Fast Statistical Analysis of Rare Circuit Failure Events in High-Dimensional Variation Space

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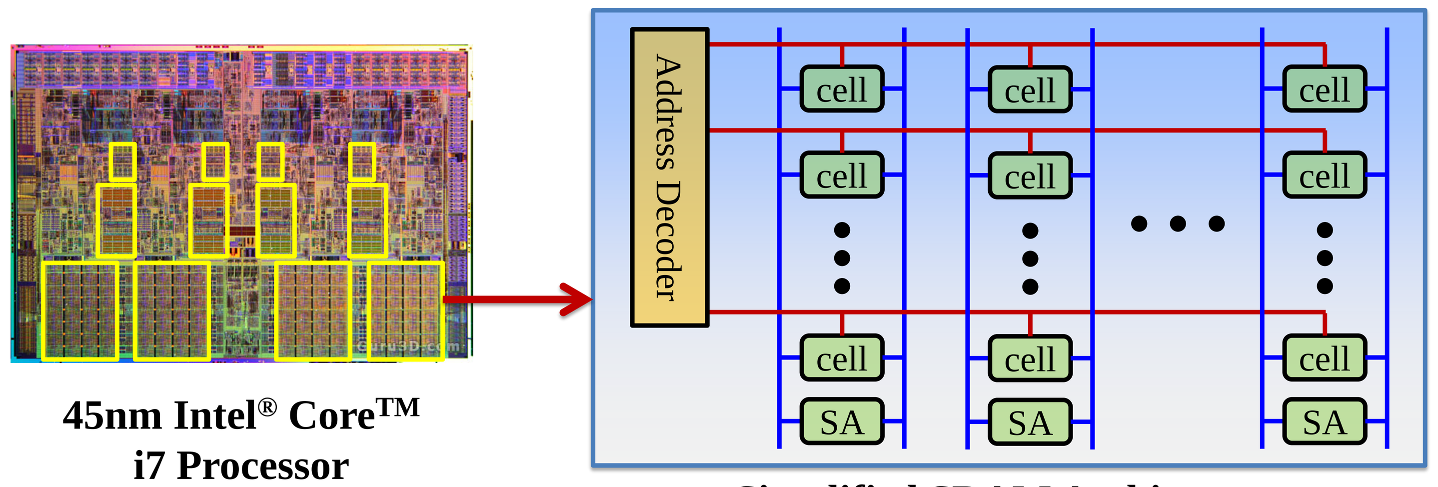
Electrical & Computer ENGINEERING

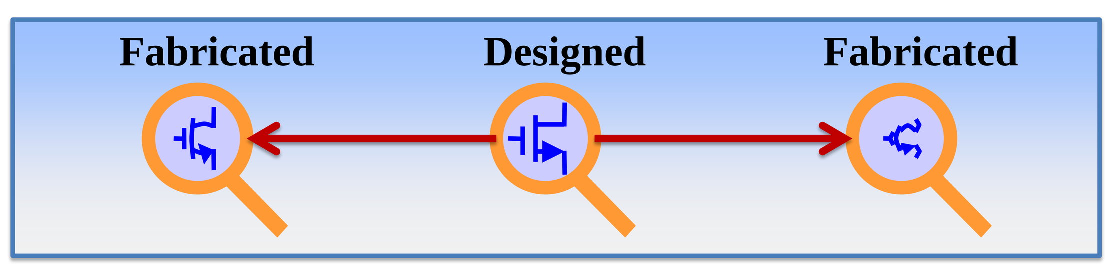
Motivation

- ✓ **Scaling**: a large number of replicated components are integrated on the chip



45nm Intel® Core™ i7 Processor **Simplified SRAM Architecture**

- ✓ **Challenge**: all the components need to function correctly under large process variations



