YOSHINDO SUZUKI. *A complete classification of the Δ^1_2-functions.* ***Bulletin of the American Mathematical Society,*** vol. 70 (1964), pp. 246–253.

One of the classical problems of descriptive set theory is that of providing a complete hierarchical classification of the Δ^1_2 sets of real numbers (sometimes referred to as the B_2 or $PCA \cap CPCA$ sets) or, equivalently, of the Δ^1_2 sets of functions of natural numbers. While attempts to construct such a hierarchy have failed, Shoenfield (XXXIV 515) and Suzuki have constructed hierarchies for an analogous class, that of the Δ^1_2 functions.

Suzuki shows, with the aid of the Novikov-Kondô-Addison uniformization theorem, that every Δ^1_2 function is hyperarithmetical in a Π^1_1 singleton, i.e., in a function which is the unique member of a Π^1_1 set of functions, and that the set of hyperdegrees of Π^1_1 singletons is well ordered by the relation "hyperarithmetical in"; furthermore, the successor of the hyperdegree of a Π^1_1 singleton in this ordering is just its hyperjump. To establish these results, ordinals are assigned to Π^1_1 singletons in a manner which refines the ordering of their hyperdegrees. This ordering of the Π^1_1 singletons induces the complete classification of the larger set of all Δ^1_2 functions.

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A. MOSTOWSKI. *Quelques applications de la topologie à la logique mathématique.* ***Topologie,*** Volume I, 4th edn., by Casimir Kuratowski, Państwowe Wydawnictwo Naukowe, Warsaw 1958, pp. 470–477.

This note gives a compact survey of a wide variety of uses of topology in logic, including the analogy between Borel sets and recursive sets, the Rasiowa-Sikorski proof of the Skolem-Gödel theorem, and topological interpretations of many-valued, intuitionistic, and modal logic, with many references.

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RICHARD MONTAGUE. *Theories incomparable with respect to relative interpretability.* ***The journal of symbolic logic,*** vol. 27 no. 2 (for 1962, pub. 1963), pp. 195–211.

It is shown that there is an infinite recursive set of sentences Φ_i ($i < \omega$) in N (the set of true sentences of elementary number theory) such that for each i, $\Phi_i \not\preccurlyeq Cn(\{\Phi_j\}_{j \neq i})$, where $\preccurlyeq$ is the relation of relative interpretability. It follows that there is a continuum of subtheories of N which are mutually incomparable with respect to $\preccurlyeq$. The method of proof is by an extension of the self-referential lemma to prescribed joint self-reference by an infinity of sentences. For the application, each Φ_i can be thought of as expressing that if there is a verification of a relative interpretation of itself in the remaining sentences, then there is already one such for some Φ_k with $k \neq i$.

S. FEFERMAN

JERZY SŁUPECKI and WITOLD A. POGORZELSKI. *A variant of the proof of the completeness of the first order functional calculus.* English with Polish and Russian summaries. ***Studia logica,*** vol. 12 (1961), pp. 125–134.

The authors fix a set of axioms and inference rules for the first-order predicate calculus with negation, disjunction, and universal quantification as primitive (and without identity and operation symbols), and define $Cn'(X)$, for a set of formulas X, as the set of formulas α such that for some finite sequence $(\alpha_1, \cdots, \alpha_n)$, we have $\alpha_n = \alpha$, and for all $i \in \{1, \cdots, n\}$ either $\alpha_i \in X$, or for some j, k less than i, α_i is $(\alpha_j \vee \alpha_k)$, or for some $j < i$, $(\neg\alpha_i \vee \alpha_j)$ is a theorem, or for some $j < i$, α_j has the form $\forall x\phi(x)$ and α_i has the form $\phi(y)$ for some y such that the substitution is proper and y does not occur free in $(\alpha_1 \vee \cdots \vee \alpha_{i-1})$. (More precisely, they use separate letters for free and for bound variables, so that the substitution is automatically proper.) $(\alpha_1, \cdots, \alpha_n)$ is thought of as a proof that α is false when all the members of X are assumed to be false.

This Cn' satisfies the axioms of Tarski's XXXIV 99(7), so by Lindenbaum's theorem (Theorem 12 in Tarski's paper) every "consistent" set of formulas has a "complete" and "consistent" extension, i.e., a maximal superset Y such that $Cn'(Y)$ is not the set of all formulas.

If α is a formula which is not a theorem, and $X = \{\alpha\}$, then X is "consistent" since $Cn'(X)$ contains no theorems. (It can be proved by induction on n that whenever $(\alpha_1, \cdots, \alpha_n)$ is as above, the formula $(\alpha_1 \vee \cdots \vee \alpha_n)$ is not a theorem.) Let Y be a "complete" and "consistent" extension of X. For all formulas ϕ and ψ, $(\phi \vee \psi) \in Y$ iff $\phi \in Y$ and $\psi \in Y$; $\neg\phi \in Y$ iff $\phi \notin Y$, and