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Stability of Networked Control Systems with Dynamic Controllers in the Feedback Loop

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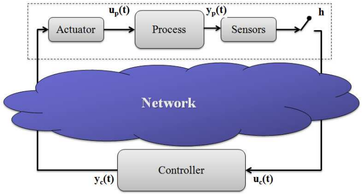
Outline

Networked Control System



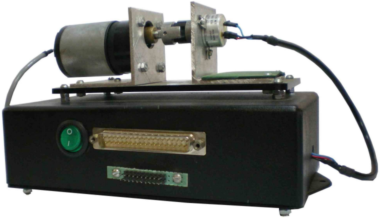
- Advantages
  - Lower instalation costs;
  - Reduced system wiring;
  - Greater flexibility;
  - Higher reliability.

- Network-Induced Delay
  - Time-varying;
- Packet Loss
  - Retransmitted or discarded;

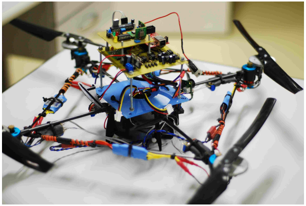
Degrade system performance;
System instability.

Applications

- Control over sensor networks



- Unmanned aerial vehicles



- Remote surgery

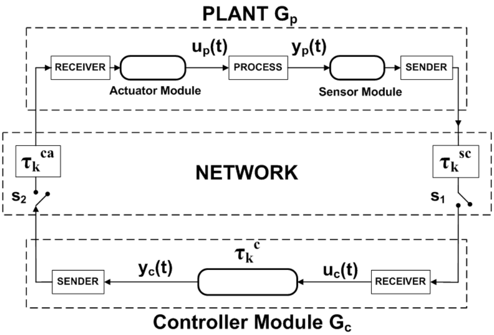
Delay-dependent stability results for systems with delays

- Model transformations (see Fridman and Shaked (2001))
- Free-weighting matrices (see Wu *et al.* (2004));
- Jensen's inequality (see Gu *et al.* (2003), and Zhu and Yang (2008));

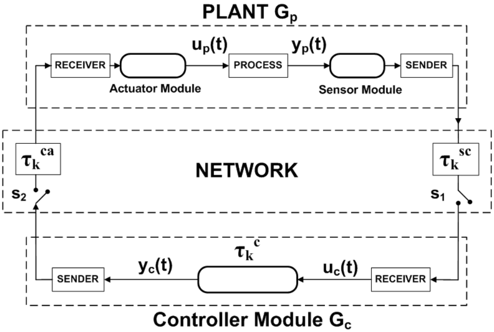
Pionner contribution (Halevi *et al.*, 1988);

One-channel feedback NCS	×	**Two-channel feedback NCS**;
No packet loss	×	**Packet loss considered**;
Constant delays	×	**Time-varying delays**;
Only delay's upper bound considered	×	**Delay's lower and upper bounds**;
Delays derivative varying with given bounds	×	**No restrictions are cast upon the derivatives**;

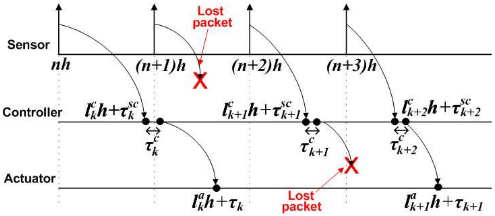
1. The new stability criterion presented here is able to deal with:
   - Dynamic controllers in the feedback loop;
   - Time-varying network delay;
   - Packet losses.
2. The theorem presented gives less conservative results compared with previous works for the network delay's upper bound.
3. It is also an extension of previous works that only consider proportional state feedback controllers in the stability analysis, being able to deal with a much larger set of control systems.



1. Closed-loop NCS with the possibility of dropping data and disordering;
2. Single packet transmission: all data lumped into one network packet;
3. Sensor module is *clock-driven* with sampling period h;
4. Controller and actuator modules are *event-driven*;
5. Actuator uses the latest avaiable control input.