Process Variation

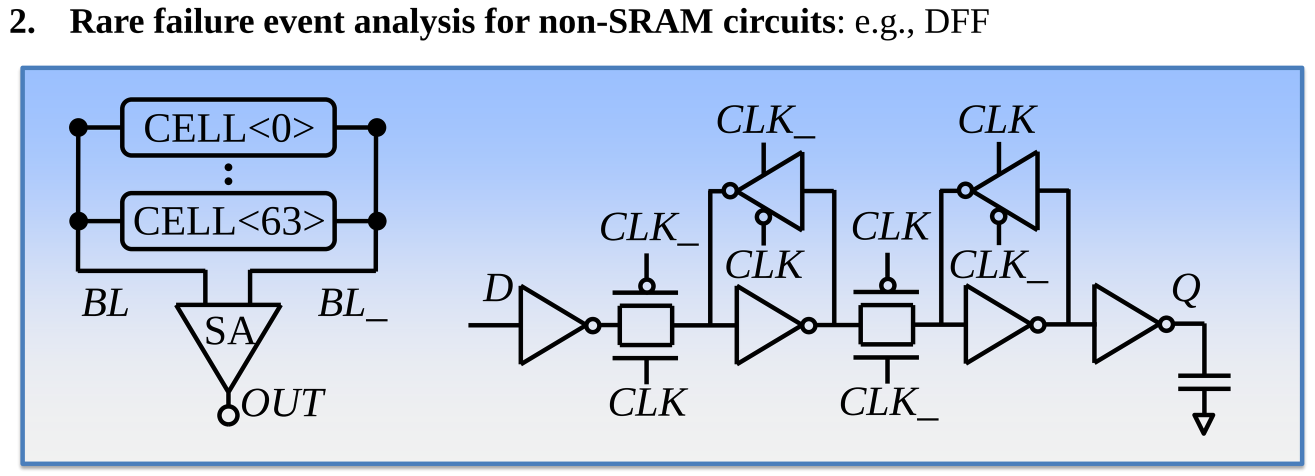
- ✓ **Yield requirement**: each component must be extremely robust under process variations
  - The failure event of each component must be rare
- ✓ **Time to market**: **fast** statistical tools are highly desired to analyze the **rare** failure event

Existing Approaches

- ✓ **Brute-force Monte Carlo (MC)** [1]
  - Pros: no dimensionality issue
  - Cons: **not efficient**
- ✓ **Importance sampling (IS)** [2]: bias the sampling distribution
  - Pros: efficient in low-D space
  - Cons: difficult to find an appropriate biased sampling distribution in **high-D** space
- ✓ **Statistical blockade** [3]: classifier based
  - Pros: efficient in low-D space
  - Cons: expensive to construct an accurate classifier in **high-D** space
- ✓ **Deterministic approach** [4]: integrate the failure region in the variation space
  - Pros: efficient in low-D space
  - Cons: expensive to accurately describe the failure region in **high-D** space

Challenge: High-Dimensionality

- ✓ In the past, rare failure event analysis was mainly focused on SRAM bit cell ← **low-D**
- ✓ Recently, rare failure event analysis in high-D becomes more and more important
  1. **Dynamic SRAM bit cell stability related to peripherals**: many transistors from multiple SRAM bit cells and their peripheral circuits must be considered
  2. **Rare failure event analysis for non-SRAM circuits**: e.g., DFF



SRAM **DFF**

Subset Simulation (SUS): Continuous Performance Metric

- ✓ **Idea**: find K subsets $\{\Omega_k; k = 1, 2, \cdots, K\}$ in **x**-space, and estimate the rare failure rate P_F by the conditional probabilities $\{P_k; k = 1, 2, \cdots, K\}$

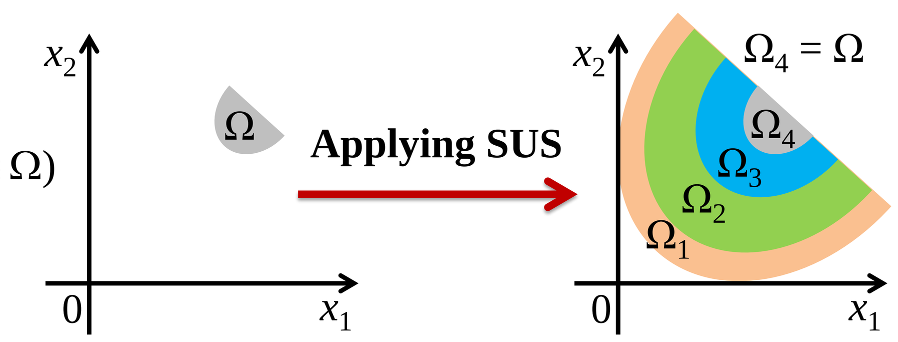
$$\Omega_1 \supseteq \Omega_2 \supseteq \cdots \supseteq \Omega_{K-1} \supseteq \Omega_K = \Omega$$

