Hybrid Data Fusion for 3D Localization Under Heavy Disturbances

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***Abstract*— This work addresses the problem of stochastic data fusion for systems liable to heavy disturbances, which denote environmental perturbations strong enough to modify the system's internal structure, including signal interference, sensor faults, physical structure modification, and many other sources of disturbance. In these such cases, traditional filtering methods usually fail to provide reliable estimates because of the highly corrupted sensor measurements. This work proposes to model the data fusion problem for heavily disturbed systems through a hybrid systems modeling framework and presents an online hybrid stochastic filter capable of tracking the system's state in unfavorable operating conditions. Simulated and experimental results compare the proposed filter's performance with the traditionally used Extended Kalman Filter (EKF) and show its usefulness as a robust localization filter for an Unmanned Aerial Vehicle (UAV) designed for aerial power lines inspection.**

I. INTRODUCTION

NAVIGATION and 3D localization for robotic systems are problems of utmost importance [1], specially for robots operating in outdoor, uncontrolled environments. Providing reliable estimates of the system's pose involves combining data, by means of filtering algorithms, from proprioceptive and exteroceptive sensors, which may provide information about position, orientation, velocities or any other spatial variable of interest. Depending on the application and the types of sensors available, different strategies may be used for robots localization. For example, [2] applies Kalman filtering to the problem of positioning and heading control of ships and offshore rigs using inertial, GPS, and compass measurements, whilst [3] extracts information from a stereo visual system in order to simultaneously localize a robot and build a map of its surroundings. These examples and most works concerning data fusion implicitly assume that "two sensors are better than one", an idea made famous in the robotics field by [4]. However, this statement is true only for sensors working correctly, i.e., with unbiased measurements. In the case of robots operating in real life, faulty measurements from just one sensor may degrade the whole localization system's performance.

Detecting failures and anomalous behavior in dynamic systems have long been a matter of great interest, as can be seen in the survey presented in [5]. One way of dealing with disturbed measurements consists of considering different sets of measurement equations in order to describe a given system's output. Nevertheless, having different kinds of measurement equations introduces the problem of determining which set is the most likely to be true at each time instant. In the field of Simultaneous Localization and Mapping (SLAM), for example, [6] and derived works define the set of measurement equations based on the correspondence between local and global maps, which is determined by means of statistical tests that are independent from the localization filter. On the other hand, the hybrid systems approach proposed in this work elegantly incorporates measurement compatibility tests, which naturally determine the most adequate set of measurement equations at each time instant without the need of additional verifications.

Hybrid systems denote a class of dynamical systems whose behavior combines continuous and discrete state variables [7] and extensive work has been done in the field of state estimation for this kind of system. For instance, [8], [9] apply particle filters as state estimators for hybrid systems, while [10] uses robust Kalman filtering techniques. One of the most famous multiple model estimation algorithms, the Interacting Multiple Model (IMM), is introduced in [11], [12] and target tracking applications using hybrid systems are presented in [13], [14].

Motivated by the problem of 3D localization for an Unmanned Aerial Vehicle (UAV) designed for aerial power lines inspection, this contribution lies in the description of the data fusion problem for heavily disturbed systems through the hybrid systems modeling framework [15]. Due to the strong electromagnetic interference generated by the power lines and occasional sensor faults, the traditional stochastic filters first evaluated for state estimation using the aircraft's sensors were not able to provide reliable information. Instead of redesigning the instrumentation system, leading to higher costs and delays in the project, the solution to cope with environmental disturbances was developing a robust localization filter able to deal with highly corrupted measurements, making it capable to track the UAV's state in conditions where other filters fail.

This paper is organized as follows. Section II models the localization system designed to track the UAV's state. The hybrid data fusion algorithm is presented in Section III and simulated as well as experimental results comparing its performance with the Extended Kalman Filter (EKF) are shown in Section IV. Finally, conclusions are presented in Section V.