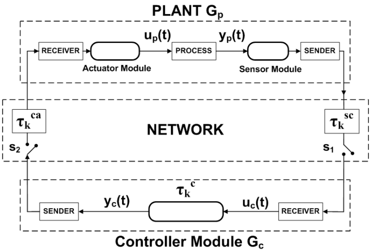
1. τ_k^{sc}: delay from sensor to controller module for the k-th network packet;
2. τ_k^{c}: computation delay for the k-th network packet;
3. τ_k^{ca}: delay from controller to actuator module for the k-th network packet;
4. τ_k: total delay from sensor to actuator module for the k-th network packet.
5. The switches S_1 and S_2 model the possibility of packet loss.



Remark

Dynamic controllers $\rightarrow$ SC and CA channels considered separately.
Proportional controllers $\rightarrow$ Only overall delay and package dropout considered.

1. Sensor module samples data at instants nh;
2. l_k^c: index for the k-th packet received at the controller;
3. l_k^a: index for the k-th packet received at the actuator;
4. $\{l_1^c, \ldots, l_n^c, \ldots\} = \{1, \ldots, n, \ldots\}$ $\rightarrow$ no packet dropout or disordering from the sensor to the controller;
5. $\{l_1^a, \ldots, l_n^a, \ldots\} = \{l_1^c, \ldots, l_n^c, \ldots\}$ $\rightarrow$ no packet dropout or disordering occured from the controller to the actuator;



- Plant's model

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t),$$
$$y_p(t) = C_p x_p(t),$$

- Controller's model

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t),$$
$$y_c(t) = C_c x_c(t - \tau_k^c) + D_c u_c(t - \tau_k^c),$$

- Controller's input

$$u_c(t) = y_p(l_k^c h) = C_p x_p(l_k^c h),$$
$$\forall k \in \mathbb{N}^*, t \in [l_k^c h + \tau_k^{sc},\ l_{k+1}^c h + \tau_{k+1}^{sc})$$

- Plant's input

$$u_p(t) = y_c(l_k^a h + \tau_k^{sc} + \tau_k^c)$$
$$= C_c x_c(l_k^a h + \tau_k^{sc}) + D_c C_p x_p(l_k^a h)$$
$$\forall k \in \mathbb{N}^*, \tau_k = \tau_k^{sc} + \tau_k^c + \tau_k^{ca},$$
$$t \in [l_k^a h + \tau_k, l_{k+1}^a h + \tau_{k+1}).$$

- Delay constraints

$$(l_k^c - l_{k-1}^c)h + \tau_{\mathbf{k}}^{\mathbf{sc}} \leq \eta_{\mathbf{1}},$$
$$(l_k^a - l_{k-1}^a)h + (\tau_{\mathbf{k}}^{\mathbf{c}} + \tau_{\mathbf{k}}^{\mathbf{ca}}) \leq \eta_{\mathbf{2}},$$
$$(l_k^a - l_{k-1}^a)h + \tau_{\mathbf{k}} \leq \eta_{\mathbf{3}},$$
$$\tau_{\mathbf{1}} \leq \tau_{\mathbf{k}}^{\mathbf{sc}}, \quad \tau_{\mathbf{2}} \leq \tau_{\mathbf{k}}^{\mathbf{c}} + \tau_{\mathbf{k}}^{\mathbf{ca}}, \quad \tau_{\mathbf{3}} \leq \tau_{\mathbf{k}}, \forall k \in \mathbb{N}^*.$$

- Defining $x(t) = [x_p^T(t), x_c^T(t)]^T \in \mathbb{R}^{n_p+n_c}$, the closed loop NCS can be described as:

$$\dot{x}(t) = Ax(t) + Bx(t - d_1(t)) + Cx(t - d_2(t)) + Dx(t - d_3(t)),$$
$$x(t) = \phi(t), t \in [t_1 - \eta_3,\ t_1],$$

with

$$\tau_j \leq d_j(t) \leq \eta_j, j \in \{1, 2, 3\},$$
$$\dot{d}_1(t) = 1, t \in [l_k^c h + \tau_k^{sc}, l_{k+1}^c h + \tau_{k+1}^{sc}),$$
$$\dot{d}_2(t) = \dot{d}_3(t) = 1, t \in [l_k^a h + \tau_k, l_{k+1}^a h + \tau_{k+1}),$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ B_c C_p & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & B_p C_c \\ 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} B_p D_c C_p & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} d_1(t) =& t - l_k^c h, t \in [l_k^c h + \tau_k^{sc}, l_{k+1}^c h + \tau_{k+1}^{sc}), \\ d_2(t) =& t - l_k^a h - \tau_k^{sc}, t \in [l_k^a h + \tau_k, l_{k+1}^a h + \tau_{k+1}), \\ d_3(t) =& t - l_k^a h, t \in [l_k^a h + \tau_k, l_{k+1}^a h + \tau_{k+1}). \end{aligned}$$

