MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

Tutorial for the week of December 2nd - December 6th

Alex's Office Hours
Monday 4-6pm
Tuesday 5-7pm

Important Due Dates:

- Problem Set 10 due on Wednesday. (Last one!!!)
- Lab 3 due on Friday

Today

1. A little more on Z-Transforms
2. Work through elements of Prob. 8 on PS 10.

Intentionally blank....

[Example]

Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[n+3] * x_2[-n+1]$$

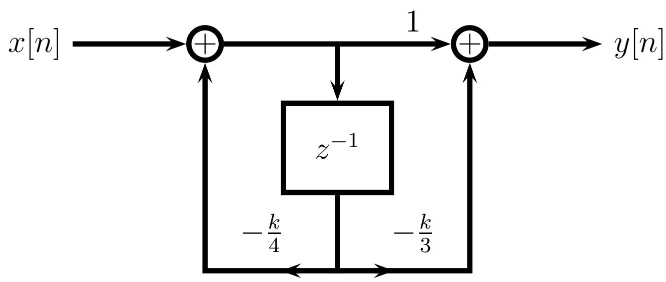
where

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \quad \text{and} \quad x_2[n] = \left(\frac{1}{3}\right)^n u[n]$$

Use the properties of the z-transform to determine the z-transform, $Y(z)$, of $y[n]$.

[Example]

Consider the causal digital filter structure shown below



(a) Find $H(z)$ for this filter. Plot the pole-zero pattern and indicate the region of convergence.

(b) For what values of the k is the system stable?

(c) Determine $y[n]$ if $k = 1$ and $x[n] = (2/3)^n$ for all n.

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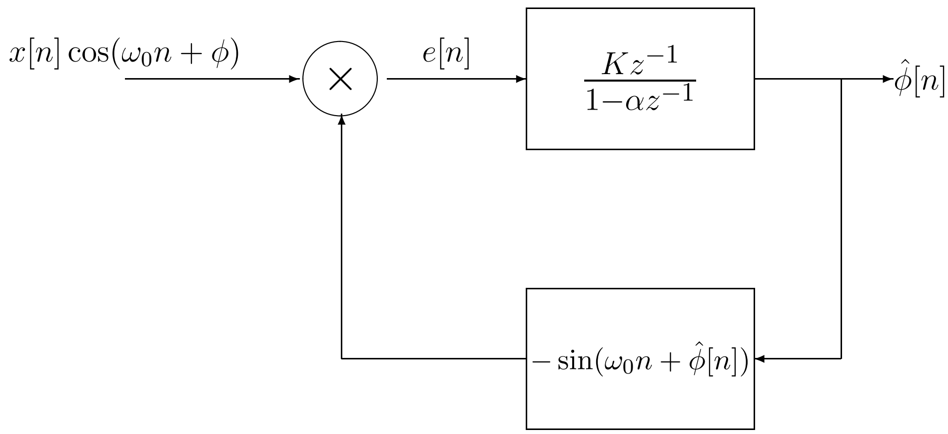
Problem 8

To refresh your memory, the problem is briefly restated below:

Suppose we have a band-limited DT waveform which is being modulated by a cosine carrier of frequency ω_0 and has an unknown phase ϕ.

Our goal is to use a feedback loop to get an estimate of ϕ. In the diagram below, this estimate is denoted as $\hat{\phi}[n]$.



Notice that if $x[n]$ is constant (say at 10), then

$$e[n] = -10\cos(\omega_0 n + \phi)\sin(\omega_0 n + \hat{\phi})$$

We then use the angle sum identity (remember your high school trig!!!)

$$-2\sin(B)\cos(A) = \sin(A+B) + \sin(B-A)$$

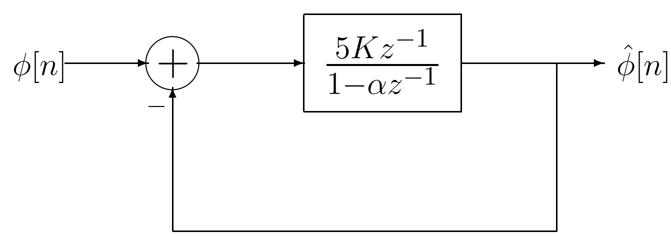
If we let $A = \omega_0 n + \phi$ and $B = \omega_0 n + \hat{\phi}$, then we get that

$$\begin{aligned} e[n] &= -10\cos(A)\sin(B) \\ &= 5(\sin(A+B) + \sin(B-A)) \\ &= 5\sin(2\omega_0 n + \phi + \hat{\phi}) + 5\sin(\hat{\phi} - \phi) \end{aligned}$$

Since the second term can be approximated for very small $\hat{\phi} - \phi$,

$$\sin(\hat{\phi} - \phi) \approx \hat{\phi} - \phi$$

we can linearize the system for slowly varying $\hat{\phi}[n]$.



1. Determine $H(z) = \frac{\hat{\Phi}(z)}{\Phi(z)}$ and $G(z) = \frac{E(z)}{\Phi(z)}$. For what range of values of K is the closed-loop system stable?

2. Suppose that the input $\phi[n]$ is a step, i.e., $\phi[n] = \phi_0 u[n]$ for some ϕ_0. Find an α for which

$$\lim_{n \to \infty} e[n] = 0$$

3. Suppose now that the input $\phi[n]$ is given by $\phi[n] = \cos(\Omega_0 n)$, where $\Omega_0 \approx \pi/2$. Then the output $\hat{\phi}[n]$ is of the form $A \sin(\omega_0 n + \psi)$. Show that

$$A \approx \frac{|5K|}{\sqrt{1 + (5K - 1)^2}}$$

4. Going back to the original, non-linearized system, based on your answer to the previous part, how should we choose K to filter out the harmonic components at $2\omega_0$ in $e[n]$.

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