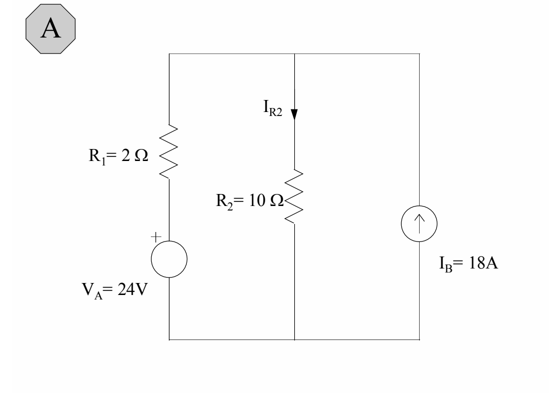
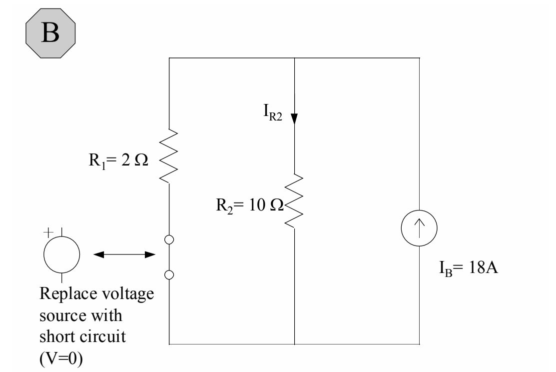


SUPERPOSITION

The circuit shown contains two sources, a voltage source V_A and a current source I_B. To determine the voltages and currents in the circuit with the two sources present is difficult.

Instead, we analyse the circuit with just one source present, taking each source in turn. This gives us the voltage and current components caused by each individual source. These can be simply added to find the voltages and currents in the circuit with all the sources present.

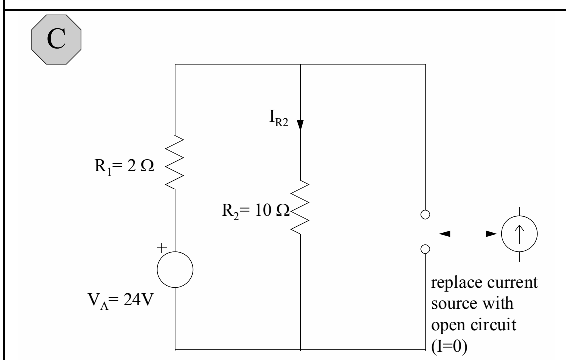


With just I_B present

To view the effect of I_B acting alone, the effect of V_A must be removed. To remove the effect of a voltage source, its voltage is set to zero. A component with a voltage drop of zero across it is equivalent to a short circuit, so this is what the voltage source effectively becomes.

For example, the component of I_{R2} caused by I_B is:

$$I_{R2} = \frac{R_1}{R_1 + R_2} I_B = \frac{2}{2+10} 18 = 3A$$

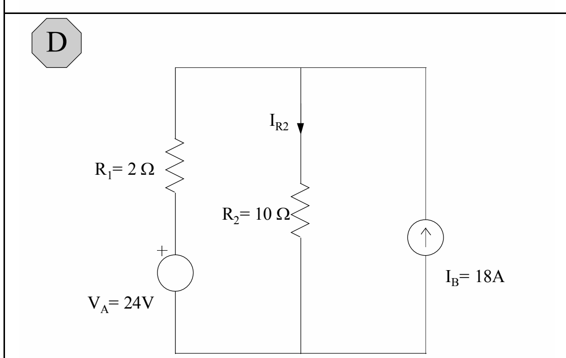


With just V_A present

To view the effect of V_A acting alone, the effect of I_B must be removed. To remove the effect of a current source, its current is set to zero. A component with zero current flowing through it is equivalent to a short circuit, so this is what the current source effectively becomes.