Lyapunov function candidate

Considering the Lyapunov function candidate

$$V(t) = \sum_{i=1}^{6} V_i(t),$$

where

$$V_1(t)=x^T(t)Px(t), \qquad V_2(t)=\sum_{i=1}^{3}\int_{t-\tau_i}^{t}\left[x^T(s)M_i x(s)\right]ds,$$

$$V_3(t)=\sum_{i=1}^{3}\int_{t-\eta_i}^{t}\left[x^T(s)N_i x(s)\right]ds, \qquad V_4(t)=\sum_{i=1}^{3}\int_{t-\alpha_i d_i(t)}^{t}\left[x^T(s)Q_i x(s)\right]ds,$$

$$V_5(t)=\sum_{i=1}^{3}\int_{-\eta_i}^{0}\int_{t+\beta}^{t}\left[\dot{x}^T(s)S_i\dot{x}(s)\right]dsd\beta, \quad V_6(t)=\sum_{i=1}^{3}\int_{-\eta_i}^{-\tau_i}\int_{t+\beta}^{t}\left[\dot{x}^T(s)Z_i\dot{x}(s)\right]dsd\beta,$$

and matrices $P = P^T > 0,\ Q_j = Q_j^T \geq 0,\ M_j = M_j^T \geq 0,\ N_j = N_j^T \geq 0,$ $Z_j = Z_j^T > 0,\ S_j = S_j^T > 0,\ j \in \{1, 2, 3\}$, the following stability criterion was derived.

Theorem 1

For given scalars $0 \leq \tau_i < \eta_i$ and $0 < \alpha_i < 1$, $i \in \{1,2,3\}$, the NCS previously described is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_j = Q_j^T \geq 0$, $M_j = M_j^T \geq 0$, $N_j = N_j^T \geq 0$, $Z_j = Z_j^T > 0$, $S_j = S_j^T > 0$, $j \in \{1,2,3\}$, such that

$$\Omega = \begin{bmatrix}
\Omega_{1,1} & \Omega_{1,2} & \Omega_{1,3} & \Omega_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{S_1}{\alpha_1\eta_1} & \frac{S_2}{\alpha_2\eta_2} & \frac{S_3}{\alpha_3\eta_3} \\
* & \Omega_{2,2} & \Omega_{2,3} & \Omega_{2,4} & \frac{Z_1}{\eta_1-\tau_1} & 0 & 0 & \Omega_{2,8} & 0 & 0 & \frac{S_1}{(1-\alpha_1)\eta_1} & 0 & 0 \\
* & * & \Omega_{3,3} & \Omega_{3,4} & 0 & \frac{Z_2}{\eta_2-\tau_2} & 0 & 0 & \Omega_{3,9} & 0 & 0 & \frac{S_2}{(1-\alpha_2)\eta_2} & 0 \\
* & * & * & \Omega_{4,4} & 0 & 0 & \frac{Z_3}{\eta_3-\tau_3} & 0 & 0 & \Omega_{4,10} & 0 & 0 & \frac{S_3}{(1-\alpha_3)\eta_3} \\
* & * & * & * & \Omega_{5,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Omega_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Omega_{7,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{8,8} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Omega_{9,9} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Omega_{10,10} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Omega_{11,11} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Omega_{12,12} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \Omega_{13,13}
\end{bmatrix} < 0$$

holds, where

$$U_1 = \left[\sum_{i=1}^{3} \eta_i S_i\right],\ U_2 = \left[\sum_{i=1}^{3} (\eta_i - \tau_i) Z_i\right],$$

$$U_3 = \sum_{i=1}^{3} \left(M_i + N_i + Q_i - \frac{1}{\alpha_i \eta_i} S_i\right),$$

$$V_i = \left(\frac{1}{(1-\alpha_i)\eta_i} + \frac{1}{\eta_i - \tau_i}\right) S_i,\ i \in \{1,2,3\},$$

