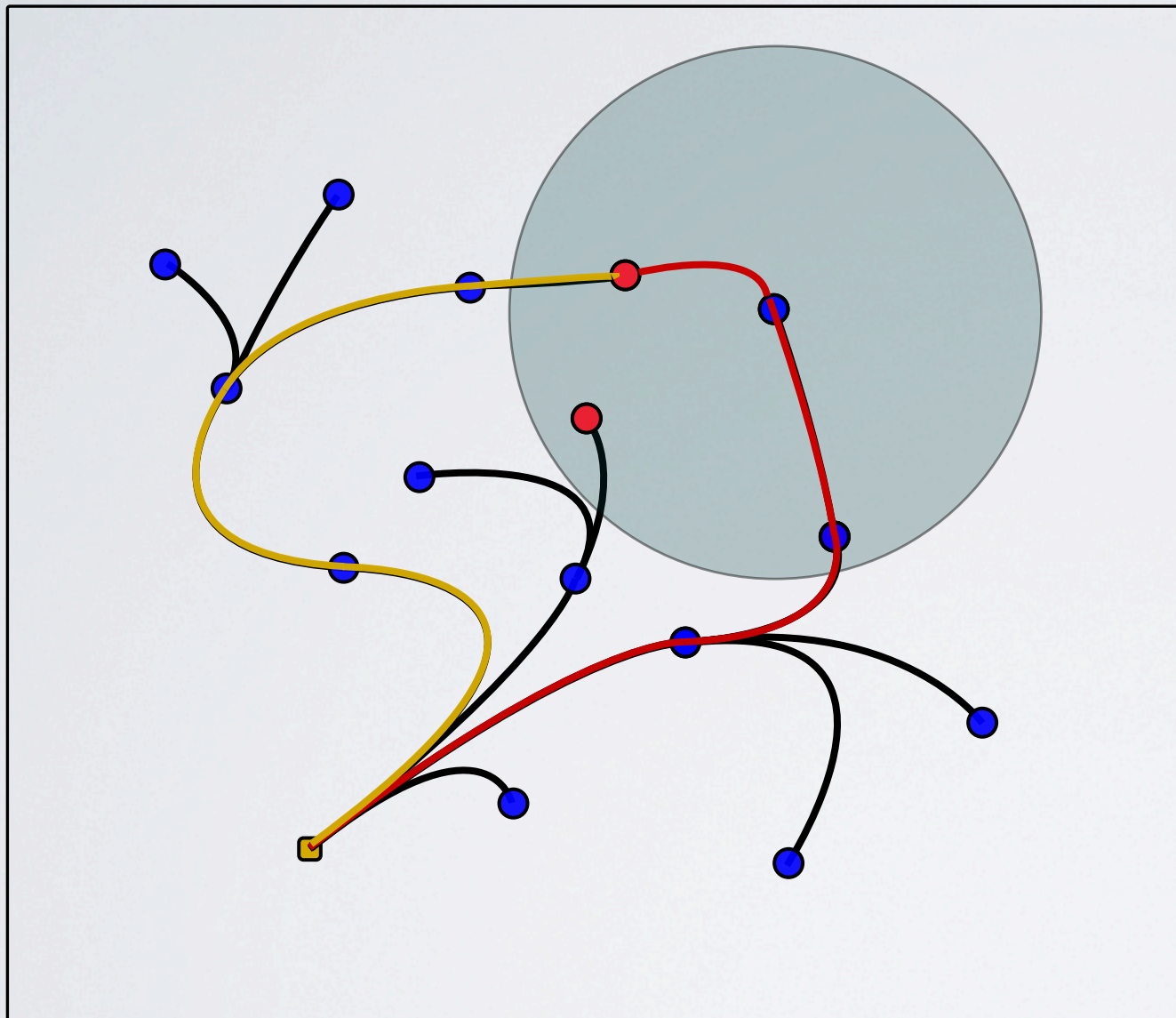


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

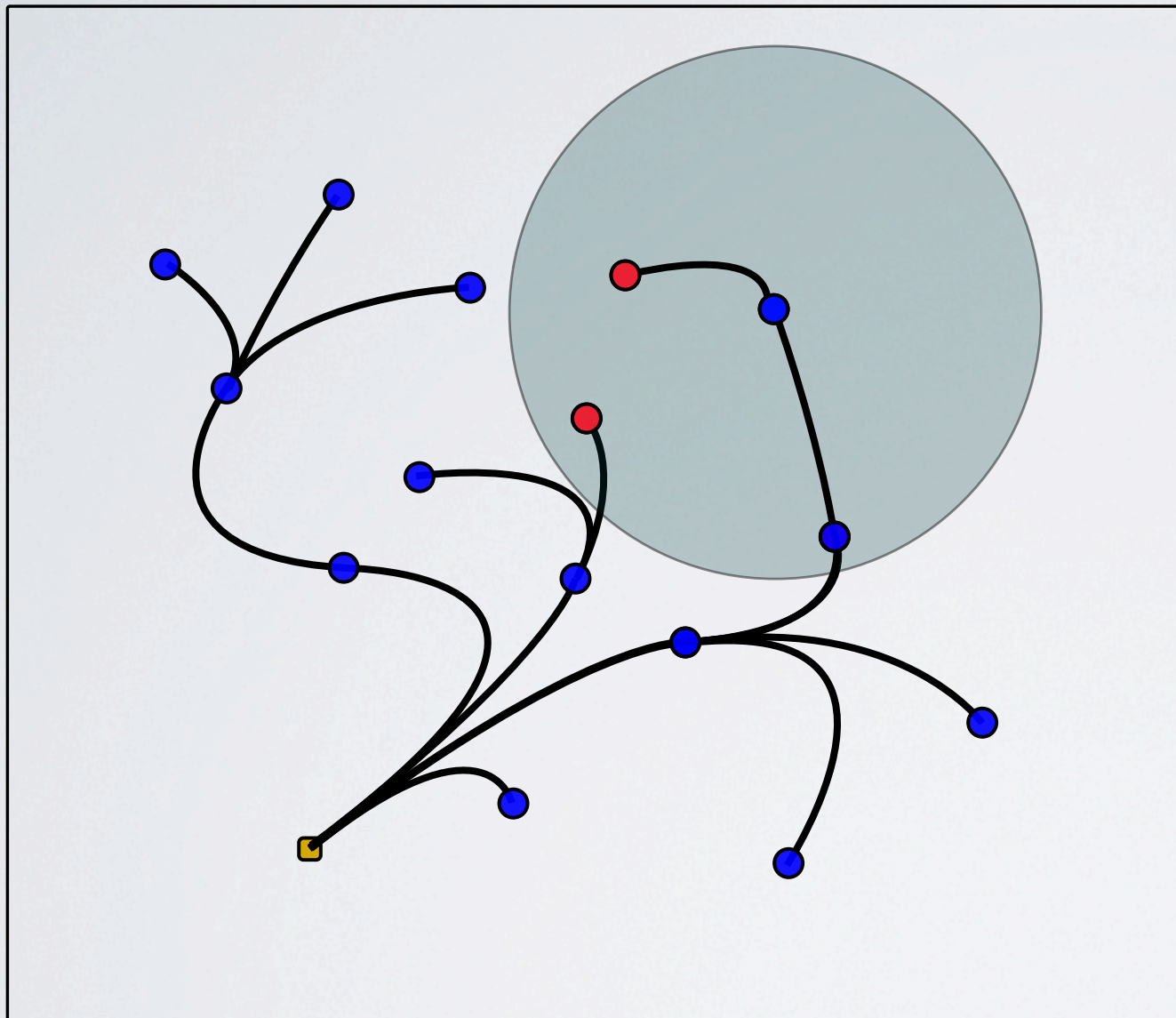


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

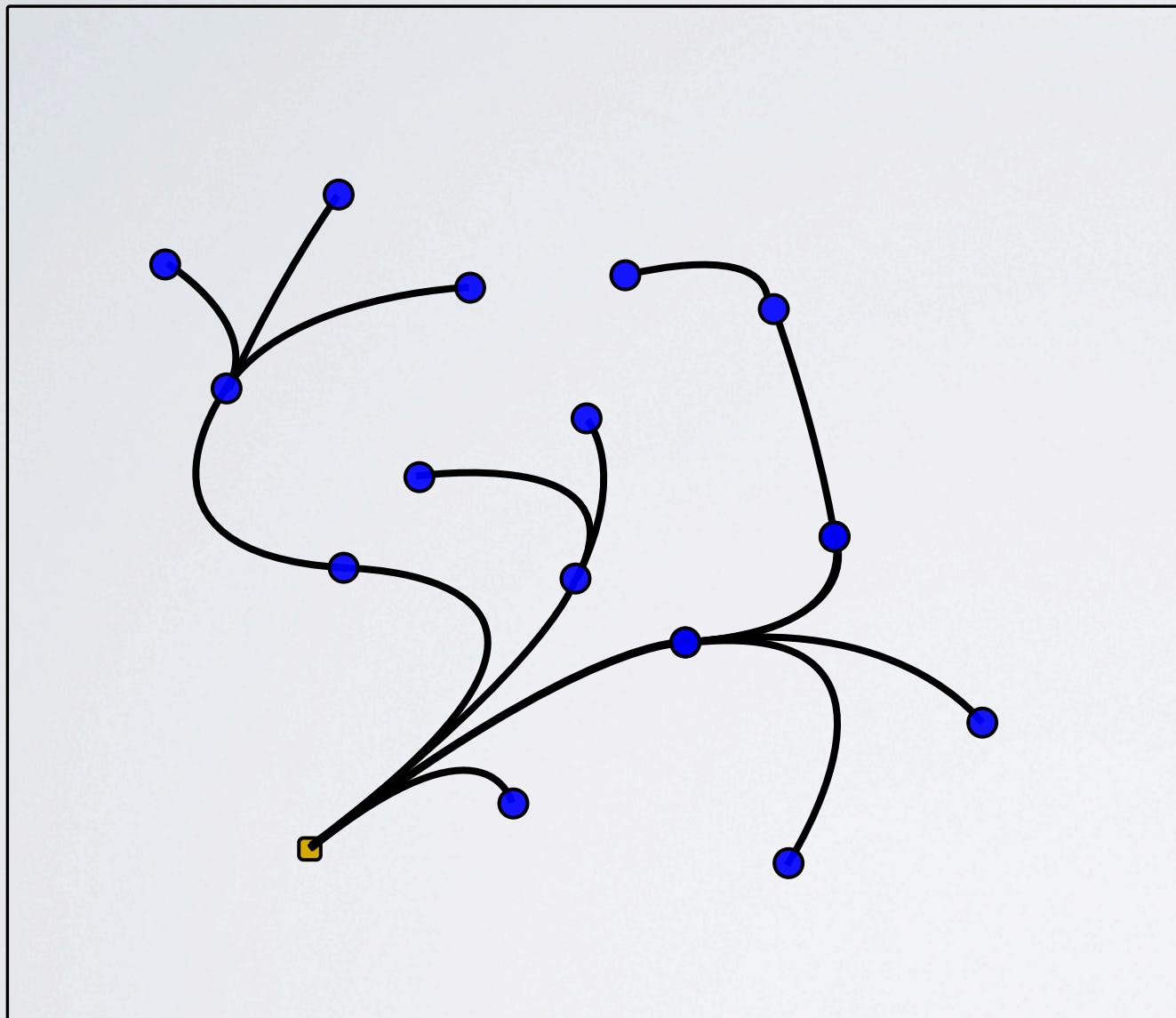


```

1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*

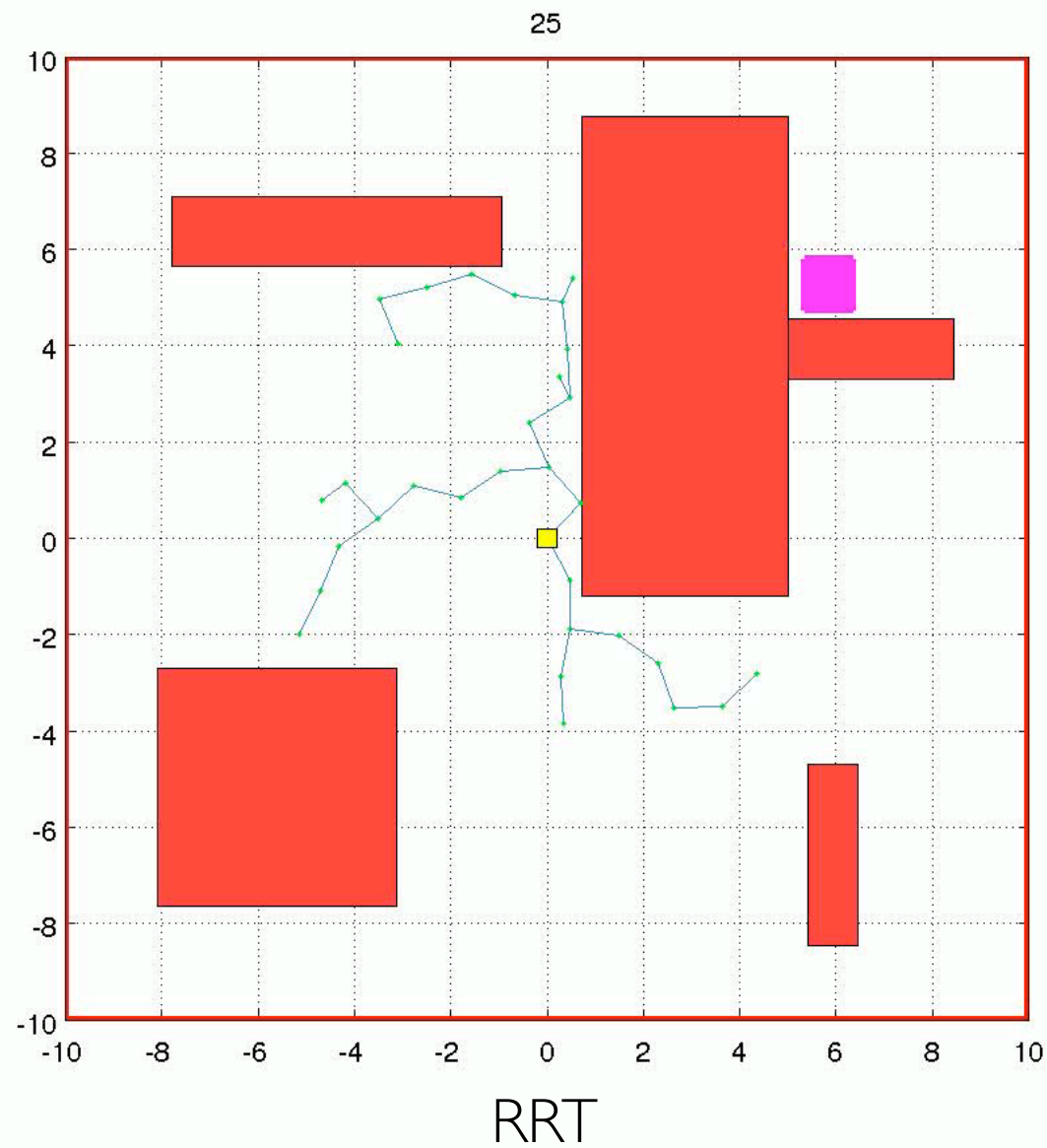


```

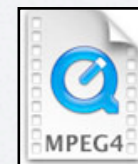
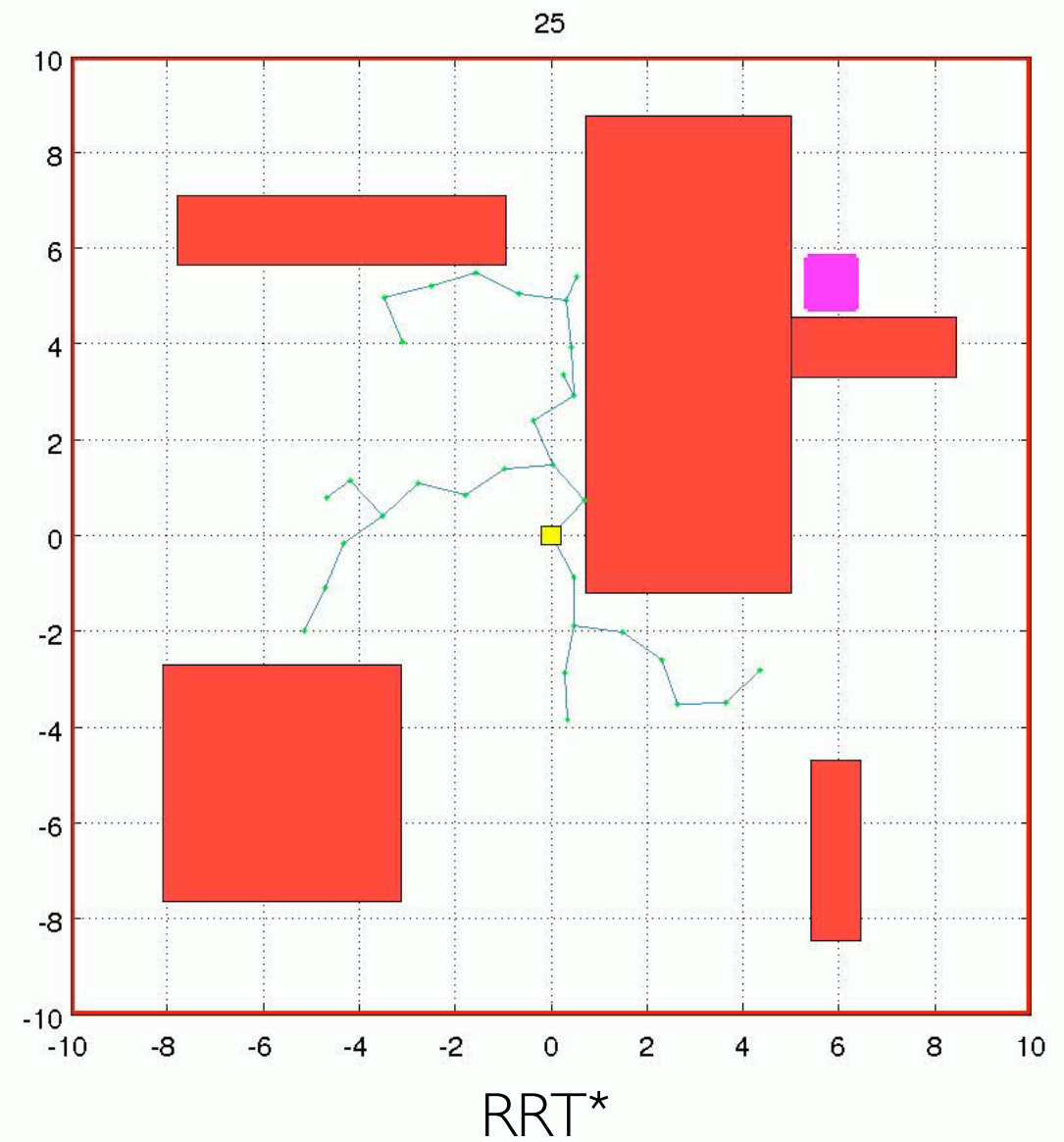
1  $\mathcal{T} \leftarrow \text{InitializeTree}();$ 
2  $\mathcal{T} \leftarrow \text{InsertNode}(\emptyset, z_{\text{init}}, \mathcal{T});$ 
3 for  $i = 1$  to  $i = N$  do
4    $z_{\text{rand}} \leftarrow \text{Sample}(i);$ 
