Q1

for $i = 1 \ldots m$

$\quad sum = 0$

$\quad$ for $y = 1 \ldots K$

$\quad\quad sum = sum + P(x^i, y \mid \theta^{t-1})$

$\quad$ for $y = 1 \ldots K$

$$P(y \mid x^i; \theta^{t-1}) = \frac{P(x^i, y \mid \theta^{t-1})}{sum}$$

$$\overline{Count}(y) = \overline{Count}(y) + P(y \mid x^i; \theta^{t-1})$$

$\quad\quad$ for $w \in V$

$$\overline{Count}(y, w) = \overline{Count}(y, w) + P(y \mid x^i; \theta^{t-1})$$

for $y = 1 \ldots K$

$$P(y) = \frac{\overline{Count}(y)}{m}$$

$\quad$ for $w \in V$

$$P(w \mid y) = \frac{\overline{Count}(y, w)}{\sum_{w'} \overline{Count}(y, w')}$$

Q2 for $i = 1 \ldots m$

$sum = 0$

for $y = 1 \ldots k$

$$Sum = sum + P(w_2^i, y | w_1^i, \theta^{t-1})$$

for $y = 1 \ldots k$

$$P(y | w_1^i, w_2^i \theta^{t-1}) = \frac{P(w_2^i, y | w_1^i, \theta^{t-1})}{sum}$$

$$\overline{Count}(w_2^i | y) = \overline{count}(w_2^i | y) + P(y | w_1^i, w_2^i, \theta^{t-1})$$

$$\overline{count}(y | w_1^i) = \overline{count}(y | w_1^i) + P(y | w_1^i, w_2^i, \theta^{t-1})$$

$$P(w|y) = \frac{\overline{Count}(w|y)}{\sum_{w'} \overline{Count}(w'|y)}$$

$$P(y|w) = \frac{\overline{Count}(y|w)}{\sum_{y'} \overline{Count}(y', w)}$$

Q3

(a) $$\frac{\alpha_1(2).a_{12}.b_1(o_2).a_{21}.b_2(o_3).\beta_1(4)}{Z}$$

(b) $$\sum_p \sum_q \frac{\alpha_1(2).a_{1p}.b_1(o_2).a_{pq}.b_p(o_3).a_{q1}.b_q(o_4).\beta_1(5)}{Z}$$

(c) base cases are the same

$$\alpha'_p(j+1) = \max_q \left(\alpha'_q(j).a_{qp}.b_q(o_j)\right)$$

$$\beta'_p(j) = \max_q \left(a_{pq}.b_p(o_j).\beta'_q(j+1)\right)$$

$$\max_{y:y_3=1} P(y|x,\theta) = \frac{\alpha'_1(3)\,\beta'_1(3)}{Z}$$

in all cases $Z = \sum_p \alpha_p(j)\beta_p(j)$

where j can be any index in the sequence

Q4

a) $$L(\theta) = \sum_{i=1}^{m} \log \sum_{z \in f(w_i, w_i')} P(w_i, w_i', z \mid \theta)$$

b) $$\overline{\text{Count}}^{t}(r) = \sum_{i=1}^{m} \sum_{z \in f(w_i, w_i')} P(z \mid w_i, w_i', \theta)\, C(z, r)$$

where r can be (c, c'), (ϵ, c), (c, ϵ) or $\#$

and $C(z, r)$ is the count of r in sequence z

c) $\alpha(0, 0) = 1$ $\qquad$ $\beta(p+1, q+1) = P(\#)$

d) $$\alpha(k, l) = \alpha(k-1, l) P(c_k, \epsilon) + \alpha(k, l-1) P(\epsilon, c_l) + \alpha(k-1, l-1)\, P(c_k, c_l)$$

there are special cases for $k=0$ and $l=0$

e) $O(pq)$

f) $$\overline{\text{count}}^{t}(c, c') = \sum_{i=1}^{m} \sum_{k: c_k = c} \sum_{l: c_l = c} \frac{\alpha(k-1, l-1)\, P(c, c')\, \beta(k+1, l+1)}{\beta(0, 0)}$$

$$\overline{\text{count}}^{t}(c, \epsilon) = \sum_{i=1}^{m} \sum_{k: c_k = c} \sum_{l} \frac{\alpha(k-1, l-1)\, P(c, \epsilon)\, \beta(k+1, l)}{\beta(0, 0)}$$

$$\overline{\text{count}}^{t}(\#) = m$$

e) $$P^{t}(c, c') = \frac{\overline{\text{count}}^{t}(c, c')}{\sum_{a, a'} \overline{\text{count}}^{t}(a, a') + \sum_{a} \overline{\text{count}}^{t}(a, \epsilon) + \sum_{a} \overline{\text{count}}^{t}(\epsilon, a) + m}$$