$$P_F = \Pr(\mathbf{x} \in \Omega) = \underbrace{\Pr(\mathbf{x} \in \Omega_1)}_{P_1} \prod_{k=2}^{K} \underbrace{\Pr(\mathbf{x} \in \Omega_k | \mathbf{x} \in \Omega_{k-1})}_{P_k} = \prod_{k=1}^{K} P_k$$

x : process variation
Ω : interested failure region
P_F : interested failure rate

Example

$P_F = \Pr(\mathbf{x} \in \Omega) = 10^{-4}$

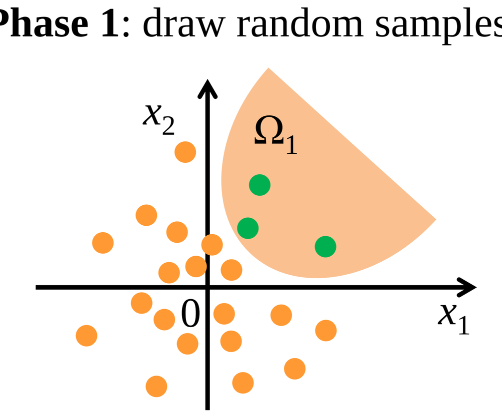


$P_1 = \Pr(\mathbf{x} \in \Omega_1) = 0.1$
$P_2 = \Pr(\mathbf{x} \in \Omega_2 | \mathbf{x} \in \Omega_1) = 0.1$
$P_3 = \Pr(\mathbf{x} \in \Omega_3 | \mathbf{x} \in \Omega_2) = 0.1$
$P_4 = \Pr(\mathbf{x} \in \Omega_4 | \mathbf{x} \in \Omega_3) = 0.1$

Estimate P_F → **Estimate P_1, P_2, P_3, P_4**

- ✓ **Question**: how to estimate these conditional probabilities $\{P_k; k = 1, 2, \cdots, K\}$?

Phase 1: draw random samples from PDF $f(\mathbf{x})$ and estimate $P_1 = \Pr(\mathbf{x} \in \Omega_1)$

