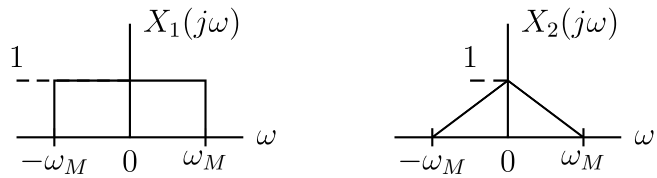
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

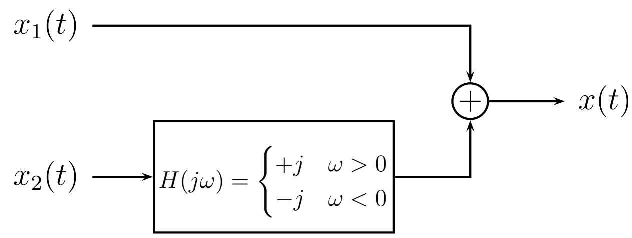
Quiz question

In this problem, we use symmetry to send two real signals within the same frequency band.

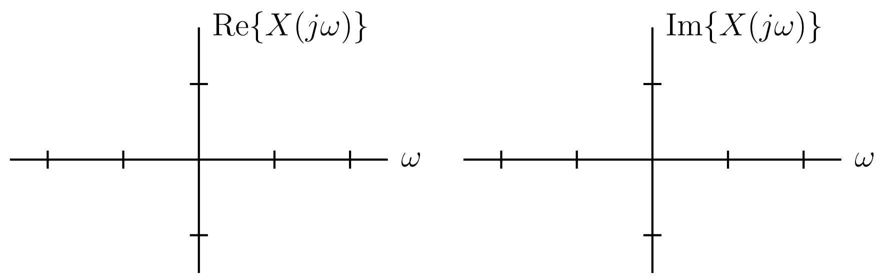
Let $x_1(t)$ and $x_2(t)$ both be *real* and *even* signals which are band-limited to ω_M. The Fourier Transforms of each are shown below:



Suppose we are given the following system which combines $x_1(t)$ and $x_2(t)$ into a single signal:



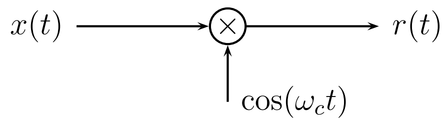
(a) Draw the real and imaginary parts of $X(j\omega)$



(b) Is $x(t)$ completely real, completely imaginary, or neither?

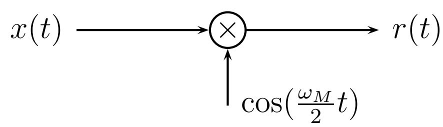
$x(t)$ is **REAL** **IMAGINARY** **NEITHER**

(c) Suppose we want to transmit a modulated version of $x(t)$ using a cosine carrier signal as shown below:

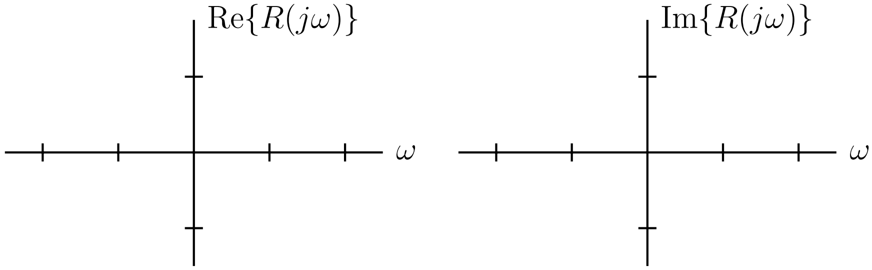


What is the minimum value of ω_c such that there is no aliasing from replicated components of $X(j\omega)$.

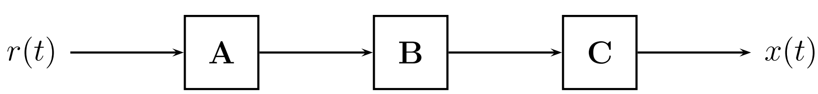
(d) Now suppose that $\omega_c = \frac{\omega_M}{2}$.



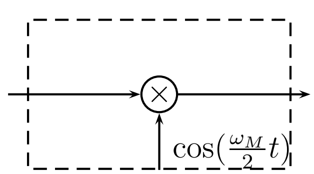
Draw the real and imaginary parts of $R(j\omega)$.



(e) Given the block operations shown below (1,2,3), find the correct order (A,B,C) in which to perform the operations in order to recover $x(t)$ from $r(t)$.

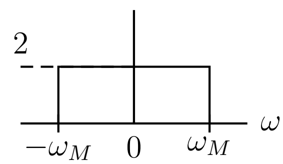


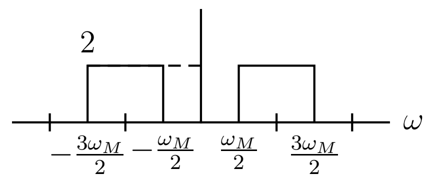
1. Modulation



2. Low-pass Filtering

3. Band-pass Filtering





Operation 1 (Modulation) goes in

BLOCK A **BLOCK B** **BLOCK C**

Operation 2 (Low Pass Filtering) goes in

BLOCK A **BLOCK B** **BLOCK C**

Operation 3 (Band Pass Filtering) goes in

BLOCK A **BLOCK B** **BLOCK C**

(f) Assuming that we have perfectly recovered $x(t)$, then how can we recover $x_1(t)$?

$x_1(t)$ is

$\text{Re}(x(t))$ $\text{Im}(x(t))$ $\text{Odd}(x(t))$ $\text{Even}(x(t))$