Physics becomes the computer

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Physics becomes the computer

Emulating Physics
  » Finite-state, locality, invertibility, and conservation laws

Physical Worlds
  » Incorporating comp-universality at small and large scales

Spatial Computers
  » Architectures and algorithms for large-scale spatial computations

Nature as Computer
  » Physical concepts enter CS and computer concepts enter Physics
Looking at nature as a computer
Looking at computation as physics
Looking at nature as a computer
Introduction

As we zoom in on a digital image,
Introduction

As we zoom in on a digital image, we begin to notice that there isn’t an infinite amount of resolution:
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As we zoom in on a digital image, we begin to notice that there isn’t an infinite amount of resolution: We begin to see the pixels.
Introduction

Something similar happens in nature. A box full of particles doesn’t have an infinite number of possible configurations:
Something similar happens in nature. A box full of particles doesn’t have an infinite number of different configurations: the number of distinct configurations is finite.
Introduction

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Similarly, the rate at which a finite system can transition from one distinct state to another is also finite. *Thus a finite physical system is much like a computer.*
Introduction

- Physics studies macro properties of finite information systems.
- Basic quantities such as Entropy and Energy are informational:

\[ dQ = TdS \]

\[ \text{Entropy}_{\text{MAX}} = \text{Info}_{\text{MAX}} \]
\[ \text{KineticE}_{\text{MAX}} = \text{Ops}_{\text{MAX}} \]
Introduction

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- Basic quantities such as Entropy and Energy are informational:

\[ \text{Entropy}_{\text{MAX}} = \text{Info}_{\text{MAX}} \]
\[ \text{KineticE}_{\text{MAX}} = \text{Ops}_{\text{MAX}} \]

(1996, with Levitin)
In this talk…

**Review:**
- info (Entropy) in physics

**Discuss:**
- statistical description of computation (→ QM)
- energy and action in comp
- what does QM add?
- physics as computation
What is Info?

- number of bits system can hold, given its constraints
- system with $2^n$ possible states can represent $n$ bits
- focus on classical info:
  » survives in macro limit
  » substitute micro dynamics when QM is invisible
  » ordinary macro quantities have classical info interp

$$\text{Info} = -\sum_i p_i \log p_i$$

% equally probable states,

$$\text{Info} = -\sum_{i=1}^{\Omega} \frac{1}{\Omega} \log \frac{1}{\Omega}$$

$$= \log \Omega$$
What is Entropy?

- Formal parameter in thermo (irreversibility)
- Boltzmann and Gibbs understood as counting
- Mixing neat → mess
- Mixing mess → mess
- Entropy is log of #states that fit with constraints
Classical Entropy

- For particles in a box, can introduce some coarseness
- This allows relative probabilities to be calculated
- (Also do the same thing for momentum)
Infinite Entropy?

- Thermo of EM radiation in cavity led to QM
- General state is a superposition of waves with integer number of peaks
- Any amplitude, can put unit of energy into any wave (*infinite info!*)
- Planck proposed $E = nh\nu$ (*finite info!*)

*EM radiation in a cavity (periodic boundaries)*
Looking at nature as a computer

- With QM, every finite system has finite state
- Dynamics of finite state systems is familiar
- Develop QM from computer viewpoint!
- Begin by discussing computer logic in statistical situations
Looking at computation as physics

• With QM, every finite system has finite state
• Dynamics of finite state systems is familiar
• Develop QM from computer viewpoint
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Statistical Dynamics

- To give a complete dynamics, we say what happens to each state in a fixed time
- Weighted sum of states (superposition) describes an ensemble
- Probability of initial state applies to corresponding final state

\[
\begin{align*}
U_{\text{XOR}}|00\rangle &= |00\rangle \\
U_{\text{XOR}}|01\rangle &= |01\rangle \\
U_{\text{XOR}}|10\rangle &= |11\rangle \\
U_{\text{XOR}}|11\rangle &= |10\rangle \\

a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle &\rightarrow \\
a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle
\end{align*}
\]
Statistical Dynamics

• Better to use square roots of probabilities (amplitudes)