II. LOCALIZATION SYSTEM

This section describes the mathematical model used in the UAV's localization system. An inertial navigation system (INS) composed of a 6 degrees of freedom (DOF) Inertial Measurement Unit (IMU) measures angular and linear rates

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through accelerometers and gyros. A three-axis magnetometer and a barometric altimeter are used, respectively, to correct the aircraft's attitude and altitude. Finally, a global navigation satellite system (GNSS) provides measurements of position and velocity through a GPS receiver. For the localization system's equations, consider the coordinate frames shown in Figure 1.

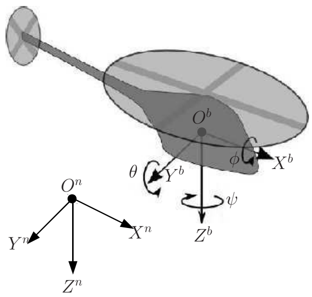


Fig. 1. Body (b) and reference (n) coordinate frames.

A. Translation

Let p^b be the representation of the IMU's position vector in the body-fixed frame b and p^n be the same vector described in the earth-fixed frame n. Considering that b translates and rotates with respect to n, it follows that

$$p^n = C_n^b p^b + t_n^b, \tag{1}$$

where C_n^b denotes the rotation matrix from b to n and $t_n^b = O^b - O^n$ is the displacement between the origins of b, O^b, and of n, O^n. Since the IMU's accelerometers provide measurements in b, a transformation is necessary to describe these accelerations in n. Differentiating (1) twice yields

$$v^n = \dot{p}^n = \dot{C}_n^b p^b + C_n^b \dot{p}^b + \dot{t}_n^b, \tag{2}$$
$$a^n = \dot{v}^n = \ddot{C}_n^b p^b + 2\dot{C}_n^b \dot{p}^b + C_n^b \ddot{p}^b + \ddot{t}_n^b. \tag{3}$$

In most cases, p^b is a fixed point in the structure, yielding $\dot{p}^b = \ddot{p}^b = 0$. In order to further simplify (1)-(3), p^b can be chosen to coincide with O^b and the body's center of mass, making $p^b = 0$. In these such cases, (3) can be rewritten as

$$a^n = \ddot{t}_n^b = C_n^b a^b, \tag{4}$$

where $a^b = [a_x^b \ a_y^b \ a_z^b]^T$ is the acceleration measured in b. Since accelerometers measure the specific force f^b acting on the body instead of real accelerations, (4) is changed to

$$a^n = C_n^b f^b + g^n, \tag{5}$$

where f^b corresponds to the IMU readings and g^n is the local gravitational field measured in n.

Choosing $[p^n \ v^n]^T$ as the state vector to represent the motion of O^b with respect to n, the body's translation can be described as

$$\begin{bmatrix} \dot{p}^n \\ \dot{v}^n \end{bmatrix} = \begin{bmatrix} 0 & \mathbb{I} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p^n \\ v^n \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C_n^b & \mathbb{I} \end{bmatrix} \begin{bmatrix} f^b \\ g^n \end{bmatrix} + \epsilon_t, \tag{6}$$

where $\mathbb{I}$ denotes the identity matrix of appropriate dimensions. The term ϵ_t models disturbances due to modeling errors and sensor noise.

B. Rotation

The body's attitude, i.e., the orientation of b with respect to n, is represented by means of hypercomplex quaternions vectors [16]. Although this representation is not as intuitive as Euler angles, which directly denote the body's rotation angles around each axis of b, using quaternions exhibits many advantages concerning computational costs and singularities in the representation of rotations [1].

Let $q_n^b = [q_0 \ q_1 \ q_2 \ q_3]^T$, $\|q_n^b\| = 1$, be the quaternion representing the orientation of b with respect to n. The equation relating q_n^b to its corresponding rotation matrix is

$$C_n^b(q_n^b) = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}.$$

As the body rotates, the IMU's gyros provide angular rate measurements ω_x, ω_y, and ω_z around axis X^b, Y^b, and Z^b, respectively (Figure 1). Thus, the body's rotation can be described as

$$\begin{aligned} \dot{q}_n^b &= -\frac{1}{2} \begin{bmatrix} 0 & \omega_x & \omega_y & \omega_z \\ -\omega_x & 0 & -\omega_z & \omega_y \\ -\omega_y & \omega_z & 0 & -\omega_x \\ -\omega_z & -\omega_y & \omega_x & 0 \end{bmatrix} q_n^b + \epsilon_q, \\ &= -\frac{1}{2} W q_n^b + \epsilon_q. \end{aligned} \tag{7}$$

Similarly to (6), a disturbance term ϵ_q is added to cope with modeling errors and sensor noise.

