

# The growth of cryptography

Ronald L. Rivest

Viterbi Professor of EECS  
MIT, Cambridge, MA

James R. Killian Jr. Faculty Achievement Award Lecture  
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# Outline

Some pre-1976 context

Invention of Public-Key Crypto and RSA

Early steps

The cryptography business

Crypto policy

Attacks

More New Directions

What Next?

Conclusion and Acknowledgments

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Euclid – 300 B.C.



There are infinitely many primes:  
2, 3, 5, 7, 11, 13, ...

## Euclid – 300 B.C.



There are infinitely many primes:  
2, 3, 5, 7, 11, 13, ...

The greatest common divisor of two  
numbers is easily computed  
(using “Euclid’s Algorithm”):  
 $\text{gcd}(12, 30) = 6$

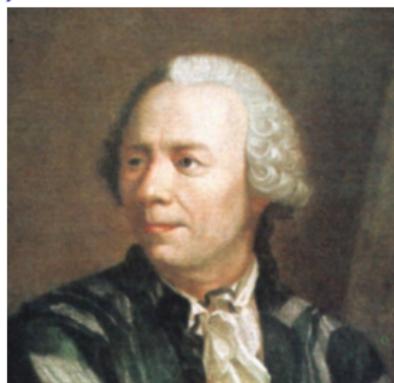
## Greek Cryptography – The Scytale



An unknown *period* (the circumference of the scytale) is the secret key, shared by sender and receiver.

Pierre de Fermat (1601-1665)

Leonhard Euler (1707–1783)



**Fermat's Little Theorem** (1640):

For any prime  $p$  and any  $a$ ,  $1 \leq a < p$ :

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**Euler's Theorem** (1736):

If  $\gcd(a, n) = 1$ , then

$$a^{\phi(n)} = 1 \pmod{n},$$

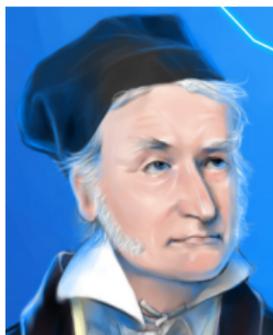
where  $\phi(n) = \#$  of  $x < n$  such that  $\gcd(x, n) = 1$ .

# Carl Friedrich Gauss (1777-1855)



Published *Disquisitiones Arithmeticae* at age 21

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“The problem of *distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors* is known to be one of the most important and useful in arithmetic. . . . the dignity of the science itself seems to require solution of a problem so elegant and so celebrated.”

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Factored by Derrick Lehmer in 1903. (89681 \* 96079)

## World War I – Radio

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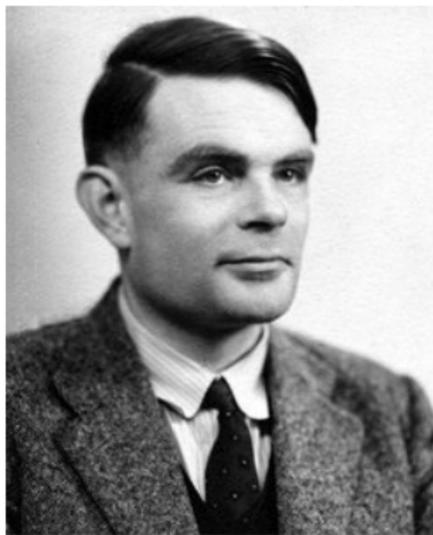
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TELEGRAM RECEIVED.  
From SAC from London # 5747.  
We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of America neutral. In the event of this not succeeding, we have Mexico a proposed of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and Arizona. The settlement in detail is left to you. You will inform the President of the above most secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, ~~call~~ Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ADOLF HITLER.

Decipherment of  
*Zimmermann Telegram* by  
British made American  
involvement in World War I  
inevitable.

## Alan Turing (1912–1954)



Developed foundations of theory of computability (1936).

# World War II – Enigma, Purple, JN25, Naval Enigma



- ▶ Cryptography performed by (typically, rotor) *machines*.

