ALGORITHM 99
# EVALUATION OF JACOBI SYMBOL

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procedure  Jacobi (n, m, r);  value n, m;
integer   n, m, r;
comment   Jacobi computes the value of the Jacobi symbol (n/m),
   where m is odd, by the law of quadratic reciprocity.  The param-
   eter r is assigned one of the values −1, 0, or 1 if m is odd.  If m
   is even, the symbol is undefined and r is assigned the value 2.
   For odd m, the routine provides a test of whether m and n are
   relatively prime.  The value of r is 0 if and only if m and n have
   a nontrivial common factor.  In the special case where m is prime,
   r = −1 if and only if n is a quadratic nonresidue of m;
begin
   integer s;
   Boolean p, q;
   Boolean procedure parity (x);  value x;  integer x;
      comment  The value of the parity function is true if x is
         odd, false if x is even;
      begin
         parity := x ÷ 2 × 2 ≠ x
      end parity;
   if ¬ parity (m) then begin r := 2;  go to exit end;
   p := true;
   loop:  n := n − n ÷ m × m;
          q := false;
          if n ≦ 1 then go to done;
   even:  if ¬ parity (n) then
          begin
            q := ¬ q;
            n := n ÷ 2;
            go to even
          end n now odd;
          if q then if parity ((m↑2 − 1) ÷ 8) then p := ¬ p;
          if n = 1 then go to done;
          if parity ((m − 1) × (n − 1) ÷ 4) then p := ¬ p;
          s := m;  m := n;  n := s;  go to loop;
   done:  r := if n = 0 then 0 else if p then 1 else −1;
exit:    end Jacobi
```