5    $z_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, z_{\text{rand}});$ 
6    $(x_{\text{new}}, u_{\text{new}}, T_{\text{new}}) \leftarrow \text{Steer}(z_{\text{nearest}}, z_{\text{rand}});$ 
7   if  $\text{ObstacleFree}(x_{\text{new}})$  then
8      $Z_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, z_{\text{new}}, |V|);$ 
9      $z_{\text{min}} \leftarrow \text{ChooseParent}(Z_{\text{near}}, z_{\text{nearest}}, z_{\text{new}}, x_{\text{new}});$ 
10     $\mathcal{T} \leftarrow \text{InsertNode}(z_{\text{min}}, z_{\text{new}}, \mathcal{T});$ 
11     $\mathcal{T} \leftarrow \text{ReWire}(\mathcal{T}, Z_{\text{near}}, z_{\text{min}}, z_{\text{new}});$ 
12 return  $\mathcal{T}$ 

```

THE RRT*



video: 2011_05_10_icra_rt.mp4

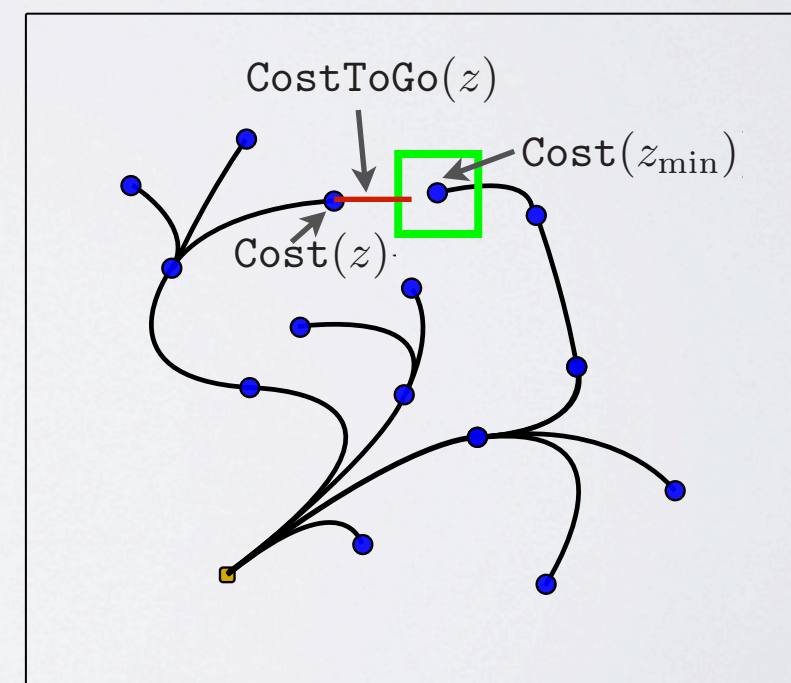
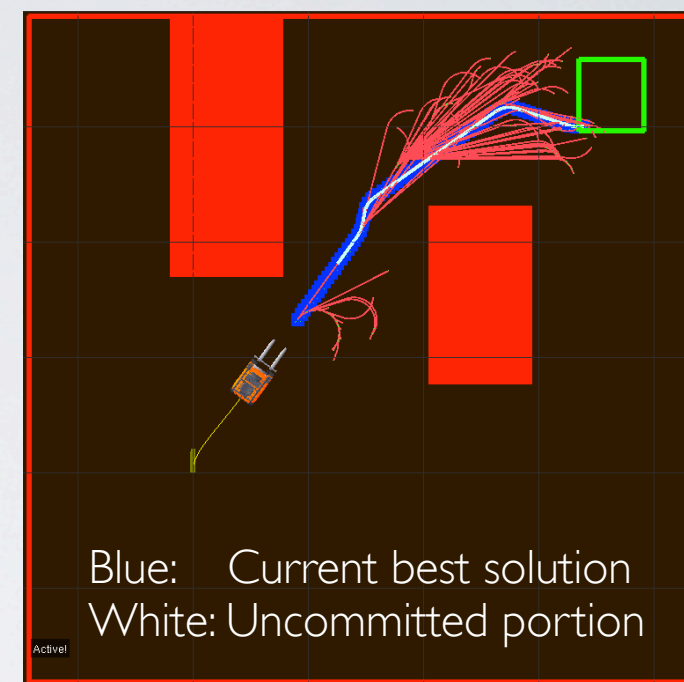


video: 2011_05_10_icra_rrtstar.mp4

ANYTIME EXTENSIONS

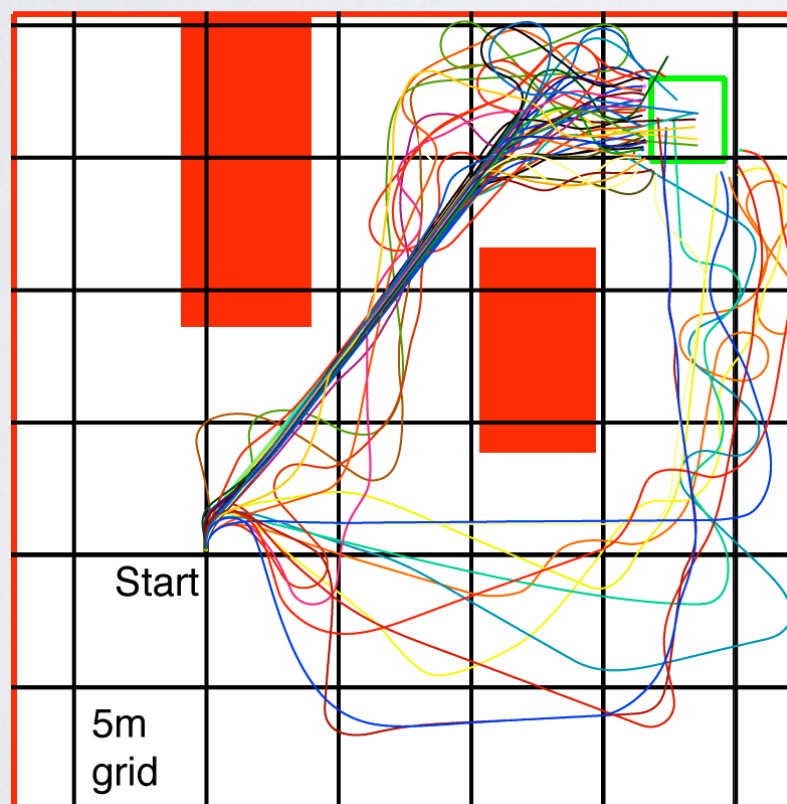
- Committed trajectory
 - Robot commits to execute immediate portion of current solution
 - Delete branches off committed trajectory, making the endpoint the new tree root
 - The planner improves paths (rewires) beyond committed trajectory
- Branch-and-bound
 - Maintain a lower-bound on the cost to get to the goal from each node in the tree (e.g., Euclidean distance)