C. Corrective measurements

Denoting

$$r_k = \left[(q_{n,k}^b)^T \ (p_k^n)^T \ (v_k^n)^T \right]^T \tag{8}$$

as the system's state vector at the k-th sample instant, where $q_{n,k}^b = [q_{0,k} \ q_{1,k} \ q_{2,k} \ q_{3,k}]^T$, $p_k^n = [x_k^n \ y_k^n \ z_k^n]^T$, $v_k^n = [v_{x,k}^n \ v_{y,k}^n \ v_{z,k}^n]^T$, the model provided by (6) and (7) allows the localization system to predict the body's current state based on the IMU's measurements. However, because of modeling errors and sensor noise, estimating the system's state using only inertial measurements quickly leads to unreliable results, making it necessary to use additional sensors in order to correct the estimates [17]. As described in the beginning of Section II, besides the IMU, the UAV's embedded instruments are a magnetometer, a GPS receiver, and a barometric altimeter. These instruments' readings are related to (8) according to

$$m_{mag,k}^b = \left(C_{n,k}^b(q_{n,k}^b) \right)^T m_E^n + \epsilon_m, \tag{9}$$
$$p_{gps,k}^n = p_k^n + \epsilon_p, \tag{10}$$
$$v_{gps,k}^n = v_k^n + \epsilon_v, \tag{11}$$
$$h_{alt,k}^n = z_k^n + \epsilon_h, \tag{12}$$

where $m_{mag,k}^b$ denotes the k-th magnetometer reading described in b and m_E^n is the local magnetic field vector; $p_{gps,k}^n$ and $v_{gps,k}^n$ are, respectively, the position and velocity samples read from the GPS receiver; $h_{alt,k}^n$ is the altitude measurement provided by the altimeter; and ϵ_i, $i \in \{m, p, v, h\}$, model sensor disturbances.

III. HYBRID DATA FUSION

A filtering algorithm capable of joining (6), (7), (9)-(12) to provide reliable estimates of (8) completes the localization system described in Section II. In standard operating conditions where the model equations are valid, the EKF and the Unscented Kalman Filter (UKF) are among the most widely adopted filtering solutions for this kind of nonlinear problem [18]. Particle filtering is another very common alternative to handle nonlinearity [8], [9], generally allowing for non-Gaussian noise. However, robots operating in outdoor environments sometimes undergo very strong disturbances, making traditional filters unable to track (8) over time. The term heavy disturbance here is used to denote environmental perturbations so strong that they are able to modify the system's internal structure. This perturbations include signal interference from outside the system, temporary and permanent sensor faults, physical structure modification and any other source of disturbance that cannot be modeled just by adding an ϵ noise term. An approach to deal with this problem is presented next.

Similarly to a well known procedure in the modeling of hybrid systems [8], [19], state estimation in dynamical systems under heavy disturbances may be modeled as a hybrid data fusion problem with equations

$$r_k = f_{m_k}(r_{k-1}, u_{k-1}, w_{k-1}), \tag{13}$$

$$y_{m_k} = h_{m_k}(r_k, v_k), k \in \mathbb{N}, \tag{14}$$

where $r_k \in \mathbb{R}^{n_r}$ is the sampled state vector; $m_k \in \mathbb{M} \triangleq \{1, 2, \ldots, \mathrm{M}\}$ is the system's operating mode, which can assume M different discrete values; $f_{m_k} : \mathbb{R}^{n_r} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w} \rightarrow \mathbb{R}^{n_r}$ is a possibly nonlinear mode-dependent process evolution function; $h_{m_k} : \mathbb{R}^{n_r} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}^{n_y}$ and $y_{m_k} \in \mathbb{R}^{n_y}$ are the mode-dependent measurement function and measurement vector, respectively; $u_{k-1} \in \mathbb{R}^{n_u}$ is the input vector; and $v_{k-1} \in \mathbb{R}^{n_v}$ and $w_{k-1} \in \mathbb{R}^{n_w}$ are noise processes. The parameter m_k is assumed to follow a Markov Chain with possibly unknown transition probability matrix (TPM)