The component of I_{R2} caused by V_A is:

$$I_{R2} = \frac{V_A}{R_1 + R_2} = \frac{24}{2+10} = 2A$$

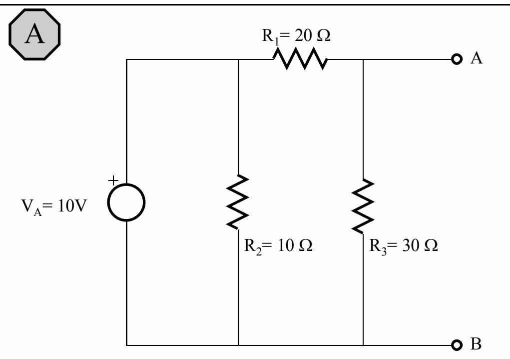


With both I_B and V_A present

The currents and voltages in the circuit with both sources present are the sum of the components caused by each source acting on its own.

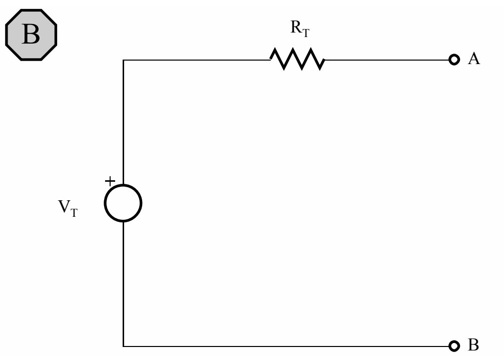
The current I_{R2} caused by both I_B and V_A acting together is:

$$I_{R2} = 3 + 2 = 5A$$



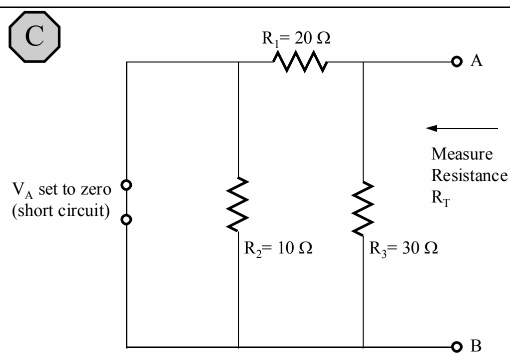
THÉVENIN EQUIVALENT CIRCUIT

The circuit shown could be a section of a larger circuit that is being analysed. If none of the currents or voltages *within* this section are of interest, then this circuit section can be replaced with a simpler arrangement that behaves the same as this one at the terminals A and B (which is all that matters to the external circuit). One way of making this replacement is to use a Thévenin Equivalent Circuit.



The Equivalent Circuit

The circuit shown is the Thévenin equivalent circuit. With a suitable choice of V_T and R_T this circuit section will behave identically to the original circuit at the terminals A and B.
If the source in this circuit is set to zero, and the impedance from A to B is measured, the result is R_T. Since this circuit is equivalent to the original circuit, R_T is also the resistance measured between A and B in the original circuit when all sources are set to zero.
V_T is the voltage measured when no current is drawn from the terminal (and the voltage drop across R_T is therefore zero).

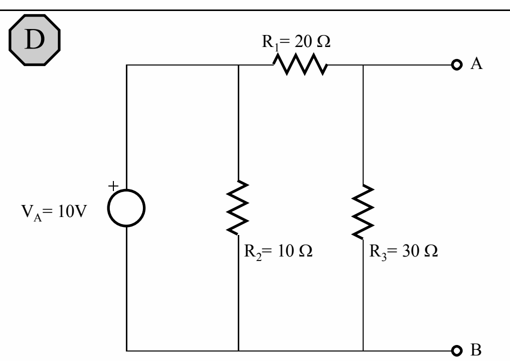


Equivalent resistance

To find the resistance R_T all sources in the circuit section must be set to zero, and then the resistance can be measured from A to B.