 - Delete nodes for which

$$\text{Cost}(z) + \text{CostToGo}(z) \geq \text{Cost}(z_{\min})$$

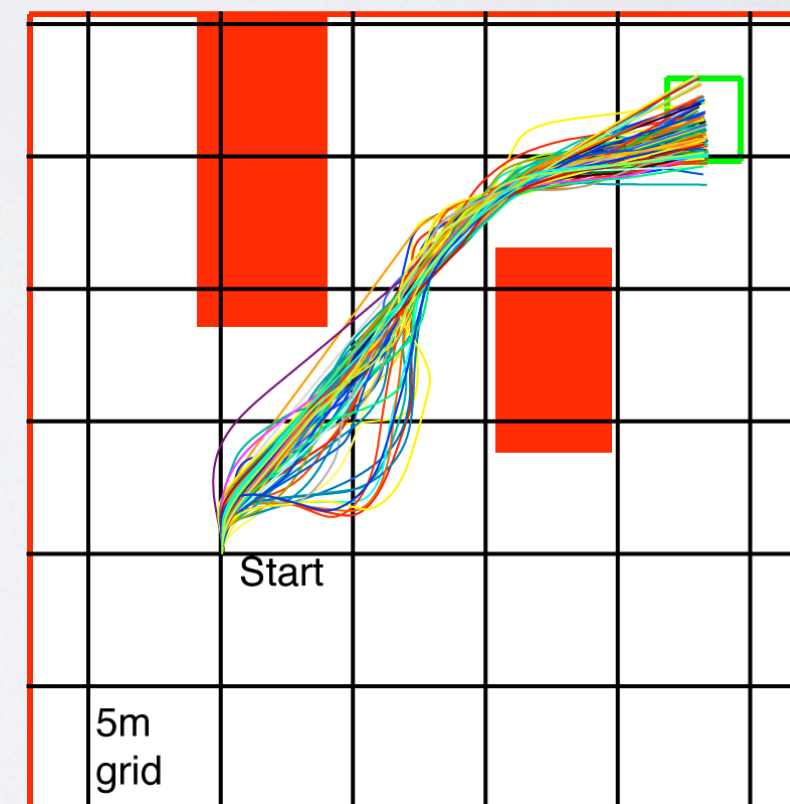


PERFORMANCE ANALYSIS

- Series of Monte Carlo simulations of a non-holonomic vehicle
 - Compare our anytime RRT* with anytime RRT
 - Both planners utilized committed trajectory and branch-and-bound heuristics
 - Both planners were allowed to maintain the tree until robot reached the goal
 - High-fidelity forklift dynamics model



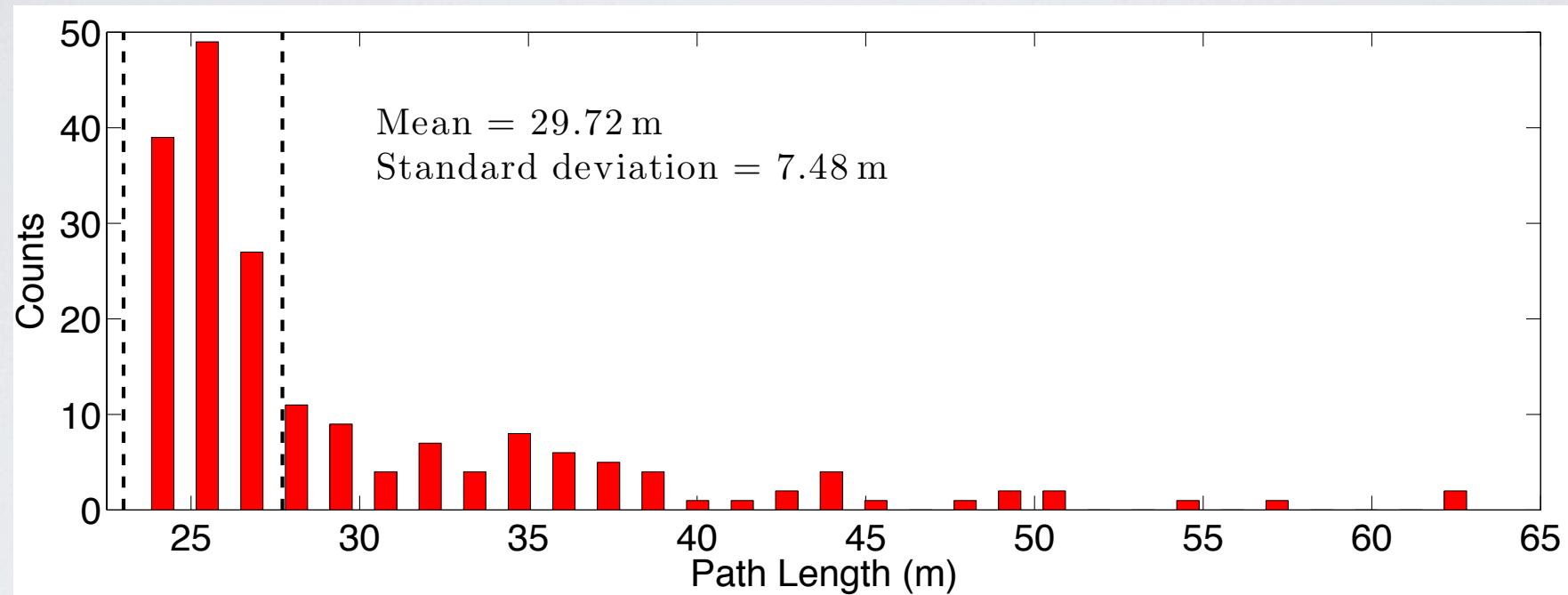
Anytime RRT



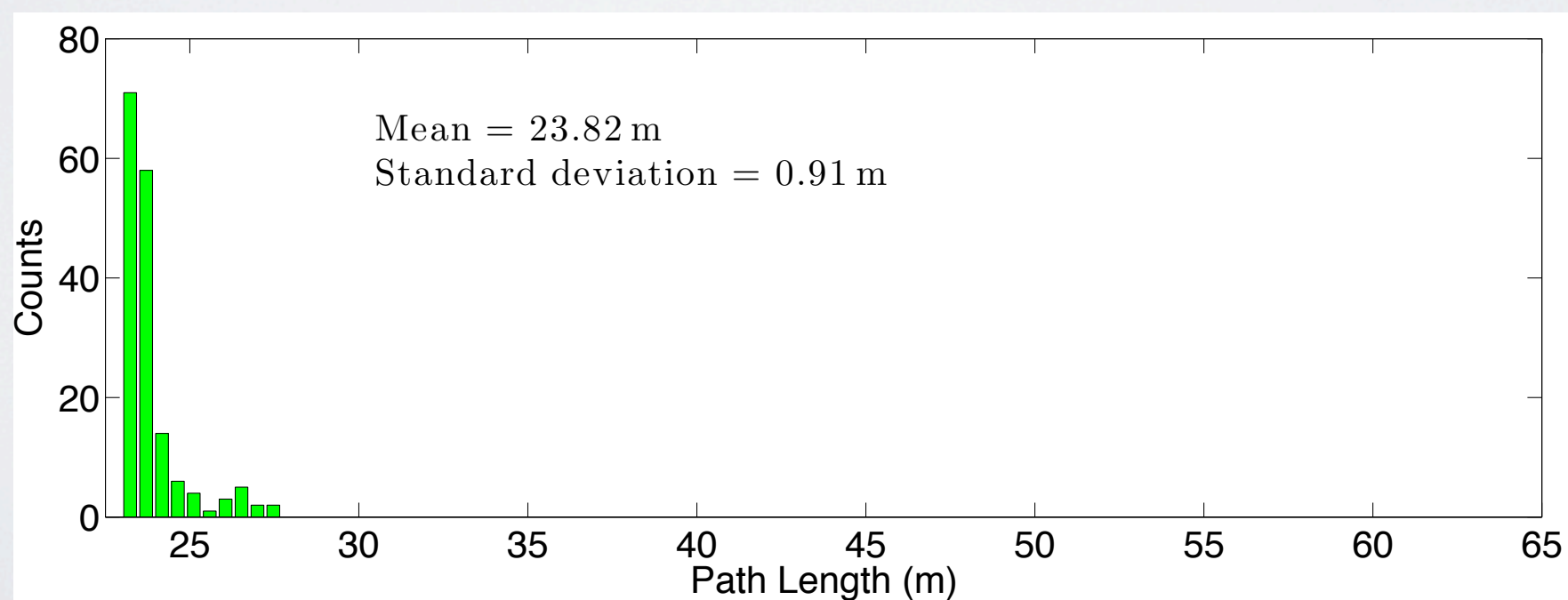
Anytime RRT*

PERFORMANCE ANALYSIS