$$W_i = (1-\alpha_i) Q_i + \left(\frac{1}{\alpha_i \eta_i} + \frac{1}{(1-\alpha_i)\eta_i}\right) S_i,\ i \in \{1,2,3\},$$

$$\Omega_{1,1} = A^T P + PA + U_3 + A^T (U_1 + U_2) A,$$

$$\Omega_{1,2} = PB + A^T (U_1 + U_2) B,$$

$$\Omega_{1,3} = PC + A^T (U_1 + U_2) C,$$

$$\Omega_{1,4} = PD + A^T (U_1 + U_2) D,$$

$$\Omega_{2,2} = B^T (U_1 + U_2) B - V_1 - \frac{2}{\eta_1 - \tau_1} Z_1,$$

$$\Omega_{2,3} = B^T (U_1 + U_2) C,$$

$$\Omega_{2,4} = B^T (U_1 + U_2) D,$$

$$\Omega_{2,8} = \frac{1}{\eta_1 - \tau_1} (S_1 + Z_1),$$

$$\Omega_{3,3} = C^T (U_1 + U_2) C - V_2 - \frac{2}{\eta_2 - \tau_2} Z_2,$$

$$\Omega_{3,4} = C^T (U_1 + U_2) D,$$

$$\Omega_{3,9} = \frac{1}{\eta_2 - \tau_2} (S_2 + Z_2),$$

$$\Omega_{4,4} = D^T (U_1 + U_2) D - V_3 - \frac{2}{\eta_3 - \tau_3} Z_3,$$

$$\Omega_{4,10} = \frac{1}{\eta_3 - \tau_3} (S_3 + Z_3),$$

$$\Omega_{5,5} = -M_1 - \frac{1}{\eta_1 - \tau_1} Z_1,$$

$$\Omega_{6,6} = -M_2 - \frac{1}{\eta_2 - \tau_2} Z_2,$$

$$\Omega_{7,7} = -M_3 - \frac{1}{\eta_3 - \tau_3} Z_3,$$

$$\Omega_{8,8} = -N_1 - \frac{1}{\eta_1 - \tau_1} (S_1 + Z_1),$$

$$\Omega_{9,9} = -N_2 - \frac{1}{\eta_2 - \tau_2} (S_2 + Z_2),$$

$$\Omega_{10,10} = -N_3 - \frac{1}{\eta_3 - \tau_3} (S_3 + Z_3),$$

$$\Omega_{11,11} = -W_1,\quad \Omega_{12,12} = -W_2,\quad \Omega_{13,13} = -W_3.$$

What is the maximum delay that we guarantee will keep the system stable?

- $u_p(t) = Kx_p(l_k^a h) \rightarrow$ Proportional state-feedback controller

Same NCS example presented in Zhang *et al.* (2001)

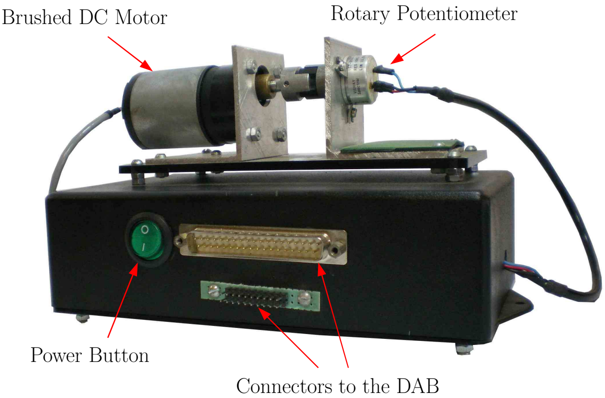
$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \ B=0, \ C=0, \ D= \begin{bmatrix} 0 & 0 \\ -0.375 & -1.15 \end{bmatrix}.$$

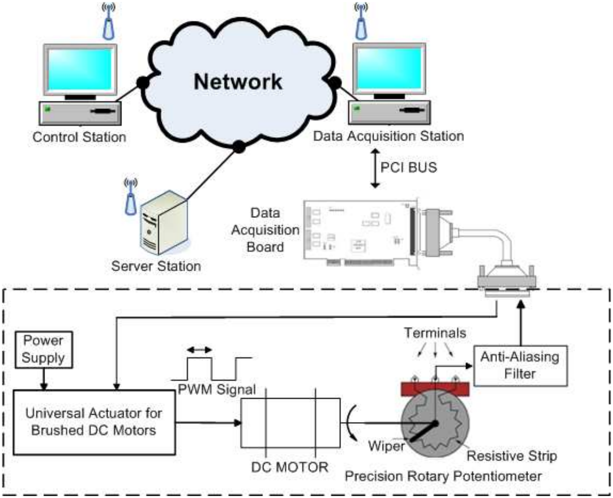
Table 1: Maximum delay's upper bounds for different criteria.