## World War II – Enigma, Purple, JN25, Naval Enigma



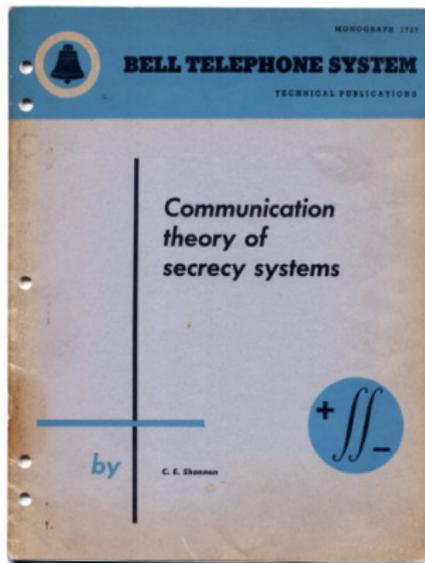
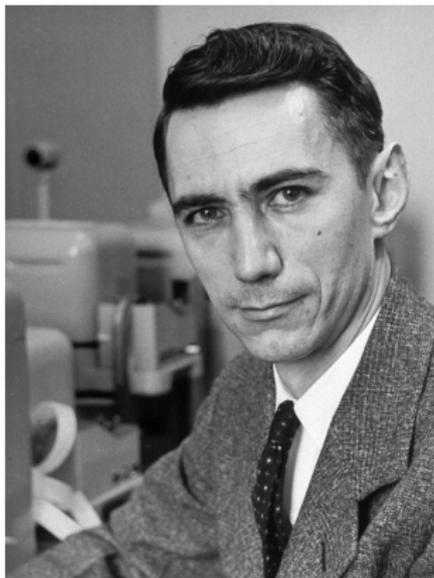
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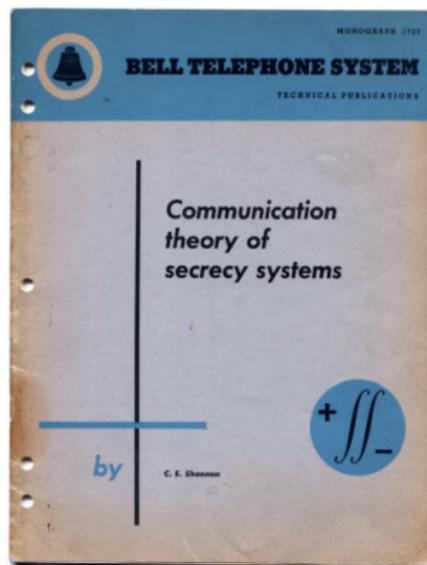
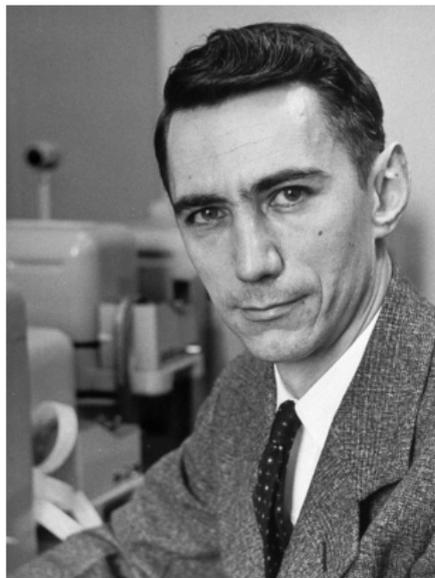
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- ▶ Cryptanalytic effort involved development and use of early computers (Colossus).

# Claude Shannon (1916–2001)



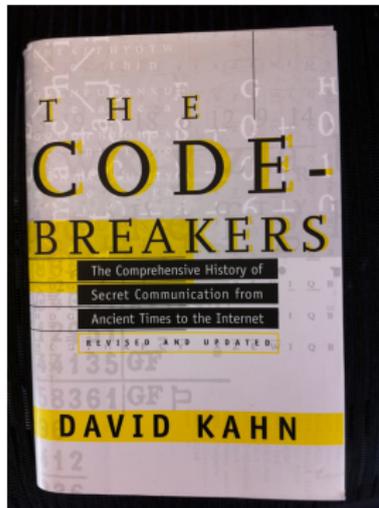
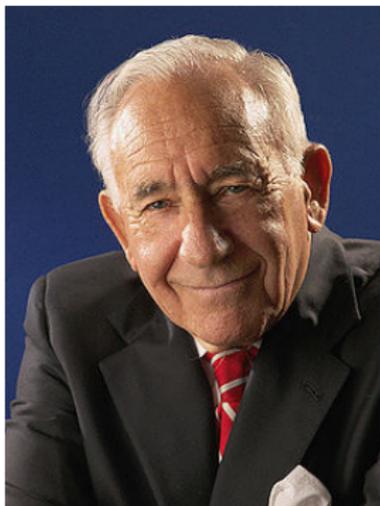
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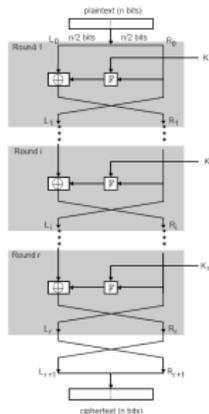
- ▶ “Communication Theory of Secrecy Systems” Sept 1945 (Bell Labs memo, classified).
- ▶ Information-theoretic in character—proves unbreakability of one-time pad. (Published 1949).