Histogram of final path lengths



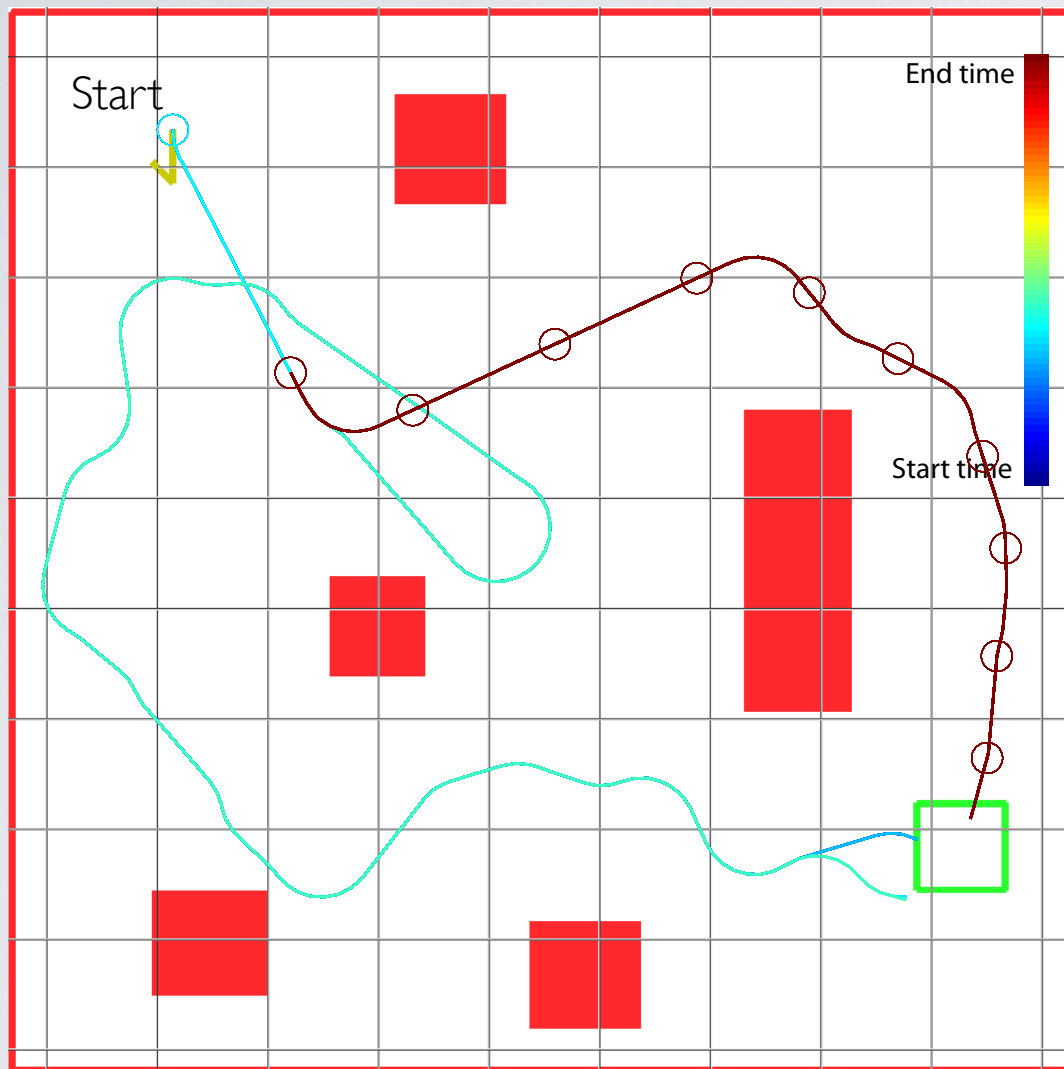
Anytime RRT



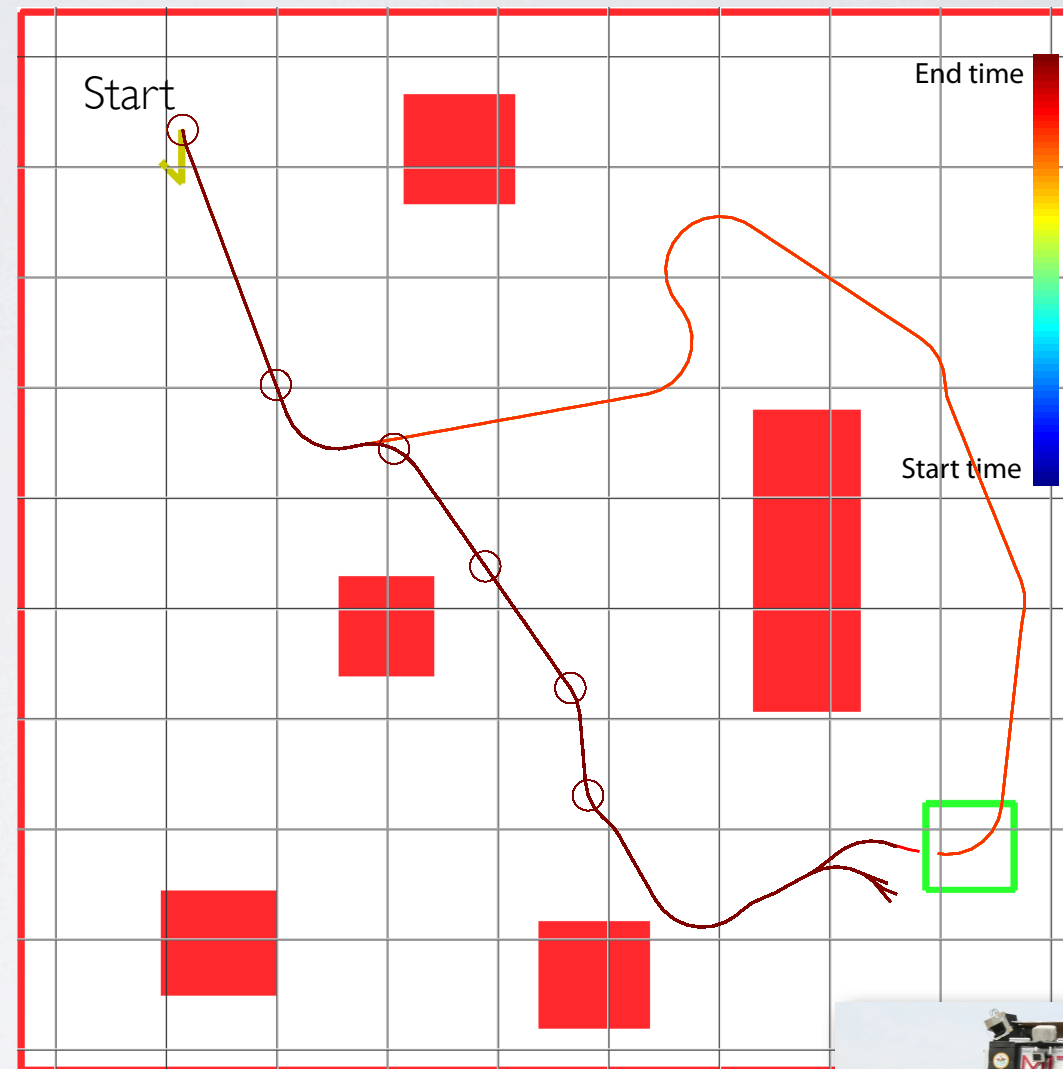
Anytime RRT*

FORKLIFT EXPERIMENTS: ANYTIME RRT

Run 1



Run 2



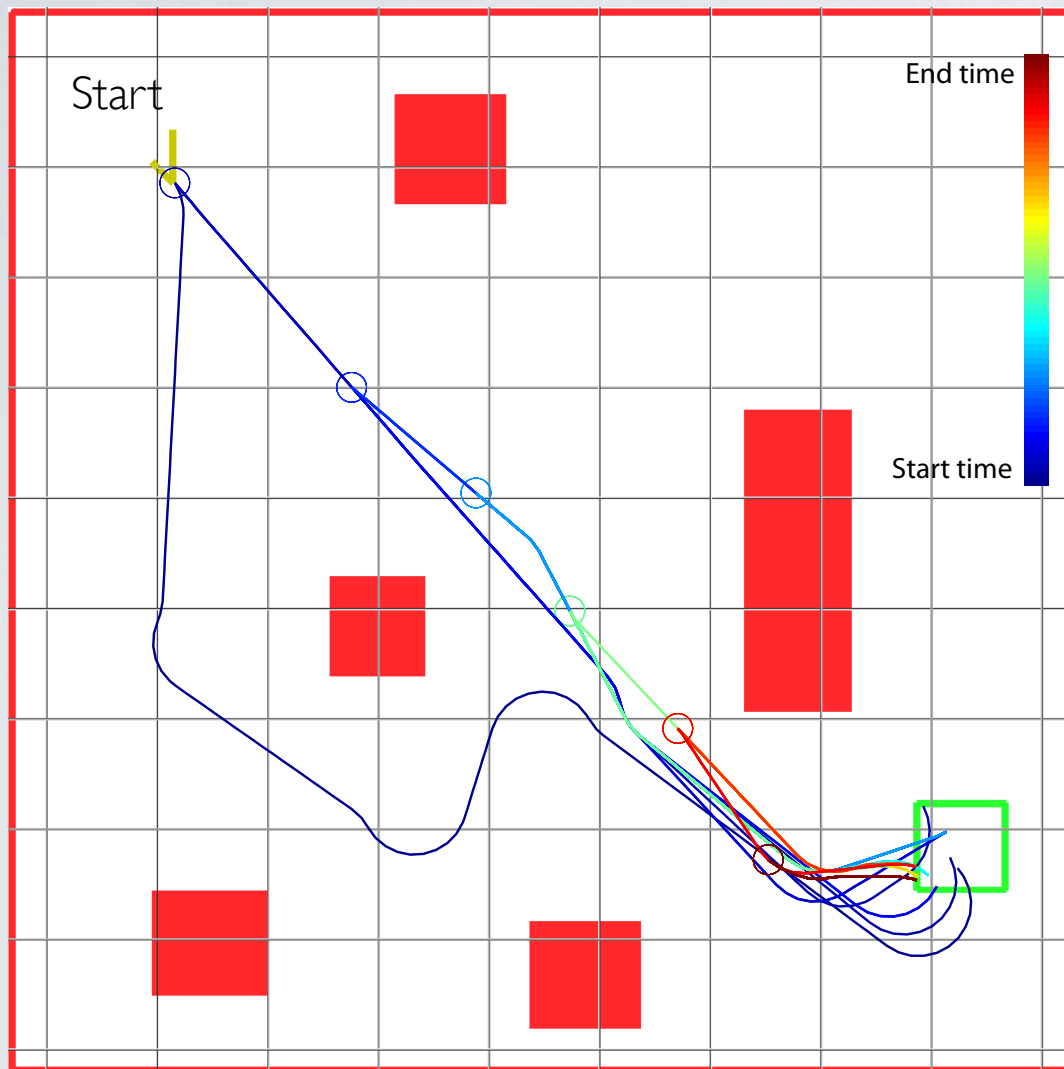
Increasing
plan/execution
time

Circles denote initial positions of the planned (uncommitted) paths

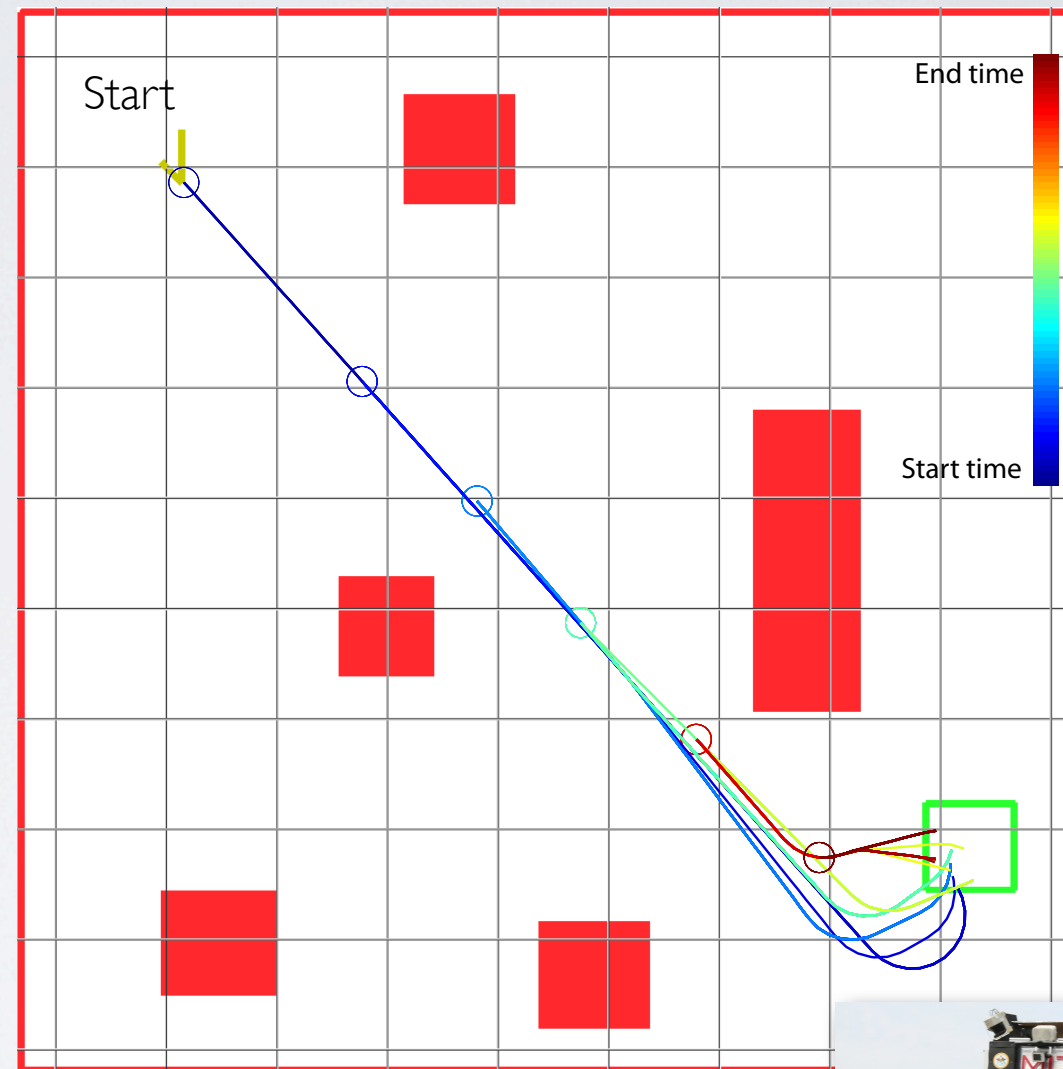


FORKLIFT EXPERIMENTS: ANYTIME RRT*

Run 1



Run 2



Increasing
plan/execution
time

Circles denote initial positions of the planned (uncommitted) paths



CONCLUSION

The algorithm demonstrates the desired anytime properties:

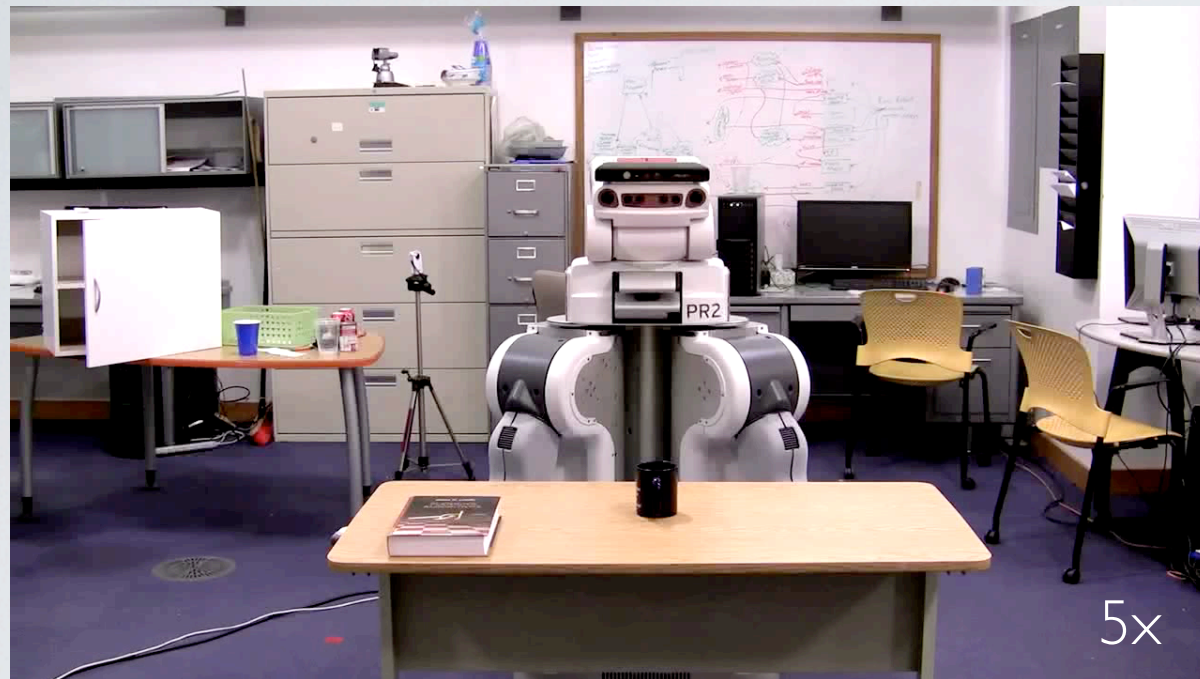