	η
Zhang *et al.* (2001)	$0.45ms$
Park *et al.* (2002)	$53.8ms$
Yue *et al.* (2004)	$869.5ms$
Naghshtabrizi *et al.* (2009)	$870ms$
Yue *et al.* (2005)	$887.1ms$
Theorem 1[a], Zhu *et al.* (2008)	$1008ms$

[a] Choosing $\alpha_1 = \alpha_2 = 0$, $\alpha_3 = 0.75$, $\tau_1 = \tau_2 = \tau_3 = 0$ s and $\eta_1 = \eta_2 = \frac{\eta_3}{2}$.

Numerical Examples 2




Network
Control Station
Data Acquisition Station
PCI BUS
Data Acquisition Board
Server Station
Power Supply
Universal Actuator for Brushed DC Motors
PWM Signal
DC MOTOR
Terminals
Anti-Aliasing Filter
Wiper
Resistive Strip
Precision Rotary Potentiometer

What is the maximum delay (η_3) that we guarantee will keep the system stable for a given delay lower bound (τ_3)?

PI Controller: $G_c(s) = K_p + \frac{1}{s}K_I$

$$\dot{x}(t) = Ax(t) + Bx(t - d_1(t)) + Cx(t - d_2(t)) + Dx(t - d_3(t))$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ B_cC_p & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & B_pC_c \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} B_pD_cC_p & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & -36.17 \end{bmatrix}, B_p = \begin{bmatrix} 0 \\ 36.3 \end{bmatrix}, C_p = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$A_c = 0$, $B_c = K_I = -47.45$, $C_c = 1$, $D_c = K_p = -11.86$

Table 2: Maximum total delay boundary for different values of τ_3

τ_3	0s	20ms	40ms	50ms	68.5ms
η_3	59.85ms	61.63ms	64.34ms	65.79ms	68.58ms

[a] Choosing $\tau_1 = \tau_2 = \frac{\tau_3}{2}$, $\eta_1 = \eta_2 = \frac{\eta_3}{2}$, $\alpha_1 = \alpha_2 = 0.5\alpha_3 = 0.375$.

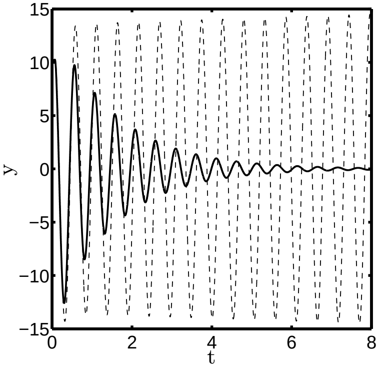


Figure 1: Simulation results using $\tau_3 = \eta_3 = 0.07s$ (solid) and $\tau_3 = \eta_3 = 0.078s$ (dashed).

Maximum total delay boundary for $\tau_3 \approx \eta_3$, obtained with Theorem 1, is **68.58** ms.

[a] Choosing $\tau_1 = \tau_2 = \frac{\tau_3}{2}$, $\eta_1 = \eta_2 = \frac{\eta_3}{2}$, $\alpha_1 = \alpha_2 = 0.5\alpha_3 = 0.375$.

1. Being able to deal with dynamic controllers in the feedback loop is this work's main contribution, since it extends previous works' results and allows the consideration of a much larger set of controllers;
2. This work aids the analysis of many practical control systems that were not considered previous criteria. For example, the widely used PID controllers, which are very common in industrial environments, could not be modeled by those works;
3. The experimental apparatus shown and simulated here is already under testing and results about its stability will be published soon;
4. We are currently working on extending our previously published results in robust stability and stabilization of NCSs in order to deal with dynamic controllers.

Acknowledgments

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Questions?