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Robust Stability of Networked Control Systems

L.F.C. Figueredo
lfc.figueredo@yahoo.com.br
E.S. Alves
ealves@ieee.org
G.A. Borges
gaborges@unb.br
P.H.R.Q.A. Santana
phrqas@ieee.org
J.Y. Ishihara
ishihara@ene.unb.br
A. Bauchspiess
adolfobs@unb.br

Robotics and Automation Laboratory (LARA)
Robotics, Automation and Computational Vision Group (GRAV)
Department of Electrical Eng. – University of Brasília (UnB)

1. Introduction
2. System Description
3. Stability Analysis
4. Numerical Examples
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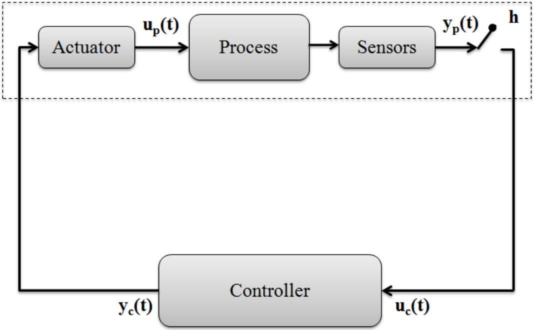
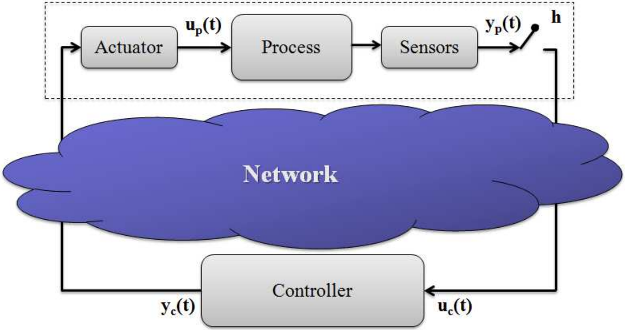
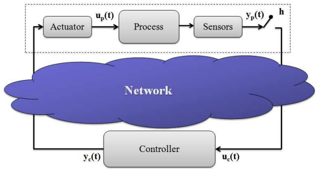
Discrete Control System



Networked Control System



Networked Control Systems

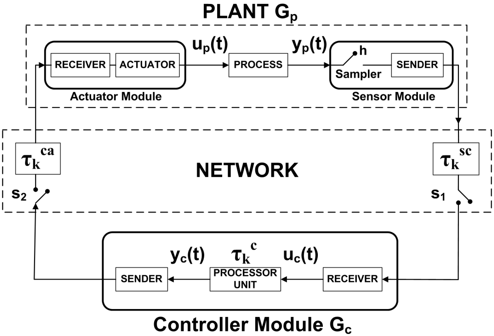


- Applications
  - Control over sensor networks;
  - Remote surgery;
  - Automated highway systems;
  - Unmanned aerial vehicles.

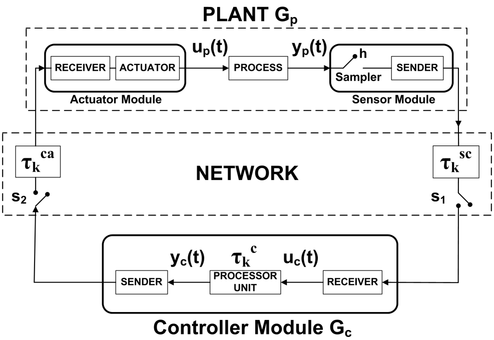
- Network-Induced Delay
  - Time-varying;

- Packet Loss
  - Retransmitted or discarded;

According to Zhang *et al.* (2001), the networked delay and the packet dropout can degrade the performance of control systems and can even cause destabilization.

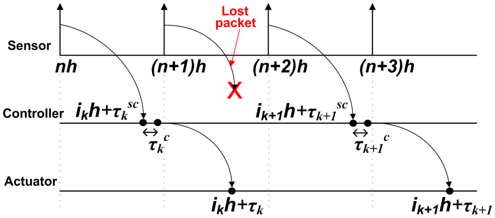


1. Closed-loop NCS with the possibility of dropping data and disordering;
2. Single packet transmission: all data lumped into one network packet;
3. Sensor module is *clock-driven* with sampling period h;
4. Controller and actuator modules are *event-driven*;
5. Actuator uses the latest avaiable control input.



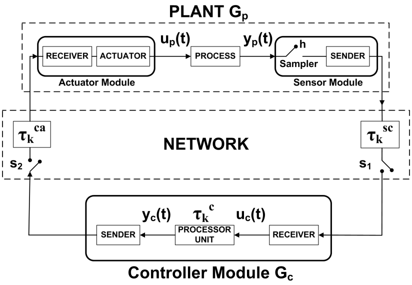
1. τ_k^{sc}: delay from sensor to controller module for the kth network packet;
2. τ_k^{c}: computation delay for the kth network packet;
3. τ_k^{ca}: delay from controller to actuator module for the kth network packet;
4. τ_k: total delay from sensor to actuator module for the kth network packet.
5. The switches S_1 and S_2 model the possibility of packet loss.