- Quickly find a feasible solution that the agent can begin to execute
- Take advantage of valuable execution time to asymptotically improve to optimal

CURRENT WORK

- Quickly find a solution that is feasible, but not necessarily optimal
- Asymptotic optimality: Exploit execution time to incrementally converge towards optimal solution
- (Probabilistic) completeness
- **Computationally efficiency (limited resources)**
- **Plan despite incomplete, imperfect knowledge**
- **Accommodate dynamic environments**

OPTIMAL MANIPULATION PLANNING

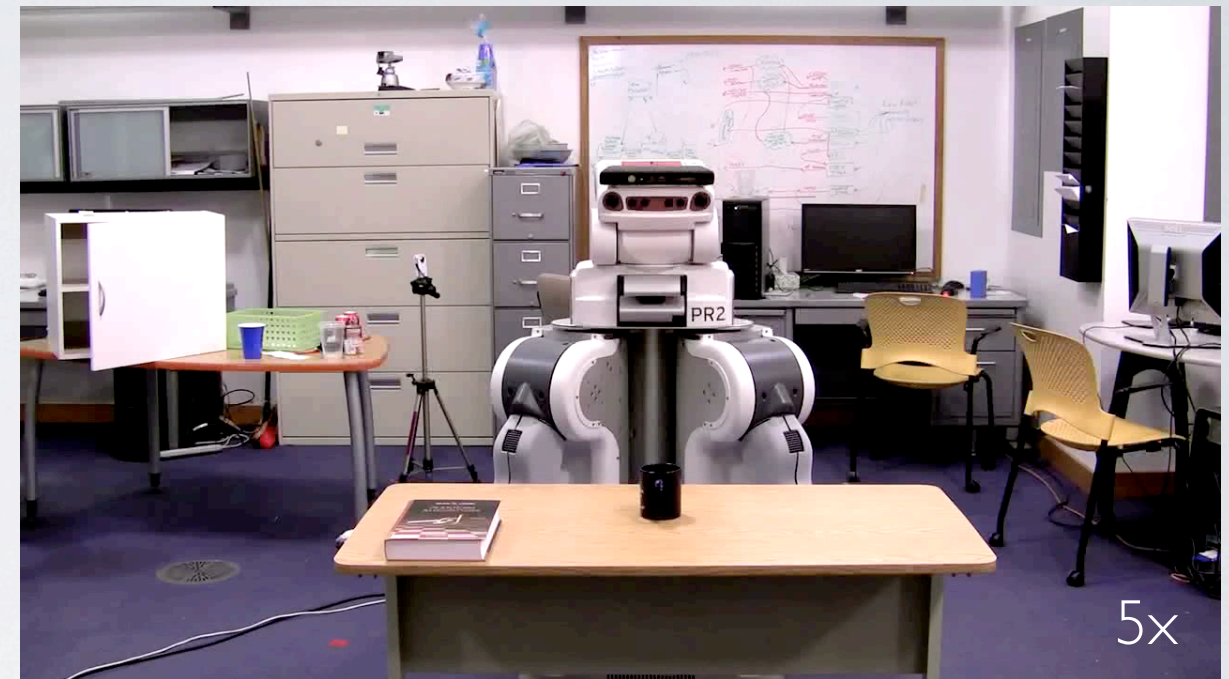
12-DOF pre-grasp planning on the PR2