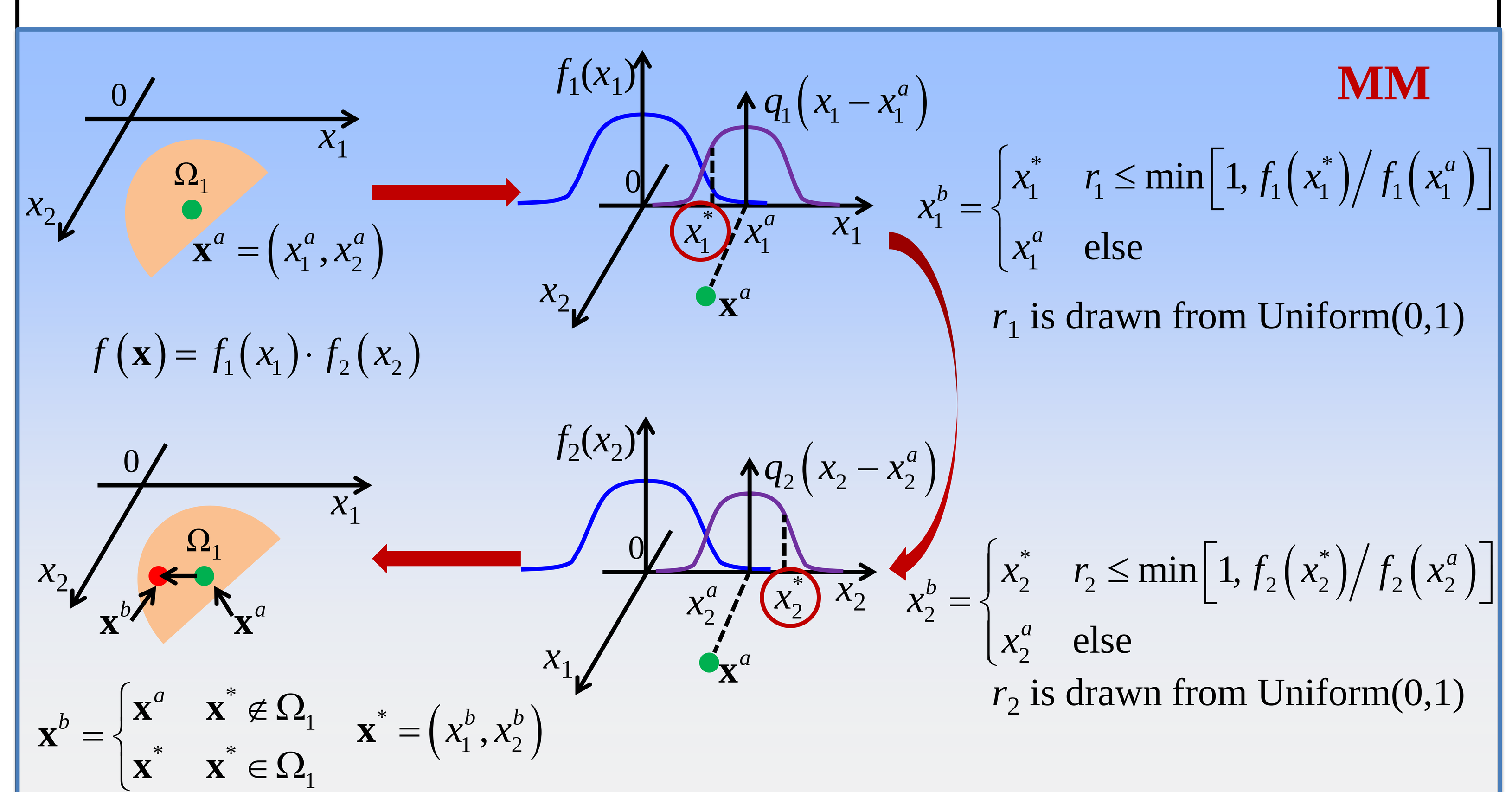


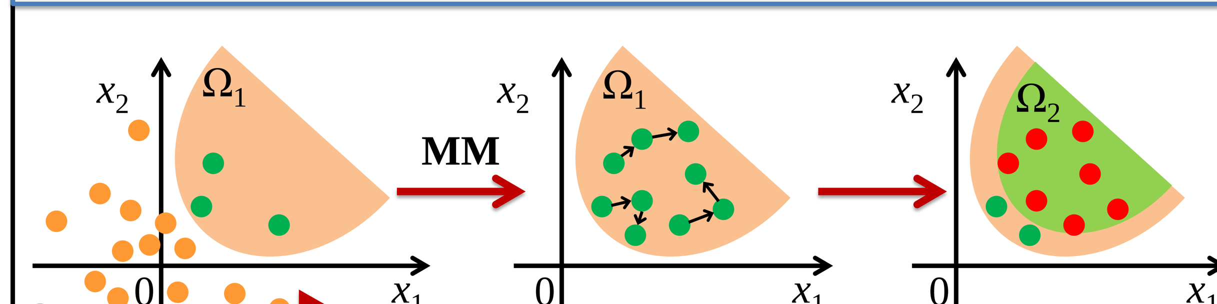
x_1, x_2 are generally modeled as independent Normal random variables

N_Y: # of yellow points N_G: # of green points

$$P_1^{SUS} = \frac{N_G}{N_G + N_Y}$$

Phase 2: draw random samples from $f(\mathbf{x} | \mathbf{x} \in \Omega_1)$ and estimate $P_2 = \Pr(\mathbf{x} \in \Omega_2 | \mathbf{x} \in \Omega_1)$
$f(\mathbf{x} | \mathbf{x} \in \Omega_1)$ is **unknown** in advance. **Modified Metropolis** (MM) algorithm is applied to generate random samples that follow $f(\mathbf{x} | \mathbf{x} \in \Omega_1)$