• Evolution preserves vector length

• Lets us analyze system in other bases

\[
\begin{align*}
U_{\text{XOR}}|00\rangle &= |00\rangle \\
U_{\text{XOR}}|01\rangle &= |01\rangle \\
U_{\text{XOR}}|10\rangle &= |11\rangle \\
U_{\text{XOR}}|11\rangle &= |10\rangle \\
\sqrt{a}|00\rangle + \sqrt{b}|01\rangle + \sqrt{c}|10\rangle + \sqrt{d}|11\rangle &\rightarrow \\
\sqrt{a}|00\rangle + \sqrt{b}|01\rangle + \sqrt{c}|11\rangle + \sqrt{d}|10\rangle
\end{align*}
\]
Energy Basis

\[ U_\tau : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle \]

\[ |E_0\rangle = \frac{1}{\sqrt{N}} (|X_0\rangle + |X_1\rangle + \cdots + |X_{N-1}\rangle) \]

\[ U_\tau |E_0\rangle = \frac{1}{\sqrt{N}} (|X_1\rangle + |X_2\rangle + \cdots + |X_0\rangle) = |E_0\rangle \]

- Suppose \( U_\tau \) represents one clock period of a reversible computer
- Add together all configs in orbit
- This state has equal prob for any config
- Time evolution leaves this state unchanged!
Energy Basis

\[ A \xrightarrow{\text{NOT}} \bar{A} \]

\[ U_\tau |0\rangle = |1\rangle, \quad U_\tau |1\rangle = |0\rangle \]

\[ |E_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |E_1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \]

\[ U_\tau |E_0\rangle = |E_0\rangle, \quad U_\tau |E_1\rangle = -|E_1\rangle \]

- Example: suppose computer only has one bit, and \( U_\tau \) just flips it.
- Form new 2-state basis by adding and subtracting configs
- Magnitudes of amplitudes of energy states don’t change with time
Energy Basis

\[ |E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi i mn/N} |X_m\rangle, \]

\[ U_\tau |E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi i mn/N} |X_{m+1}\rangle \]

\[ = e^{-2\pi i n/N} |E_n\rangle \]

\[ \langle E_j | E_k \rangle = \frac{1}{N} \sum_{m,m'} e^{2\pi i (km-jm')/N} \langle X_{m'} | X_m \rangle \]

\[ = \frac{1}{N} \sum_m e^{2\pi i (m-j)/N} = \delta_{j,k} \]

- **In general:** use complex amplitudes to form new orthogonal basis
- \( |a\rangle \) is like a column vector of components
- \( \langle a | \) is like a row vector of complex conjugates

\[ U_\tau : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle \]
Energy Basis

\[ |E_n\rangle = \frac{1}{\sqrt{N}} \sum_{m} e^{2\pi inm/N} |X_m\rangle, \]

\[ |X_m\rangle = \frac{1}{\sqrt{N}} \sum_{n} e^{-2\pi inm/N} |E_n\rangle. \]

\[ U_\tau |E_n\rangle = \frac{1}{\sqrt{N}} \sum_{m} e^{2\pi inm/N} |X_{m+1}\rangle \]

\[ = e^{-2\pi in/N} |E_n\rangle \]

For a cycle:

\[ 2\pi = 2\pi \frac{n}{N} \times \frac{\tau_n}{\tau} \]

- Energy basis is Fourier Transform of config basis
- \( |E_n\rangle \) cycles with a frequency of \( \nu_n = \nu(n/N) \), where \( \nu = 1/\tau \)
- We will call \( h\nu_n \) the Energy of the state \( |E_n\rangle \), i.e. \( E_n = h\nu_n \)
Energy Basis

\[ |E_n\rangle = \frac{1}{\sqrt{N}} \sum_m e^{2\pi i nm/N} |X_m\rangle, \]

\[ |X_m\rangle = \frac{1}{\sqrt{N}} \sum_n e^{-2\pi i mn/N} |E_n\rangle. \]

\[ U_\tau : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle \]

- Interpret coefficients in energy basis as probs
- Energy of any state is independent of time
- \( |X_n\rangle \) is composed of equally spaced energies, \( E_n = n\hbar\nu_1 \)
- \( E = \hbar\nu/2 \), or \( \nu = 2E/h \)
What is Energy?