## Kahn – The Codebreakers



In 1967 David Kahn published  
*The Codebreakers—The Story of Secret Writing.*  
A monumental history of cryptography.  
NSA attempted to suppress its publication.

# DES – U.S. Data Encryption Standard (1976)



DES Designed at IBM; Horst Feistel supplied key elements of design, such as ladder structure. NSA helped, in return for keeping key size at 56 bits.(?)

# Computational Complexity



- ▶ Theory of Computational Complexity started in 1965 by Hartmanis and Stearns; expanded on by Blum, Cook, and Karp.
- ▶ Key notions: polynomial-time reductions; NP-completeness.

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- ▶ In November 1976, Diffie and Hellman published *New Directions in Cryptography*, proclaiming  
“We are at the brink of a revolution in cryptography.”

## Public-key encryption (as proposed by Diffie/Hellman)

- ▶ Each party  $A$  has a *public key*  $PK_A$  others can use to encrypt messages to  $A$ :

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- ▶ It is easy to compute matching public/secret key pairs.
- ▶ **Publishing  $PK_A$  does not compromise  $SK_A$ !** It is *computationally infeasible* to obtain  $SK_A$  from  $PK_A$ . Each public key can thus be safely listed in a public directory with the owner's name.

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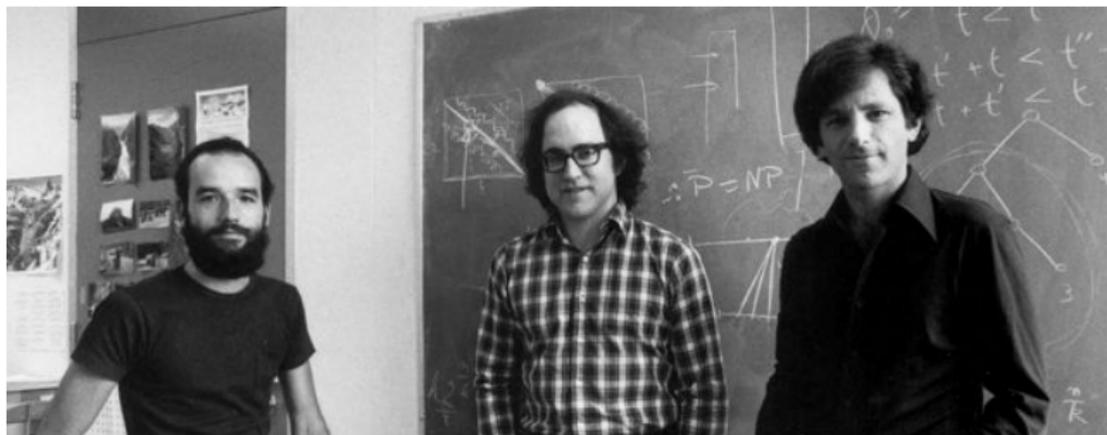
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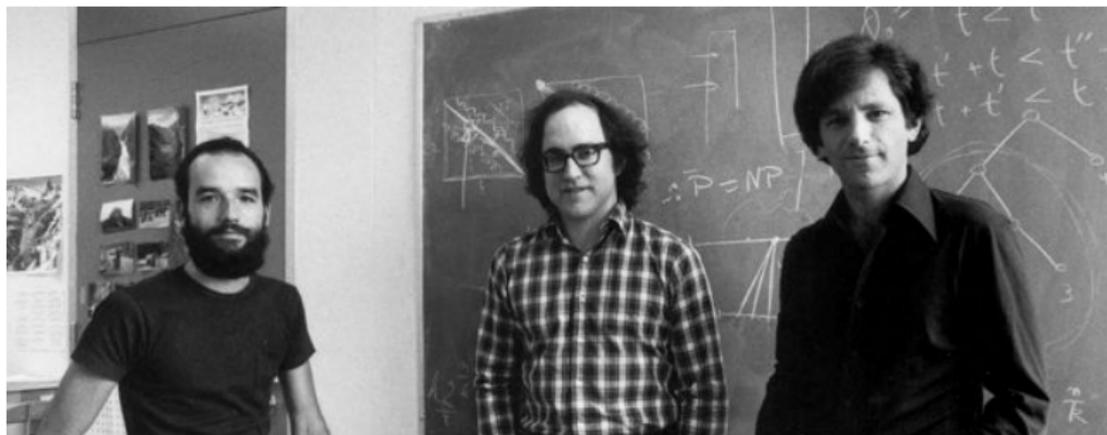
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- ▶ Amazing ideas!
- ▶ But they couldn't see how to implement them...