N_G: # of green points
N_R: # of red points

$$P_2^{SUS} = \frac{N_R}{N_G + N_R}$$

Phase 3: similar to Phase 2

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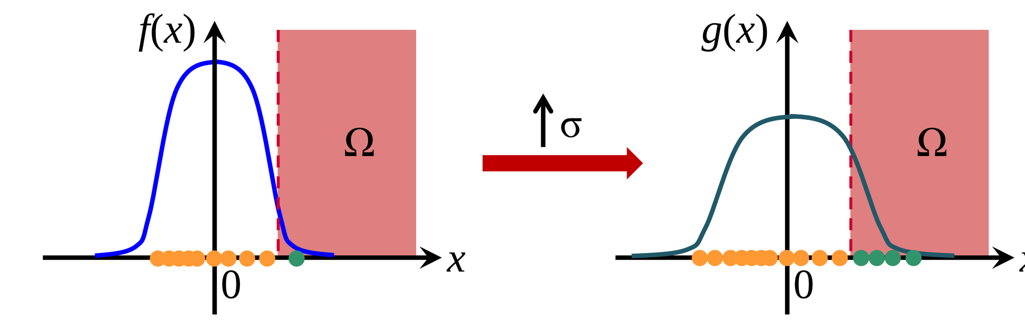
Scaled-Sigma Sampling (SSS): Discrete Performance Metric

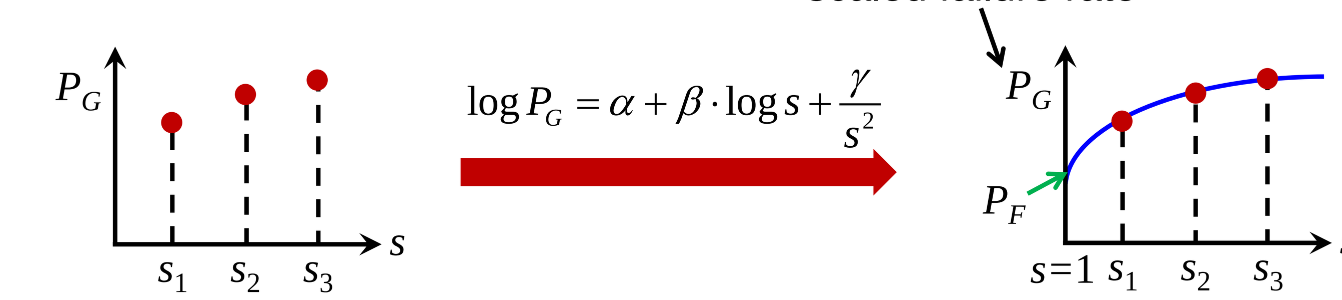
- ✓ **Idea:** it is much easier to estimate P_G than P_F if s is large enough



$\sigma_g = s \cdot \sigma_f$
scaling factor

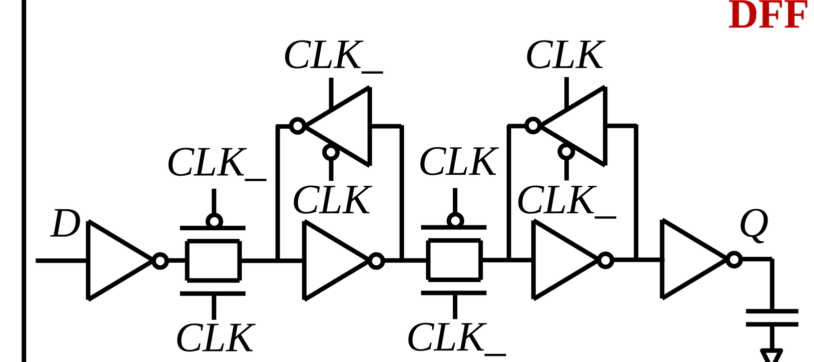
$f(x)$: original probability density function P_F : failure rate by sampling $f(x)$
$g(x)$: scaled probability density function P_G : failure rate by sampling $g(x)$
σ_f : σ of $f(x)$ σ_g : σ of $g(x)$