$$\Pi = (\pi_{i,j}), \pi_{i,j} = P\{m_k = j | m_{k-1} = i\}, i, j \in \mathbb{M}, \tag{15}$$

and initial probability vector $p(m_0)$. Each discrete mode m_k defines a measurement equation y_{m_k}, each one of them modeling a different way measurements can be used to correct the predicted estimates. In the case of heavily disturbed systems, a single measurement equation is generally not sufficient to cope with the different ways that output measurements can be generated and affected, possibly being completely degraded by the environment. In theoretical situations when all sensors work properly and signal disturbance is not strong, the model (13)-(14) can be simplified to

$$r_k = f(r_{k-1}, u_{k-1}, w_{k-1}), \tag{16}$$

$$y_k = h(r_k, v_k), k \in \mathbb{N}. \tag{17}$$

For the system (16)-(17), all sensor measurements correct the estimates. Furthermore, since the measurements are independent, a sequential correction process is commonly used. Considering that r_k is not directly measurable, the problem of stochastic state estimation can be formulated in a Bayesian framework as obtaining the *a posteriori* probability density function (p.d.f)

$$p(r_k | y_{1:k}) = \frac{p(y_k | r_k, y_{1:k-1}) p(r_k | y_{1:k-1})}{p(y_k | y_{1:k-1})} \tag{18}$$

from the sequence $y_{1:k} = \{y_1, y_2, \ldots, y_k\}$. This subject has been widely addressed in the literature both for the case of linear and nonlinear functions f and h in (16)-(17) [9], [10], [17], [20].

Assuming the possibility of not having any previous knowledge on Π and considering the hybrid model described in (13)-(14), one wishes to obtain

- $\hat{r}_k$, the estimated minimum variance state vector;
- $\hat{p}(m_k)$, the estimated mode probability vector;
- $\hat{\Pi}(k)$, the estimated TPM;

from a sequence of disturbed measurements $y_{1:k} = \{y_1, y_2, \ldots, y_k\}$. From the Total Probability Theorem [21], (18) can be rewritten as

$$p(r_k | y_{1:k}) = \sum_{i=1}^{M} p(r_k | y_{1:k}, m_k = i) P(m_k = i). \tag{19}$$

The term $P(m_k = i)$ denotes the unconditional probability of having $m_k = i$ at the k-th sample instant.

A. TPM estimation

Many works, such as [8], [11]–[13], concerning state estimation in the context of Markovian jump systems (MJS) assume prior knowledge on the mode transition probabilities, i.e., Π is a given parameter. However, this assumption is usually unrealistic, specially in the case of hybrid systems like (13)-(14) where mode transitions have unknown causes and occur at random. Choosing an incorrect *a priori* value for Π may degrade the filter's performance and lead to inaccurate values for $\hat{r}_k$ and $\hat{p}(m_k)$, making the online estimation of Π based on $y_{1:k}$ a desirable and important feature.

The algorithm presented in [22] to perform the online estimation of unknown, nonstationary TPMs models each row of Π as following a prior Dirichlet distribution and derives a Bayesian mean-variance estimator based on the fact that the Dirichlet distribution is conjugate to the multinomial distribution. However, the estimator [22] assumes perfect mode observation, which is not the case for (13)-(14). For TPM estimation, it has been used the *Quasi-Bayesian* algorithm described in [19] using just the system's measurements as inputs to the TPM estimator. This estimator, which gives an approximation to the maximum *a posteriori* estimate of the transition probabilities, is incorporated to the hybrid nonlinear filter used to track (13)-(14).