# RSA (Ron Rivest, Adi Shamir, Len Adleman, 1977)

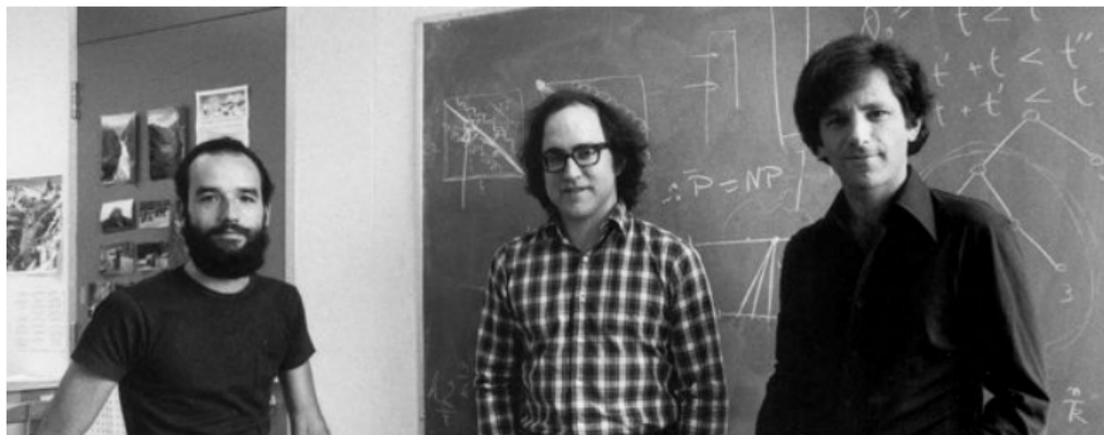


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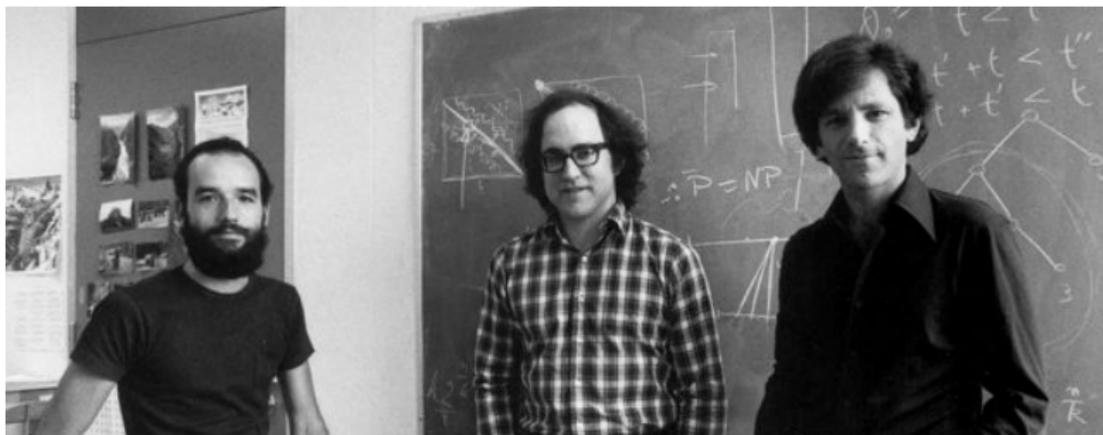
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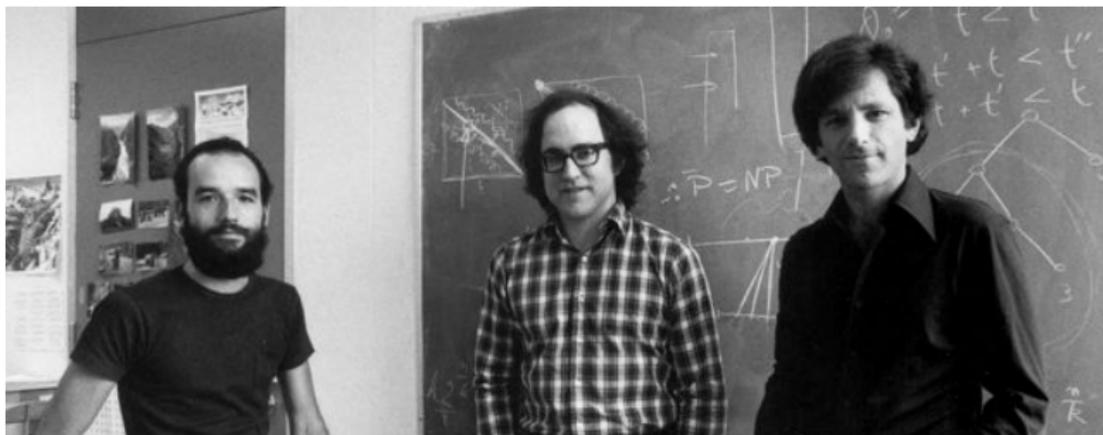
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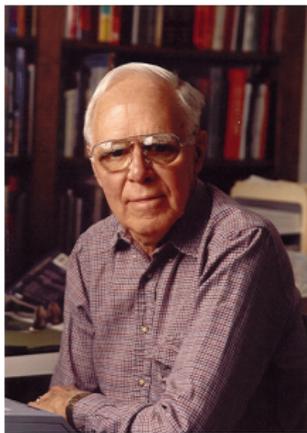


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- ▶  $SK = d$  where  $de = 1 \pmod{\phi(n)}$
- ▶ Encryption/decryption (or signing/verify) are simple:

$$C = PK(M) = M^e \pmod{n}$$

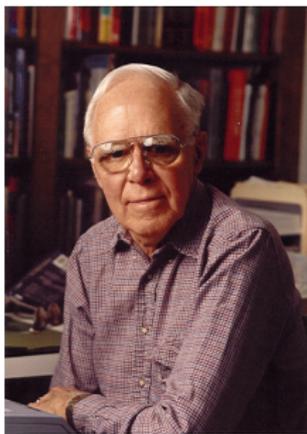
$$M = SK(C) = C^d \pmod{n}$$

# Martin Gardner column and RSA-129 challenge



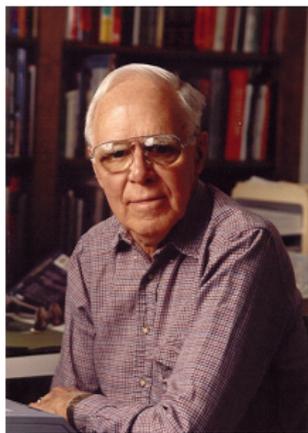
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## Martin Gardner column and RSA-129 challenge



- ▶ Described public-key and RSA cryptosystem in his Scientific American column, *Mathematical Games*
- ▶ Offered copy of RSA technical memo.
- ▶ Offered \$100 to first person to break challenge ciphertext based on 129-digit product of primes.  
(Our) estimated time to solution: 40 quadrillion years

# Publication of RSA memo and paper



## LCS-82 Technical Memo (April 1977) CACM article (Feb 1978)

Practicing S.L. Graham, R.L. Rivest\*  
Technician Editor

### A Method for Obtaining Digital Signatures and Public- Key Cryptosystems

R. L. Rivest, A. Shamir, and L. Adleman  
MIT Laboratory for Computer Science  
and Department of Mathematics

**Abstract**  
An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:

- (1) Confidentiality of communication: (a) Confidential or secret messages need not be transmitted using an encryption key publicly revealed by the intended recipient. Only the sender and recipient, who only need to know the corresponding decryption key, (b) A message can be "signed" using a privately held encryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key. Signatures cannot be forged, and a signer cannot later deny the validity of his signature. This has obvious applications to "electronic mail" and "electronic funds transfer" systems. A message is encrypted by representing it as a number  $M$ , raising  $M$  to a publicly specified power  $e$ , and then taking the remainder when the result is divided by a different, secret, power  $d$  to send, where  $e \cdot d \equiv 1 \pmod{\phi}$ . The security of the system rests in part on the difficulty of factoring the published modulus  $n$ .