For this circuit, the only source is V_A. This is set to zero, and replaced with a short circuit (zero voltage implies a short circuit). Since the resistor R_2 is now shorted,

$$R_T = R_1 || R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{20 \times 30}{20 + 30} = 12\Omega$$



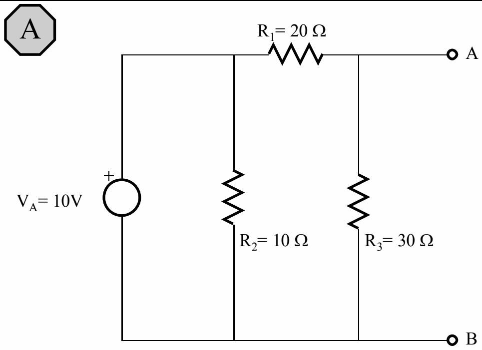
Equivalent voltage

To find the voltage V_T the voltage from A to B is measured (without drawing any current).

The voltage V_A is applied across R1 and R3 in series, so by the voltage divider rule,

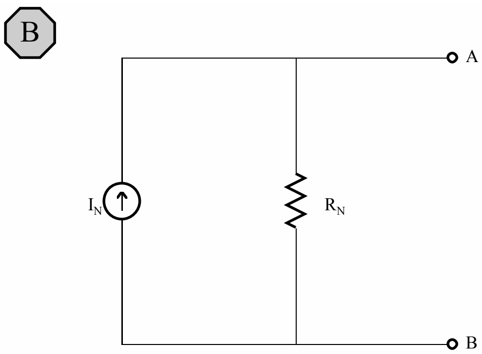
$$V_T = V_{AB} = V_A \frac{R_1}{R_1 + R_3} = 10 \frac{30}{20 + 30} = 6\text{V}$$

So the equivalent circuit is as shown in B above, with V_T being 6V and R_T being 12 Ω.



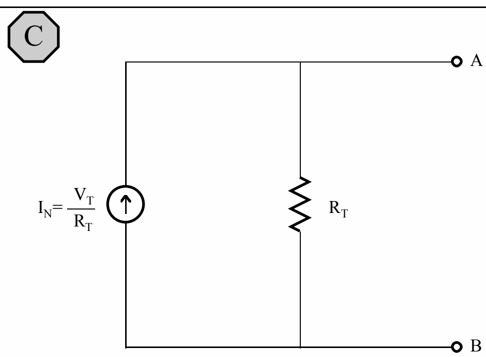
NORTON EQUIVALENT CIRCUIT

Thévenin is not the only equivalent circuit possible. Another related equivalent is the Norton equivalent circuit, which uses a current source instead of a voltage source.



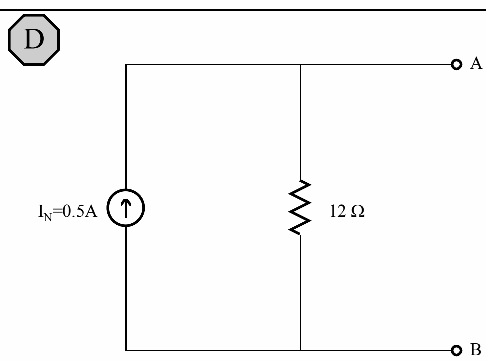
The Equivalent Circuit

The circuit shown is the Norton equivalent circuit. With a suitable choice of I_N and R_N this circuit section will behave identically to the original circuit at the terminals A and B.
If all the sources in this circuit are set to zero, and the impedance from A to B is measured, the result is R_N (since a current source of zero is equivalent to an open circuit). Note that this resistance is in fact the same as the Thévenin equivalent resistance and is found in the same way.



Norton compared with Thévenin

If the values shown here are chosen for I_N and R_N, the voltage from A to B is V_T (when no current is being drawn from the terminals), and the resistance seen from A to B with the current source removed is R_T. This is the exact same as for Thévenin's equivalent circuit, so if the Thévenin equivalent is known the Norton equivalent can be found using diagram C opposite.