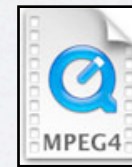
RRT



[video: 2011_02_pr2_12dof_rrt.mp4](#)



RRBT*



[video: 2011_02_pr2_12dof_rrbtstar.mp4](#)

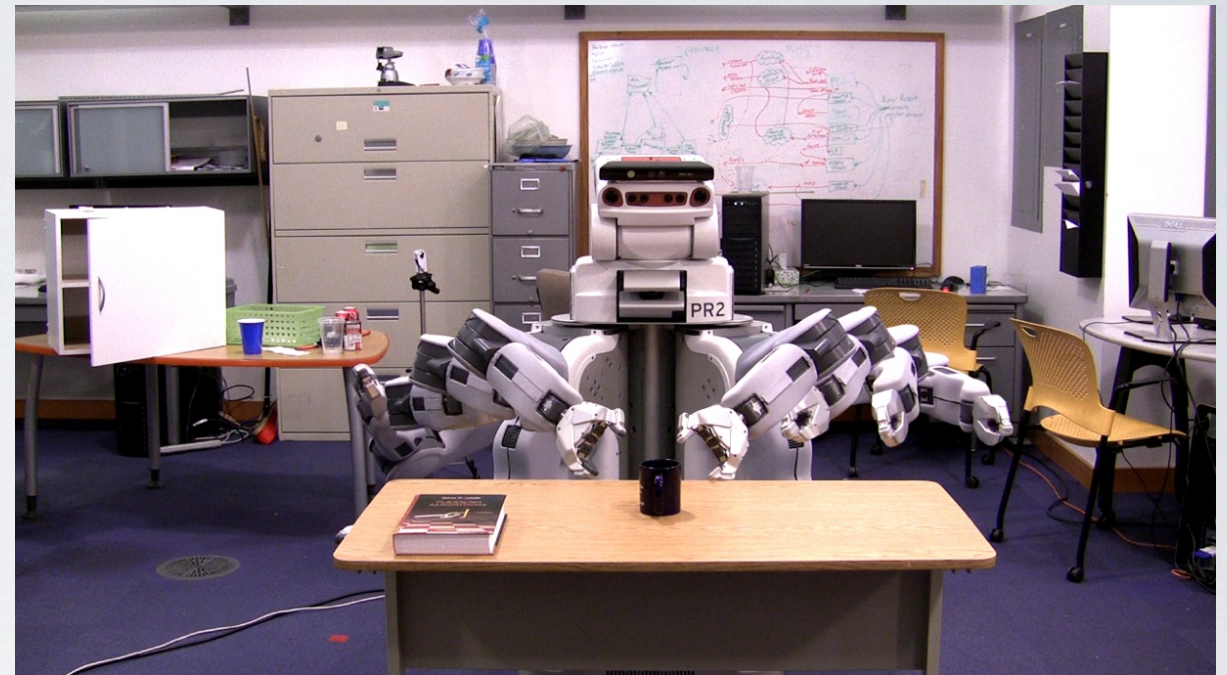
[Perez, Karaman, Walter, Shkolnik, Frazzoli, & Teller, IROS 2011 (submitted, under review)]

OPTIMAL MANIPULATION PLANNING

12-DOF pre-grasp planning on the PR2



RRT



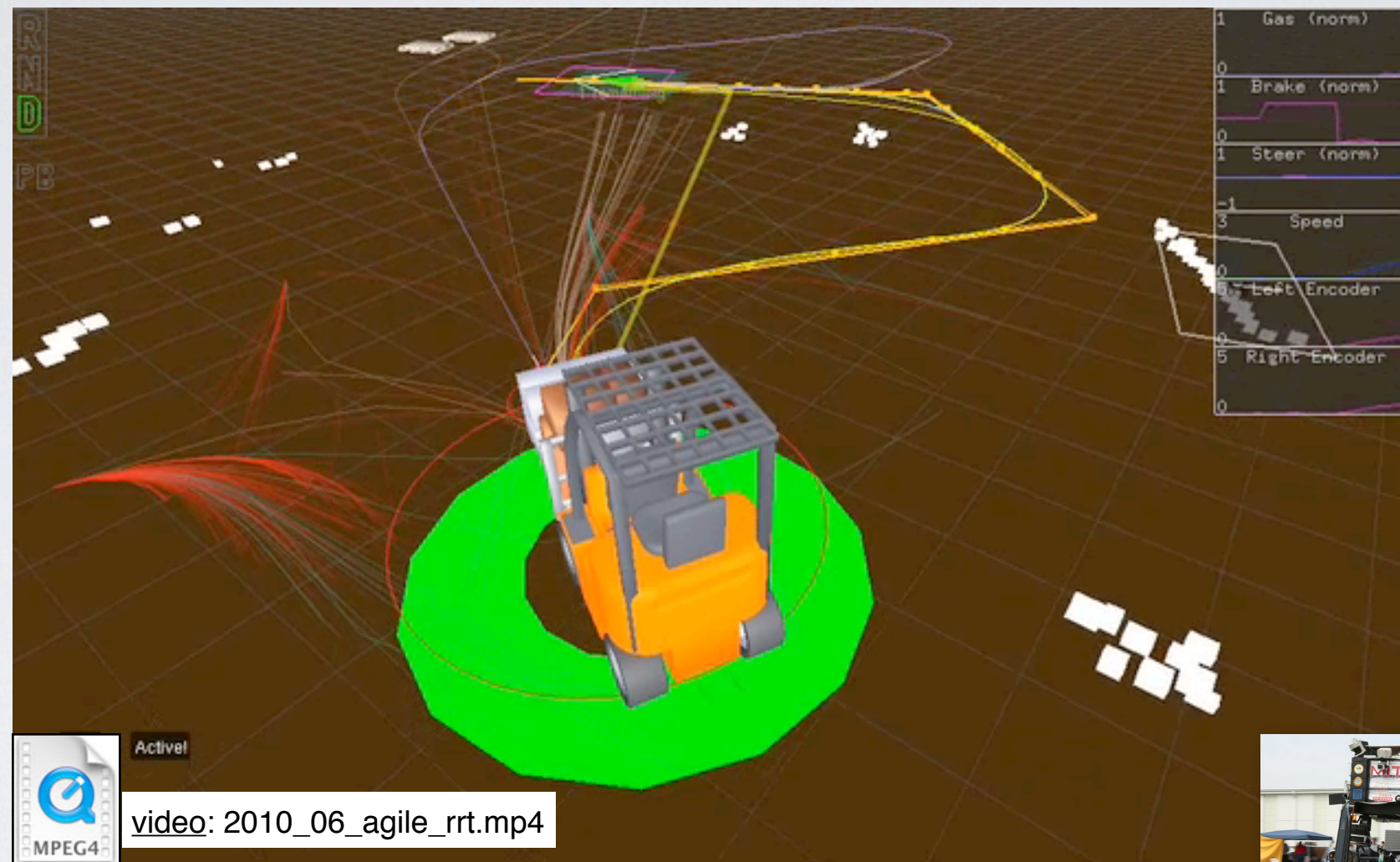
RRBT*

	RRT	RRBT*
First solution time (sec)	29.9	9.7
Cost of first solution (rad)	19.8	8.6
Cost of final solution (rad)	19.8	7.5

Averaged over several runs

[Perez, Karaman, Walter, Shkolnik, Frazzoli, & Teller, IROS 2011 (submitted, under review)]

