MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

Tutorial for the week of October 21st - October 25th

Alex's Office Hours
Monday 3-5pm
Tuesday, 6-8pm

Important Due Dates:

- Problem Set 5 due on Wednesday.
- Quiz 1 will be handed back in section on Wednesday.

Today

1. Test Recap
2. Continuous Time Fourier Transform
3. Discrete Time Fourier Transform

CT Fourier Transform

The Continuous Time Fourier transform is a natural extension of the concept of Fourier Series for continuous time periodic signals. Recall that any continuous periodic signal,

$$x(t) = x(t+T)$$

with fundamental period T, has frequency components at only discrete multiples of the fundamental frequency,

$$\pm k\omega_0 \qquad (\omega_0 = \frac{2\pi}{T})$$

The basis function for each frequency is the complex exponential, $e^{jk\omega_0 t}$, and the Fourier Series coefficient, a_k, is a measure of how much of that frequency is present in a particular signal. This is where the Fourier representation of the time domain signal comes from:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

If we look beyond periodic signals to general signals which are made up of more than just frequencies at multiples of some fundamental frequency, then we get to the Fourier Transform.

Property	Fourier Series	Fourier Transform
Representation	a_k	$X(j\omega)$
Synthesis	$x(t) = \sum a_k e^{jk\omega_0 t}$	$x(t) = \int X(j\omega)d\omega$
Meaning	a_k amount of frequency at $k\omega_0$	$X(j\omega)$ amount of frequency at ω

Since periodic signals only have frequency components at integer multiples of ω_0. This shows us that the Fourier Series are a special case of the Fourier Transform. Periodic signals have Fourier Transforms which have discrete frequency components.

$$\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt && \text{Analysis equation} \\
x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega && \text{Synthesis equation} \\
\int_{-\infty}^{\infty} |x(t)|^2 dt &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega && \text{Parseval's relation}
\end{aligned}$$

Example

Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

1. $x_1(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$
2. $x_2(t) = (\frac{\sin(2t)}{\pi t})^2$

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DT Fourier Transform

Although the concepts of periodicity and frequency are the same for Continuous Time and Discrete Time, there are two important differences.

(1) The DTFT is periodic with period 2π. (Just like the DTFS were periodic)

(2) The DTFT analysis equation is a sum of discrete complex exponentials, rather than an integral.

$$\begin{aligned}
X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} && \text{Analysis equation} \\
x[n] &= \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega && \text{Synthesis equation} \\
\sum_{n=-\infty}^{\infty} |x[n]|^2 &= \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega && \text{Parseval's relation}
\end{aligned}$$

Example

Compute the Fourier transform of the following signals:

1. $x_1[n] = (\frac{\sin(\pi n/5)}{\pi n})\cos(\frac{7\pi}{2}n)$.
2. $x_2[n] = (\frac{1}{2})^n u[n-2]$.

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Example

Find $x[n]$ for the following DTFT's:

1. $X_1(e^{j\omega}) = \cos 2\omega + \sin^2 \omega$.

2. $X_2(e^{j\omega}) = \dfrac{1 - \frac{1}{8}e^{-j3\omega}}{1 - \frac{1}{2}e^{-j\omega}}$.

Example

A signal $x[n]$ with Fourier transform $X(e^{j\omega})$ has the property that

$$\left(x[n] \sum_{k=-\infty}^{\infty} \delta[n-4k] \right) * \left(\frac{\sin \frac{\pi}{4} n}{\frac{\pi}{4} n} \right) = x[n].$$

1. What is the Fourier transform of $x[n] \sum_{k=-\infty}^{\infty} \delta[n-4k]$?

2. For what values of ω is it guaranteed that $X(e^{j\omega}) = 0$?

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Work Space