System Description: delays and packet dropout



1. Sensor module samples data at instants nh;
2. i_k denotes the nth sample number which is carried by the kth received network packet at the actuator's input;
3. $\{i_1, \ldots, i_n, \ldots\} = \{1, \ldots, n, \ldots\}$ $\rightarrow$ no packet dropout or disordering occured;
4. $i_{k+1} \neq i_k + 1 \longrightarrow$ a transmission failure occured.

System Description



- Delay constraints

$$
\begin{aligned}
&(i_{k+1} - i_k)h + \tau_{k+1} \leq \eta, (upper\ \ bound)\\
&\tau \leq \tau_k, \qquad \forall k \in \mathbb{N}^*, (lower\ \ bound)\\
&d(t) = t - i_k h, \dot{d}(t) = 1.
\end{aligned}
$$

- Plant's model

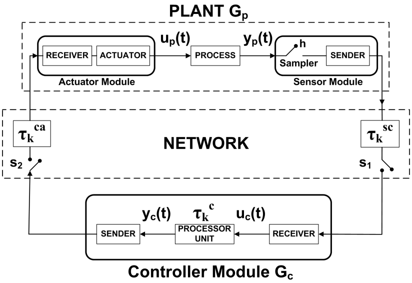
$$
\begin{aligned}
\dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t), && (1)\\
y_p(t) &= C_p x_p(t), && (2)
\end{aligned}
$$

- Controller's input

$$
u_c(t) = y_p(i_k h) = C_p x_p(i_k h), \forall\ k \in \mathbb{N}^*, \qquad (3)
$$

- Plant's input

$$
\begin{aligned}
u_p(t) &= y_c(i_k h + \tau_k^{sc} + \tau_k^{c} + \tau_k^{ca})\\
&= K C_p x_p(i_k h), && (4)
\end{aligned}
$$



- Closed loop system

$$\dot{x}(t) = Ax(t) + A_d x(t - d(t)), \quad (5)$$
$$x(t) = \phi(t), \quad t \in [t_1 - \eta, t_1] \quad (6)$$
$$\tau \leq d(t) \leq \eta \quad (7)$$

where

$$A_d = B_p K C_p$$

Closed loop system with uncertanties

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)), \quad (8)$$

where

$$\Delta A = M_A F_A N_A, \quad (9)$$
$$\Delta A_d = M_{Ad} F_{Ad} N_{Ad} \quad (10)$$

with

$$F_A^T F_A \leq I,$$
$$F_{Ad}^T F_{Ad} \leq I.$$

Lyapunov function candidate

Considering the Lyapunov function candidate

$$V(t) = \sum_{i=1}^{3} V_i(t), \tag{11}$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t),\\
V_2(t) &= \int_{t-\tau}^{t} \left[x(s)^T Q_1 x(s)\right] ds + \int_{t-\eta}^{t} \left[x(s)^T Q_2 x(s)\right] ds\\
&\quad + \int_{t-\alpha d(t)}^{t} \left[x(s)^T Q_3 x(s)\right] ds, \qquad (12)\\
V_3(t) &= \int_{-\eta}^{0}\int_{t+\beta}^{t} \left[\dot{x}(s)^T Z_1 \dot{x}(s)\right] ds d\beta\\
&\quad + \int_{-\eta}^{-\tau}\int_{t+\beta}^{t} \left[\dot{x}(s)^T Z_2 \dot{x}(s)\right] ds d\beta,
\end{aligned}$$

and matrices $P = P^T > 0$, $Q_i = Q_i^T \geq 0$, $Z_j = Z_j^T > 0$, $i \in \{1, 2, 3\}$, $j \in \{1, 2\}$, the following stability criterion was derived

Theorem (1)

For given scalars $0 \le \tau < \eta$, $0 < \alpha < 1$, $\beta_A > 0$, $\beta_{Ad} > 0$ *and* $\varepsilon > 0$ *such that*

$$\begin{bmatrix} M_A^T U M_A & M_A^T U M_{Ad} \\ M_{Ad}^T U M_A & M_{Ad}^T U M_{Ad} \end{bmatrix} - \varepsilon^{-1} I < 0,$$

the NCS described by (8) is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \ge 0$, $Z_j = Z_j^T > 0$, $i \in \{1, 2, 3\}$, $j \in \{1, 2\}$ *such that*

$$\begin{bmatrix} G^T X + XG + K & XB \\ B^T X & -I \end{bmatrix} < 0. \tag{13}$$

holds, where

Robust stability criterion for NCSs

$$U = \eta Z_1 + (\eta - \tau) Z_2,$$

$$R_q = \begin{bmatrix} A^T U M_A & A^T U M_{Ad} \\ A_d^T U M_A & A_d^T U M_{Ad} \end{bmatrix},$$

