

Question

on Problem 6 (11.59) when they ask us to prove

$$\lim_{n \rightarrow \infty} e[n]$$

why do they just evaluate the FT (at $z = 1$) and they don't even evaluate $E(z)$, they evaluate $R(z) = \frac{E(z)}{X(z)}$?

Also, can you only use this rule if the unit step is an input?

Reply

hey guys,

basically, that is an application of the final value theorem for discrete-time... Michael Pihulic pointed this out to me earlier as well. I still think that it is easier to think of the interpretation of the time domain signal of $e[n]$, but here is the relevant theorem:

Final Value Theorem for DT

$$\lim_{n \rightarrow \infty} f[n] = \lim_{z \rightarrow 1} (z - 1)F(z)$$

So if you think about the transfer function $R(z) = \frac{E(z)}{X(z)}$, when the input is a unit step, then $X(z) = \frac{z}{z-1}$, so then $E(z)$ is

$$E(z) = \frac{zR(z)}{z-1}$$

and therefore, we are evaluating

$$\begin{aligned} \lim_{n \rightarrow \infty} e[n] &= \lim_{z \rightarrow 1} (z - 1) \frac{zR(z)}{z - 1} \\ &= \lim_{z \rightarrow 1} zR(z) \end{aligned}$$

-Alex