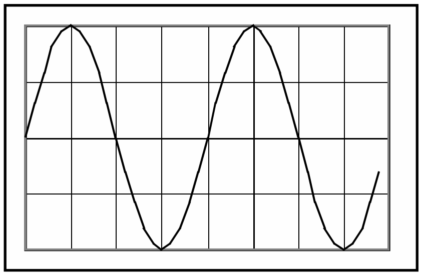
Equivalent Current and Resistance

Comparing the circuit in diagram C with the values found for the Thévenin equivalent circuit found earlier,

$$I_N = \frac{V_T}{R_T} = \frac{6}{12} = 0.5\text{A}$$

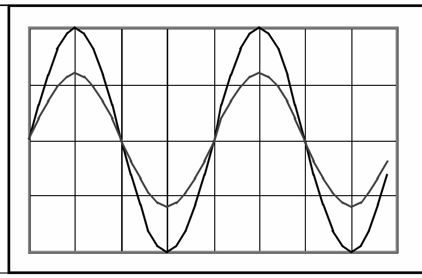
and

$$R_N = R_T = 12\Omega$$



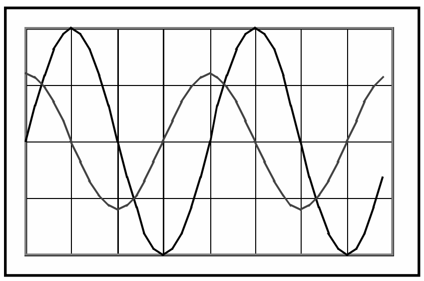
Cause of Phase Shifts

When the input voltage and currents to a circuit composed of resistors, inductors and capacitors are sinusoidal (sines or cosines), then *all* the voltages and currents in the circuit are sinusoids of the same frequency. The only changes that occur to the signals are scaling and delays. Delays are measured as “phase differences”- the original and resultant waveforms are compared, and the difference is found as a fraction of a full period (360°).



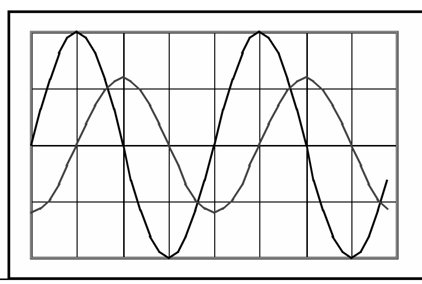
Effect of Resistors

The relationship between the current flowing through a resistor and the voltage applied across it is a simple scaling. Resistors change the magnitude of signals, but do not cause any change in phase.



Effect of Capacitors

Because the current through a capacitor is proportional to the derivative of the voltage across it, a sine voltage results in a cosine current. Since a cosine is the same as a sine moved to the left by a quarter of a full period (look at the diagram), the effect of the capacitor is to introduce a phase shift of 90°, with the current leading in front of the voltage $\cos A = \sin(A + 90°)$.



Effect of Inductors

Because the current through an inductor is proportional to the integral of the voltage across it, a cosine voltage results in a sine current. Since a sine is a cosine moved to the right by a quarter of a full period, the effect of the inductor is to introduce a phase shift of 90°, with the current lagging behind the voltage, $\sin A = \cos(A - 90°)$

Since all the signals in a linear circuit with sinusoidal inputs will also be sinusoids of the same frequency, the only properties of the sinusoid that need to be modelled are its magnitude and phase angle (measured relative to some input chosen by the analyser). Complex numbers can carry this information, since in polar form a complex number is represented as a magnitude and an angle.

Polar form	$r\angle\theta$	$\sqrt{a^2 + b^2}\angle\tan^{-1}\dfrac{b}{a}$
Rectangular form	$r\sin\theta + jr\cos\theta$	$a + jb$

- The complex impedances of capacitors and inductors ($1/j\omega C$ and $j\omega L$) contain information encoding the phase shifts that those components cause (multiplying or dividing by “j” is equivalent to a phase shift of ±90°, since $j = 1\angle 90°$).
- The magnitude of a complex number represents the amplitude of the corresponding sinusoid.
- The angle of a complex number in polar notation is the phase shift of the corresponding sinusoid.
- The frequency of the sinusoid is not represented, since it must be the same for all signals in the circuit.