- ✓ **Proposed SSS**

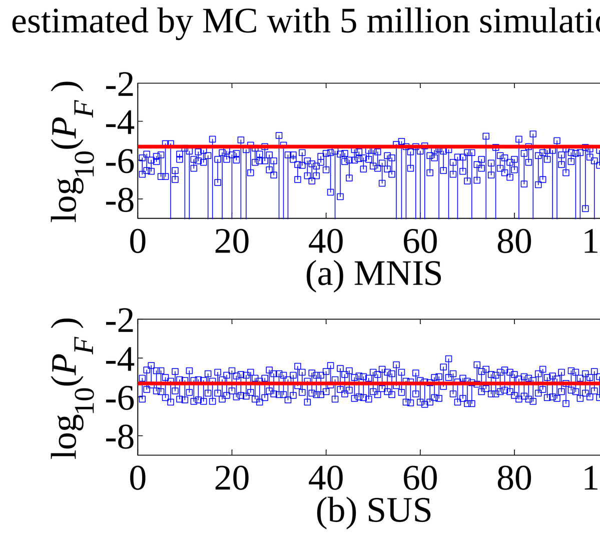


- ✓ **Experimental results**

DFF

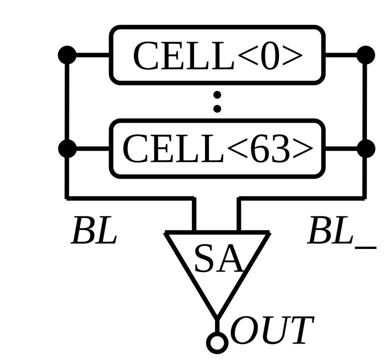


- 45nm CMOS technology
- 200 independent random variables
- The delay from CLK→Q is performance of interest
- 100 95% confidence intervals (CIs) are calculated from 100 runs with 5500 simulations in each run
- The "golden" failure rate is 4.8×10^{-6} that is estimated by MC with 5 million simulations

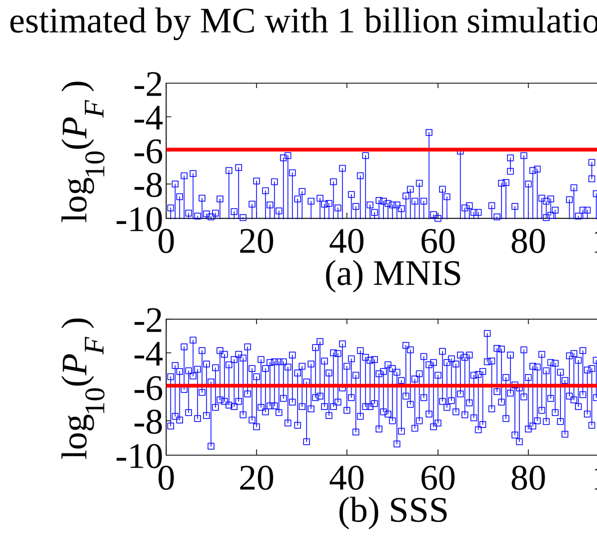


The 95% CIs (blue bars) of the DFF example for: (a) MNIS [2] and (b) SUS. The red line represents the "golden" failure rate.

SRAM



- 45nm CMOS technology
- 384 independent random variables
- The output of SA is performance of interest
- 100 95% confidence intervals (CIs) are calculated from 100 runs with 6000 simulations in each run
- The "golden" failure rate is 1.1×10^{-6} that is estimated by MC with 1 billion simulations



The 95% CIs (blue bars) of the SRAM example for: (a) MNIS [2] and (b) SSS. The red line represents the "golden" failure rate.

[1] C. Bishop, *Pattern Recognition and Machine Learning*. Prentice Hall, 2007.
[2] M. Qazi, et al., "Loop flattening & spherical sampling: highly efficient model reduction techniques for SRAM yield analysis," *IEEE DATE*, 2008, pp. 801-806.
[3] A. Singhee and R. Rutenbar, "Statistical blockade: very fast statistical simulation and modeling of rare circuit events, and its application to memory design," *IEEE TCAD*, vol. 28, no. 8, pp. 1176-1189, Aug. 2009.
[4] C. Gu and J. Roychowdhury, "An efficient, fully nonlinear, variability aware non-Monte-Carlo yield estimation procedure with applications to SRAM cells and ring oscillators," *IEEE ASP-DAC*, pp. 754-761, 2008.
[5] S. Sun, et al., "Fast statistical analysis of rare circuit failure events via scaled-sigma sampling for high-dimensional variation space," *IEEE ICCAD*, 2013, pp. 478-485.
[6] S. Sun and X. Li, "Fast statistical analysis of rare circuit failure events via subset simulation in high-dimensional variation space," *IEEE ICCAD*, 2014, accepted.