B. Hybrid fusion of filters' estimates

As can be seen in (19), estimating (18) for the hybrid system described by (13)-(14) consists of keeping track of M filters, each one of them following a model for a different mode m_k. Moreover, it is also necessary to estimate the mode probability vector $\hat{p}(m_k) = [\hat{P}(m_k = 1) \ldots \hat{P}(m_k = \mathrm{M})]^T$ in order to weight the filters' estimates according to how likely their outputs are correct. In the context of multiple models estimation, the IMM algorithm exhibits computational requirements which are nearly linear in the size of the problem (number of modes) whilst its performance is almost the same as that of an algorithm with quadratic complexity, making this algorithm one of the best choices in terms of cost and efficiency [14]. Unfortunately, many applications of the IMM algorithm assume previous knowledge of the TPM, which is rarely the case. However, joining the TPM estimation algorithm of Section III-A with the IMM algorithm's equations introduced by [11], [12] yields a hybrid data fusion system which recursively estimates $\hat{r}_k$, $\hat{p}(m_k)$, and $\hat{\Pi}(k)$.

HDFF (Hybrid Data Fusion Filter) *A set of M filters is needed to track (13)-(14), each one of them following a different system mode. Let* $\hat{r}_i(k)$ *and* $\hat{P}_i(k)$, $i \in \{1, 2, \ldots, M\}$, *be the state vector and associated covariance matrix corresponding to the filter tracking the system mode* $m_k = i$ *at the* k*-th sample instant. Let also* y_k *be the system's output vector. Denoting* $\hat{p}_i(m_k) = P(m_k = i)$ *and assuming initial conditions*

$$
\begin{aligned}
\hat{p}(m_0) &= [\hat{p}_1(m_0)\ \hat{p}_2(m_0)\ \ldots\ \hat{p}_M(m_0)], \\
\hat{r}_i(0) &= r(0), r(0) \in \mathbb{R}^{n_r}, \\
\hat{P}_i(0) &= P(0), P(0) \in \mathbb{R}^{n_r \times n_r}, \\
\hat{\Pi}(0) &= \Pi(0),
\end{aligned}
$$

the hybrid data fusion algorithm can be given by the following steps:

i *Mode probability prediction*

$$\bar{p}_i(m_k) = \sum_{j=1}^{M} \hat{\pi}_{j,i}(k-1)\hat{p}_j(m_{k-1}),$$

ii *Estimates mixing*

$$
\begin{aligned}
\underline{r}^i(k-1) &= \sum_{j=1}^{M} \frac{\hat{\pi}_{j,i}(k-1)\hat{p}_j(m_{k-1})\hat{r}_j(k-1)}{\bar{p}_i(m_k)}, \\
\underline{P}^i(k-1) &= \sum_{j=1}^{M} \frac{\hat{\pi}_{j,i}(k-1)\hat{p}_j(m_{k-1})\left[\hat{P}_j(k-1)+\delta(i,j)\right]}{\bar{p}_i(m_k)}, \\
\delta(i,j) &= \left(\hat{r}_j(k-1)-\underline{r}^i(k-1)\right)(\cdot)^T,
\end{aligned}
$$

iii *Filter-dependent prediction step*

$$\left(\underline{r}^i(k-1), \underline{P}^i(k-1)\right) \xrightarrow{\textit{Prediction}} \left(\bar{r}_i(k), \bar{P}_i(k)\right), \tag{20}$$

iv *Filter-dependent correction step*

$$\left(\bar{r}_i(k), \bar{P}_i(k)\right) \xrightarrow{\textit{Correction}} \left(\hat{r}_i(k), \hat{P}_i(k)\right), \tag{21}$$

v *Mode probability correction*

$$
\begin{aligned}
\hat{p}_i(m_k) &= \frac{p(y_k|m_k{=}i, \hat{\Pi}(k-1), y_{1:k-1})\bar{p}_i(m_k)}{c_i}, \\
\gamma_p &= \sum_{j=1}^{M} \hat{p}_j(m_k), \\
\hat{p}(m_k) &= [\hat{p}_1(m_k)\ \ldots\ \hat{p}_M(m_k)]^T \left(\frac{1}{\gamma_p}\right),
\end{aligned}
$$

vi *Output generation*

$$
\begin{aligned}
\hat{r}_k &= \sum_{i=1}^{M} \hat{p}_i(m_k)\hat{r}_i(k), \\
\hat{P}_k &= \sum_{i=1}^{M} \hat{p}_i(m_k)\left[\hat{P}_i(k) + (\hat{r}_i(k) - \hat{r}_k)(\cdot)^T\right].
\end{aligned}
$$

vii *TPM update:* $\hat{\Pi}(k-1) \xrightarrow{\textit{Algorithm [19]}} \hat{\Pi}(k)$.