$$Q_q = \left[-\varepsilon^{-1} I + \begin{bmatrix} M_A^T \\ M_{Ad}^T \end{bmatrix} U \begin{bmatrix} M_A & M_{Ad} \end{bmatrix} \right],$$

$$A_q = \begin{bmatrix} 0 & 0 & \frac{1}{\alpha\eta} Z_1 \\ \frac{1}{\eta-\tau} Z_2 & \frac{1}{\eta-\tau}(Z_1 + Z_2) & \frac{1}{(1-\alpha)\eta} Z_1 \end{bmatrix},$$

$$Z_q = \begin{bmatrix} Z_{q11} & 0 & 0 \\ 0 & Z_{q22} & 0 \\ 0 & 0 & Z_{q33} \end{bmatrix},$$

$$Z_{q11} = -Q_1 - \frac{1}{\eta - \tau} Z_2$$

$$Z_{q22} = -Q_2 - \frac{1}{\eta - \tau}(Z_1 + Z_2)$$

$$Z_{q33} = -(1-\alpha) Q_3 - \frac{1}{\alpha\eta} Z_1 - \frac{1}{(1-\alpha)\eta} Z_1$$

$$X = \begin{bmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} A & A_d & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} M_A \sqrt{\beta_A} & M_{Ad} \sqrt{\beta_{Ad}^{-1}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{12}^T & K_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & A_q & R_q \\ A_q^T & Z_q & 0 \\ R_q^T & 0 & Q_q \end{bmatrix}$$

$$K_{11} = A^T U A + \varepsilon^{-1} N_A^T N_A - \frac{1}{\alpha\eta} Z_1 + Q_1 + Q_2 + Q_3 + \beta_A^{-1} N_A^T N_A$$

$$K_{12} = A^T U A_d$$

$$K_{22} = A_d^T U A_d + \varepsilon^{-1} N_{Ad}^T N_{Ad} - \frac{1}{(1-\alpha)\eta} Z_1 - \frac{1}{\eta - \tau} Z_2 - \frac{1}{\eta - \tau}(Z_1 + Z_2) + \beta_{Ad} N_{Ad}^T N_{Ad}$$

Problem

What is the maximum delay that we guarantee will keep the system stable?

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)),$$

$$\tau \leq d(t) \leq \eta.$$

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -1.4 & 0 \\ -0.8 & -1.5 \end{bmatrix},$$

$\Delta A = 0,\ \Delta A_d = 0.$

Table 1: Maximum delay's upper bounds for different criteria ($\tau = 0.4s$)

	η
Yue *et al.* (2005)	$1.13s$
He *et al.* (2007)	$1.16s$
Zhu *et al.* (2008)	$1.17s^*$
Theorem (1) [a]	$1.17s^*$

$*$ The result in Zhu *et al.* (2008) is a corollary of Theorem (1) when no uncertainties are considered.

[a] Choosing $\alpha = 0.75$.

Problem

What is the maximum delay that we guarantee will keep the system stable?

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)),$$

$$\tau \leq d(t) \leq \eta.$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$M_A = N_A = \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix},$$

$$M_{A_d} = N_{A_d} = \begin{bmatrix} \sqrt{0.2} & 0 \\ 0 & \sqrt{0.3} \end{bmatrix},$$

$$F_A(t) = F_{A_d}(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}.$$

Table 2: Maximum delay's upper bounds for different criteria ($\tau = 0.4s$)

	η
Su *et al.* (1992)	$0.1575s$
Xu (1994)	$0.1575s$
Cao *et al.* (1998)	$0.2558s$
Jing (2004)	$0.3916s$
Yan *et al.* (2008)	$0.6090s$
Theorem (1)[b]	$0.6847s$

[b] Choosing $\alpha = 0.6$, $\epsilon = 0.9$, $\beta_A = \beta_{A_d} = 0.8$.

Problem

What is the minimum delay's influence?

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - d(t)),$$

$$\tau \leq d(t) \leq \eta.$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$M_A = N_A = \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix},$$

$$M_{A_d} = N_{A_d} = \begin{bmatrix} \sqrt{0.2} & 0 \\ 0 & \sqrt{0.3} \end{bmatrix},$$

$$F_A(t) = F_{A_d}(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}.$$

Table 3: Maximum delay's upper bounds for Theorem (1) [c]

τ	η
$0.1s$	$0.6977s$
$0.2s$	$0.7133s$
$0.3s$	$0.7336s$
$0.4s$	$0.7590s$

[c] Choosing $\alpha = 0.6$, $\epsilon = 0.9$, $\beta_A = \beta_{A_d} = 0.8$.

1. The new robust stability criterion presented here is able to deal with:
   - Bounded model uncertainties;
   - Time-varying network delay;
   - Packet losses.
2. The theorem presented gives less conservative results compared with previous works for the network delay's upper bound. It can also be seen as an extension of previous criteria that assumed perfect model's knowledge;
3. Results concerning the system's stabilization will be published soon.

Acknowledgments

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Questions?