- $v = \frac{2E}{\hbar}$, so energy is rate of change of configurations
- CA lattice can change one spot at a time for reversible rules
- Should count changes as bit changes (i.e., energy is extensive!)

$U_r : |X_0\rangle \rightarrow |X_1\rangle \rightarrow \cdots \rightarrow |X_{N-1}\rangle \rightarrow |X_0\rangle$
What is Energy?

- **Conservation Law:**
  number of ones constant

- Constrains number of spots that can change in lattice update period \( \tau_l \)

- Focus on energy of the spots that can change

- If each particle is assigned an energy \( h \nu_l \)
  max change is still \( 2E/h \)

\[
M = \text{num particles} \\
h \nu_l = \text{particle energy} \\
\nu_\Delta = 2M \nu_l = 2E/h
\]
What is Action?

- $v = 2E/h$, so $\Omega(t) = 2Et/h$
- *Action* is amount of evolution (total ops for ideal computation)
- Number of comp events in rest frame is rel scalar
- Comp energy must transform like rel energy: $2E_r t_r/h = 2(Et - px)/h$
- If $x/t = c$, then $E = cp$ so that $Et = px$ (comp stops)
What does QM add?

• Stat Comp is special case: QM allows some new kinds of operations

• Any invertible evolution which preserves vector length is okay

• Probabilities can come and go!

• Only need to add extra single-bit operations

• $\nu = 2(E - E_{\text{min}})/h$
**XOR + $\sqrt[4]{\text{NOT}}$ are universal!**

\[
U_\theta |0\rangle = \cos \theta |0\rangle - \sin \theta |1\rangle \\
U_\theta |1\rangle = \sin \theta |0\rangle + \cos \theta |1\rangle
\]

\[
\theta = \pi/2 : \quad U_\theta = U_{\text{NOT}} \\
\theta = \pi/4 : \quad U_\theta = U_{\sqrt{\text{NOT}}}
\]
**XOR + $\sqrt[4]{\text{NOT}}$ are universal!**

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**Superposition of different configurations**

No probabilities

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A XOR B XOR C = B ⊕ AC
XOR + $\sqrt[4]{\text{NOT}}$ are universal!

$U_\theta |0\rangle = \cos \theta |0\rangle - \sin \theta |1\rangle$

$U_\theta |1\rangle = \sin \theta |0\rangle + \cos \theta |1\rangle$

$A \theta A$

$c = \cos(\pi/8)$

$s = \sin(\pi/8)$

|001⟩ $\rightarrow$ $\frac{\sqrt{2}}{2} |001⟩ + \frac{\sqrt{2}}{2} |011⟩$

+ $s |011⟩$
What does QM add?

• No new kinds of computations; at most reduces effort required
• Distinction is basis dependent
• Fundamental Q: If speedup is exponential, then distinction is real!
What does this mean?

Classical: \[ \sqrt{a}|0\rangle + \sqrt{b}|1\rangle \rightarrow_{\text{NOT}} \sqrt{b}|0\rangle - \sqrt{a}|1\rangle \]

Quantum: \[ \sqrt{a}|0\rangle + \sqrt{b}|1\rangle \rightarrow_{\sqrt{\text{NOT}}} \frac{\sqrt{a} + \sqrt{b}}{\sqrt{2}}|0\rangle + \frac{\sqrt{a} - \sqrt{b}}{\sqrt{2}}|1\rangle \]
What does this mean?

Classical: \[ \sqrt{a}|0\rangle + \sqrt{b}|1\rangle \rightarrow_{\text{NOT}} \sqrt{b}|0\rangle - \sqrt{a}|1\rangle \]

Quantum: \[ \sqrt{a}|0\rangle + \sqrt{b}|1\rangle \rightarrow_{\text{NOT}} \frac{\sqrt{a} + \sqrt{b}}{\sqrt{2}}|0\rangle + \frac{\sqrt{a} - \sqrt{b}}{\sqrt{2}}|1\rangle \]
Conclusions

• On a large scale, often can’t tell if micro finite-state is QM or CM

• *Entropy, Energy* and *Action* all have comp meaning: others must

• Significant for comp and for physics

*for more information, see*  
http://www.ai.mit.edu/people/nhm/looking-at-nature.pdf