No details are given in (20) and (21) because these steps vary depending on the filter chosen to track each system mode. For example, if (13)-(14) are linear, the Kalman Filter (KF) is a sensible choice. On the other hand, the numerical results presented in Section IV for the nonlinear localization system modeled in Section II were obtained by using the EKF as the filtering solution. No matter what filter is chosen, step iii takes the mixed initial condition $(\underline{r}^i(k-1), \underline{P}^i(k-1))$ for the filter tracking the mode $m_k = i$ and yields the predicted state and covariance matrix $(\bar{r}_i(k), \bar{P}_i(k))$. Next, step iv, based on the system's current output sample y_k, provides the corrected estimates $(\hat{r}_i(k), \hat{P}_i(k))$.

IV. RESULTS

A. Simulations

This section presents the simulated tracking performance under heavy disturbances of two nonlinear filters for the localization system described in Section II. Among the UAV's embedded sensors, the magnetometer is the most affected by the electromagnetic interference generated by the aerial power lines. Furthermore, mechanical vibrations sometimes momentarily disconnect the magnetometer from the embedded computer, yielding invalid readings. At first, a single EKF was intended to perform the UAV's localization. However, poor performance was obtained during tests in a flight simulator, leading to the need of an alternative filtering method. The solution found was developing the hybrid data fusion algorithm presented in Section III, making the localization system robust to environmental disturbances.

Implementing (6) in a digital computer requires its conversion to the discrete time domain [23]. Denoting τ as the sampling period, (6) has discrete representation

$$\begin{bmatrix} p_k^n \\ v_k^n \end{bmatrix} = \begin{bmatrix} \mathbb{I} & \mathbb{I}\tau \\ 0 & \mathbb{I} \end{bmatrix} \begin{bmatrix} p_{k-1}^n \\ v_{k-1}^n \end{bmatrix} + \begin{bmatrix} C_n^b \frac{\tau^2}{2} & \mathbb{I}\frac{\tau^2}{2} \\ C_n^b \tau & \mathbb{I}\tau \end{bmatrix} \begin{bmatrix} f_{k-1}^b \\ g_{k-1}^n \end{bmatrix},$$

where the subscript $k \in \mathbb{N}$ denotes the sample taken at instant $k\tau$. Following a similar procedure for converting (7) to the discrete time domain yields

$$q_{n,k}^b = \left[\mathbb{I}_{4\times 4} \cos\left(\frac{\delta}{2}\right) - W\tau \frac{\sin\left(\frac{\delta}{2}\right)}{\delta}\right] q_{n,k-1}^b,$$

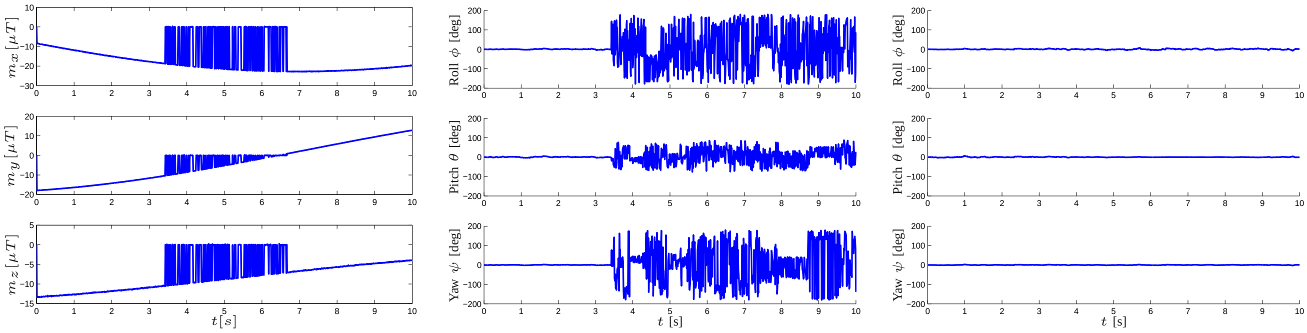
where $\delta = \left(\sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}\right)\tau$ and W is the same as in (7). In order to apply the EKF's equations, (13)-(14) were modified to

