MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2002

Tutorial for the week of November 18th - November 22nd

Alex's Office Hours
Monday 3-5pm
Tuesday, 6-8pm

Important Due Dates:

- Problem Set 8 due on Wednesday.

Today

1. Modulation Example
2. Laplace Transform

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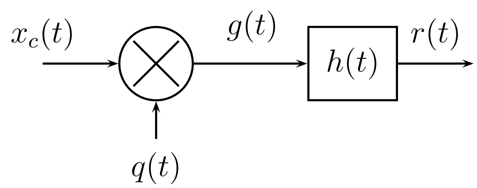
Modulation

Amplitude modulation is actually just like sampling. Start with two signals, $x(t)$ and $p(t)$, and multiply them together at each point in time. The only difference is that $p(t)$ is no longer restricted to being a periodic impulse train. We sometimes refer to $p(t)$ as the *carrier* signal.

[Example]

Suppose we are given the system shown below



The continuous time signal $x_c(t)$ is bandlimited to ω_m.

Let $g(t) = x_c(t)q(t)$, where $q(t)$ is a periodic signal with period T, defined as

$$q(t) = \begin{cases} e^{-2t} & \text{for } \; 0 \leq t \leq T/2 \\ 0 & \text{for } \; T/2 < t < T \end{cases}$$

The signal $g(t)$ is passed through an ideal bandpass filter with impulse response

$$h(t) = \cos(4\pi t/T)\frac{\sin(\pi t/T)}{\pi t/T}$$

Let $r(t)$ be the output of the bandpass filter. Assume that $\omega_s = 2\pi/T > 2\omega_c$.

(a) Find the Fourier transform of $r(t)$ in terms of $X_c(j\omega)$.

(b) Draw a block diagram of a system that can recover $x_c(t)$ from $r(t)$.

Work Space

Laplace Transform

As we saw in lecture, the Laplace transform is an extension of the continuous time Fourier Transform which allows us to deal with a wider range of signals. The Laplace transform takes the form:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Notice that this reverts back to the CTFT when we let $s = j\omega$. In general, though, s can be any complex number $\sigma + j\omega$, and is not restricted to the $j\omega$-axis. This means that we have to be careful in specifying the values of s for which the Laplace integral converges.

We specify the range of valid values of s as the Region of Convergence (ROC) for $X(s)$. Note that it is essential to include the ROC when writing out the Laplace transform for $x(t)$, otherwise, the inverse transform is not well specified (ie, two time domain signals can have the same ROC).

Finally, until you take complex analysis, it is a non-trivial task to find the inverse Laplace transform using an integral. Instead, we usually make use of tables to look up inverses given

[Example 1]

Find the Laplace Transform of the following functions:

(a) $e^{-t}u(t) + e^{2t}u(-t)$

(b) $e^{-3|t|} + e^{-4t}u(-t)$

[Example 2]

Find the inverse Laplace Transforms of the following functions.

(a) $\frac{s+1}{s^2+2s+5}$ $Re\{s\} > -1$

(b) $\frac{s^2+s}{s^2+5s+4}$ ROC is to the right of the rightmost pole.

Work Space

[Example 3]

Given an LTI system with characteristic function $H(s)$,

$$H(s) = \frac{1}{(s^2+4s+3)(s-p)}$$

1. Find the range of values for p such that $h(t)$ is causal.
2. Find the range of values for p such that $H(s)$ is stable, and also specify the ROC associated with these choices of p.

[Example 4]

Suppose that we are given the following information about an LTI system:

1. The system is causal.
2. The system function is rational and has only two poles, at $s = -1$ and $s = -10$.
3. If $x(t) = 1$, then $y(t) = 0$.
4. The value of the impulse response at $t = 0^+$ is 1.

From this information, determine the system function, $H(s)$, of the system.

Work Space