MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

TUTORIAL IN THE WEEK OF 9TH OF DECEMBER

Final Exam It will be held on 12/18 from 1:30 pm to 4:30 pm at du Pont. If you have any conflict and have not been notified by us, please let Prof. Boning as soon as possible.

Final Exam Review sessions There will be two **identical** review sessions.

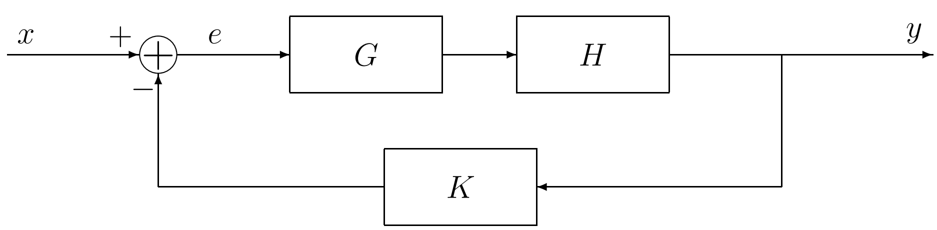
- Thursday, 12/12 in 34-101 from 3:oo pm to 5:oo pm by Matt and Ernie
- Friday, 12/13 in 34-101 from 3:oo pm to 5:oo pm by Alex and Mario

Office Hours The regularly scheduled OH this week is

- Monday 7:00 - 9:00 pm
- Wednesday 8:00 - 10:00 pm

or by appointment. Alex will hold his OH on Monday from 3:00 - 5:00 pm and Tuesday from 6:00 - 8:00 pm.

Materials Covered Feedback system - The general structure of the system considered here has the following form:



Systems can be either CT or DT as specified in each problem.

Problem 1 Find the transfer functions $\frac{Y(s)}{X(s)}$ or $\frac{Y(z)}{X(z)}$ for the following systems. Find the ranges of controller gains so that the closed loop systems are stable.

1. $H(s) = \dfrac{s-3}{(s+1)(s+10)}, \quad G(s) = 1, \quad K(s) = K.$

2. $H(s) = \dfrac{1}{s+1}, \quad G(s) = \dfrac{1}{s+5}, \quad K(s) = K_p + \dfrac{K_i}{s}.$

3. $H(z) = \dfrac{z^{-1}}{1 - \frac{1}{2}z^{-1}}, \quad G(z) = \dfrac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}, \quad K(z) = K$

Work Space

Problem 2 Consider the DT system with $H(z) = \frac{2}{z-1}$, $G(z) = 1$, and $K(z) = K$ is a positive constant to be determined.

(a) Obtain the transfer functions $Q(z) = \frac{Y(z)}{X(z)}$ and $P(z) = \frac{E(z)}{X(z)}$.

(b) For what range of K is the closed-loop system stable ?

(c) Choose K such that $e[n] = 0$ for $n \geq 1$ when $x[n] = u[n]$.

(d) Suppose $K(z) = G(z) = 1$. For this part of the problem, we assume $H(z)$ is unknown. However we know that the error to the step input, *i.e.*, $e[n]$ when $x[n] = u[n]$, is described as below:

$$e[n] = \sum_{k=0}^{N-1} a_k \delta[n-k],$$

where a_k are some constants determined from a step response experiment. Assume there is no pole-zero cancellation. What is the order of the unknown system function $H(z)$? (**Hint:** Express $H(z)$ in terms of $E(z)$).

This phenomena, convergence of the error signal to 0 in a finite time , is called dead-beat. This happens only for DT systems. (Why not for CT systems ?) This technique can be used to estimate the order of an unknown system as well as its system function.

Work space

Problem 3 Consider the following CT system with

$$H(s) = \frac{s-10}{(s+1)(s+5)},$$

and $K(s) = 1$.

(a) Sketch the pole-zero diagram and ROC of $H(s)$. Is this system stable ?

(b) In the following, suppose $G(s) = G_p$. What is the closed-loop system function $Q(s) = \frac{Y(s)}{X(s)}$?

(c) Assume that the system is stabilized. Express the natural frequency ω_n and the damping ratio ζ in terms of G_p for $Q(s)$.

(d) For what values of G_p is at least one of the poles of $Q(s)$ located on $j\omega$ axis ?

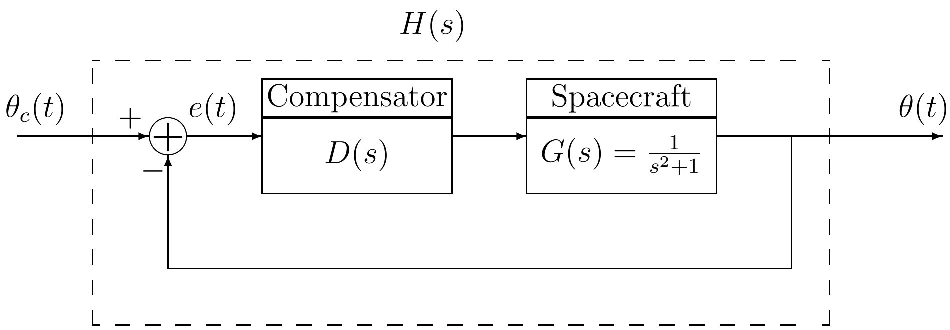
(e) For what range of G_p is $Q(s)$ stable ? Is $Q(s)$ stable when $|G_p|$ is very large ? Explain.

(f) Suppose $x(t) = u(t)$. Find the steady state value of $e(t)$, *i.e.*, $e(\infty) = \lim_{t\to\infty} e(t)$ for the two cases:

1. $G_p = -2$.
2. $G_p = 2$.

Work Space

Problem 4 A simplified model of spacecraft attitude CT control system can be defined as below:



We would like to see what kind of controller we need to meet the design specification.

(a) Is $G(s)$ stable ?

(b) Suppose $D(s) = K$, where K is a real number. This configuration is called a *Proportional (P) controller.*

1. Draw the root locus for $G(s)$.
2. Find an expression for the transfer function from $\theta_c(t)$ to $\theta(t)$, *i.e.*, $H(s) = \dfrac{\Theta(s)}{\Theta_c(s)}$.
3. Can we stabilize the system ?

Work Space

(c) Now, suppose $D(S) = K_1 s + K_2$, where K_1 and K_2 are real numbers. This configuration is called *Proportional Derivative (PD) controller*.

1. Find an expression for the transfer function $H(s)$.
2. What are the ranges of K_1 and K_2 such that the resultant closed loop system is stable ?
3. What is the steady state error, $e(t) = \theta_c(t) - \theta(t)$, where $t \to \infty$, to the unit step input $\theta_c(t) = u(t)$?

(d) Suppose $D(s) = K_1 s + K_2 + \frac{K_3}{s}$, where K_1, K_2, and K_3 are real numbers. This configuration is called *Proportional Integral Derivative (PID) controller*.

1. Find an expression for the transfer function $H(s)$.
2. What are the range of K_1, K_2, and K_3 such that the resultant closed loop system is stable ?
3. What is the steady state error, $e(t) = \theta_c(t) - \theta(t)$, where $t \to \infty$, to the unit step input $\theta_c(t) = u(t)$? Is it a function of any gains ?

Work Page