$$r_k = f(r_{k-1}, u_{k-1}) + w_{k-1}, \quad w_{k-1} \sim N(0, Q_{k-1}), \tag{22}$$

$$y_{m_k} = h_{m_k}(r_k) + v_k, \quad v_k \sim N(0, R_{m_k}). \tag{23}$$

For the system (22)-(23), steps iii and iv of Algorithm 1 are, respectively, the EKF's well-known prediction and correction steps.

In this simulation, an UAV performs a helical trajectory. The localization system was initially tested without the presence of heavy disturbances. Measurements were corrupted only by standard sensor noise and state estimation was performed by an EKF. It has been obtained a maximum of 5 degrees error in any of the attitude angles, indicating that the EKF implementation seems correct.



(a) Magnetometer measurements.

(b) Attitude angle errors for the EKF.

(c) Attitude angle errors for the HDFF.

Fig. 2. Simulation results under heavy disturbances on magnetometer measurements.

Next, two more simulations were conducted introducing heavy magnetometer disturbances during part of the experiment. The purpose of this perturbations was to simulate both signal interference induced by the aerial power lines and temporary disconnections between the magnetometer and the embedded system. During one third of the time, disturbances occurred at random, as can be seen in Figure 2(a). Once again the EKF was used to estimate the UAV's pose, yielding the attitude estimation results shown in Figure 2(b). During the first part of the experiment, where the magnetometer behaves correctly, the EKF's filtering performance is satisfactory. However, as magnetometer disturbances start to take place, the estimates provided by the EKF become completely degraded, yielding the unacceptable errors seen in Figure 2(b). In order to solve this problem, the HDFF was used to perform the UAV's state estimation. Based on (9)-(12), two mode-dependent measurement equations

$$y_1(k) = \begin{bmatrix} m^b_{mag,k} \\ p^n_{gps,k} \\ v^n_{gps,k} \\ h^n_{alt,k} \end{bmatrix} = \begin{bmatrix} \left(C^b_{n,k}(q^b_{n,k})\right)^T m^n_E + \epsilon_m \\ p^n_k + \epsilon_p \\ v^n_k + \epsilon_v \\ z^n_k + \epsilon_h \end{bmatrix}, \tag{24}$$

$$y_2(k) = \begin{bmatrix} m^b_{fault,k} \\ p^n_{gps,k} \\ v^n_{gps,k} \\ h^n_{alt,k} \end{bmatrix} = \begin{bmatrix} 0 + \epsilon_{\text{fault}} \\ p^n_k + \epsilon_p \\ v^n_k + \epsilon_v \\ z^n_k + \epsilon_h \end{bmatrix}, \tag{25}$$

were used to model the system's output. Equation (24) corresponds to the situation where all sensors are working properly, while (25) models magnetometer faults. A zero element was explicitly written in (25) because a pull-down resistor yields magnetometer measurements containing only zeros when temporary disconnections occur. The disturbance term $\epsilon_{\text{fault}} \sim N(0, P_{\text{fault}})$ models signal interference generated by the aerial power lines. Using (24)-(25) as the measurement model, the HDFF yielded the results shown in Figure 2(c). Even under magnetometer disturbances able to completely degrade the EKF's estimates, the HDFF showed a filtering performance very similar to that of the EKF in the undisturbed case, indicating its usefulness as a robust stochastic estimator.

B. Experiments

After being validated in simulation, the hybrid filter was tested in a real navigation experiment. The UAV's localization system was embedded in a car and collected sensor data while the vehicle moved around the University of Brasília's *campus* in a closed circuit. The goal was to verify if the localization system was able to provide accurate attitude and position estimates based on real sensor readings. Once again a comparison was made between the EKF and the HDFF.