PLANNING IN UNCERTAIN ENVIRONMENTS



Closed-loop RRT with false obstacle detections



[Joint work with Adam Bry and Nicholas Roy]

PLANNING IN UNCERTAIN ENVIRONMENTS

Chance-constrained optimization using incremental, sampling-based techniques

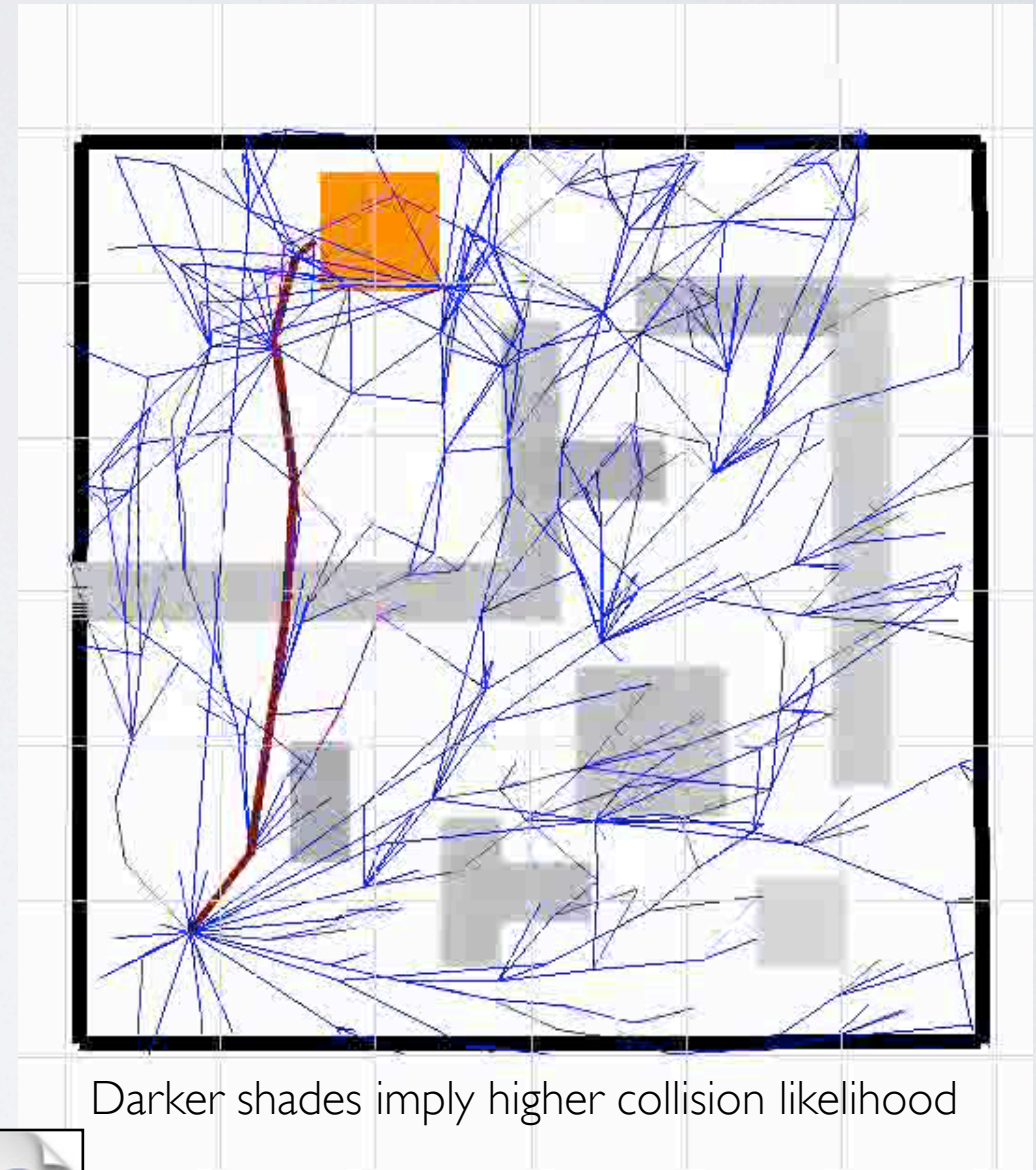
$$\min [c(\sigma)]$$

subject to:

$$P_{\text{col}} < \delta$$

$$\sigma(0) = x_{\text{init}}$$

$$\sigma(s) \in \mathcal{X}_{\text{goal}}$$



Darker shades imply higher collision likelihood



video: 2011_04_constraint_optimization.mp4

[Joint work with Adam Bry and Nicholas Roy]

QUESTIONS?

mwalter@csail.mit.edu

<http://people.csail.mit.edu/mwalter>