Attitude and 3D position estimates are shown in Figures 3(a) and 3(b) for the EKF. Similarly to the previous simulation results, the EKF's performance is poor and sensitive to sensor disturbances. Moreover, magnetometer measurements with norm beyond a chosen threshold among the experimental data had to be eliminated in order to prevent the EKF from diverging.

Differently from the EKF, the results provided by the HDFF are indeed reliable estimates of the localization system's pose during the experiment. As can be seen in Figures 4(a) and 4(b), the roll and pitch angles remained small during the whole operation, which goes in agreement with the fact that cars do not roll and pitch while moving, except for eventual suspension vibrations. At the same time, the yaw estimates follow the car heading during the course, eventually returning to zero when the car arrives at its initial position. Concerning the 3D position, the HDFF was able to correctly track the vehicle and there is little deviation between GPS measurements and estimated 3D positions, which is not the case for the EKF.

V. CONCLUSIONS

This work proposed a new approach for the data fusion problem based on the hybrid modeling of heavily disturbed systems. The localization system developed in Section II was simulated and experimentally tested for the operation of an UAV designed for aerial power lines inspection and the performance under environmental disturbances of two stochastic filters was evaluated. The classical solution using the EKF yielded good results in the undisturbed case, but failed to provide reliable estimates in the presence of measurement faults. On the other hand, the hybrid filter HDFF showed a performance, for both the disturbed and undisturbed cases, similar to the EKF in the lack of disturbances, showing the HDFF's utility as a robust state estimator for real robotic systems.

VI. ACKNOWLEDGMENTS

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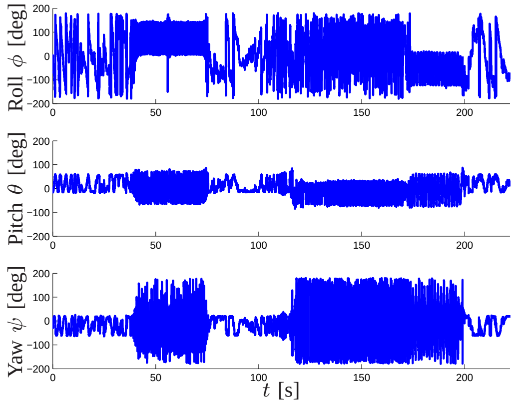
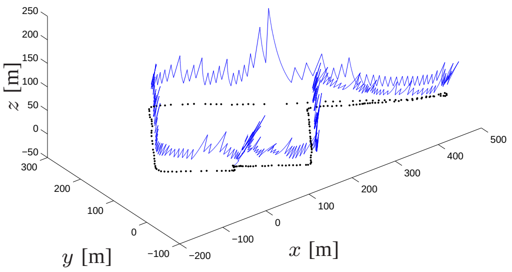
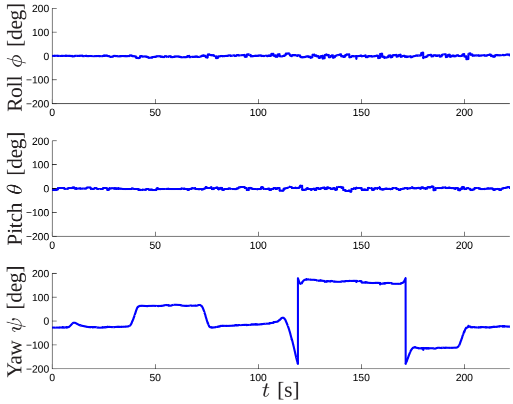


(a) Attitude estimates.

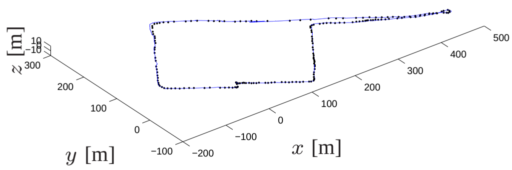


(b) Estimated position (solid) and GPS measurements (dotted).

Fig. 3. EKF's attitude and 3D position estimates in a real navigation experiment.



(a) Attitude estimates.



(b) Estimated position (solid) and GPS measurements (dotted).

Fig. 4. HDFF's attitude and 3D position estimates in a real navigation experiment.

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