**Key Words and Phrases:** digital signatures, public-key cryptosystems, private authentication, security, factorization, prime numbers, electronic mail, message-passing, electronic funds transfer, cryptography.  
**CR Categories:** 2.3.2, 3.3.3, 3.3.6, 3.31, 3.2.3

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### 1. Introduction

The use of "electronic mail" [1] has soon become an important part of our lives. We must ensure that the correct paper and system are provided; (a) messages are private, and (b) messages are not lost. We also require that the system be able to handle the large amount of data required in a distributed environment.

In the field of our research is a new encryption method. This method provides an implementation of a "public-key cryptosystem." An elegant message is sent to a public key cryptosystem [1]. This article describes our research, since they presented the concept but not any practical implementation of such a system. Readers familiar with [1] may wish to skip directly to Section 5 for a description of our method.

### II. Public-Key Cryptosystems

In a "public-key cryptosystem" each user places in a public file an encryption procedure  $E$ . That is, the public file is a directory giving the encryption procedure of each user. The user keeps secret the details of his corresponding decryption procedure  $D$ . These procedures have the following four properties:

- (1) Disrupting the unencrypted form of a message  $M$  yields  $M$ . Formally,  
 $D(E(M)) = M$  (1)
- (2) Both  $E$  and  $D$  are easy to compute.
- (3) By publicly revealing  $E$ , the user does not reveal an easy way to compute  $D$ . This means that to generate code he can decrypt messages encrypted with  $E$ , or compute  $D$  efficiently.
- (4) If a message  $M$  is first disrupted and then encrypted,  $M$  is the result. Formally,  
 $E(D(M)) = M$  (2)

An encryption (or decryption) procedure typically consists of a general method and an encryption key. The general method, under control of the key, enciphers a message  $M$  to obtain the encrypted form of the message, called the ciphertext  $C$ . Encryption can use the same general method, the security of a given procedure will rest on the security of the key. The key is a secret key, and the encryption algorithm then means revealing the key.

When the user wants to receive a message, he uses the method of computing  $D(C)$ , using all possible messages  $M$  and one such that  $D(E(M)) = C$ . If property (1) is satisfied, the number of such messages to test will be so large that this approach is impractical.

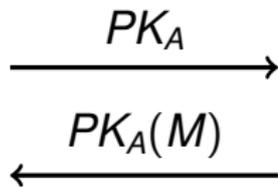
A function  $f$  satisfying (a)-(c) is a "trap-door one-way permutation." Diffie and Hellman [1] introduced the concept of trap-door one-way functions that

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Volume 31  
Number 2

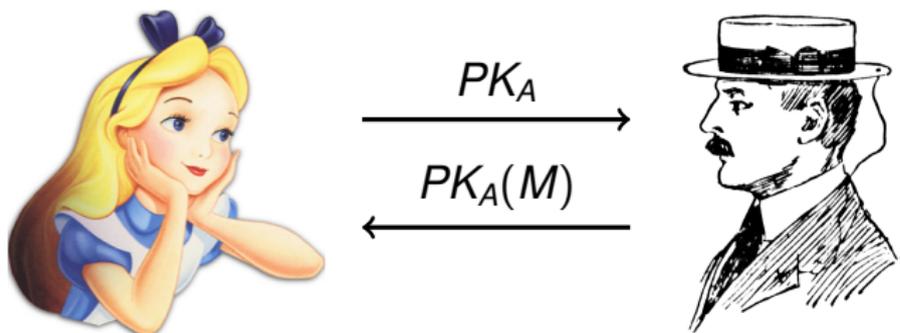
# Alice and Bob (1977, in RSA paper)



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Alice and Bob now have a life of their own—they appear in hundreds of crypto papers, in `xkcd`, and even have their own Wikipedia page:

The screenshot shows the Wikipedia article for "Alice and Bob". The page title is "Alice and Bob" and the subtitle is "From Wikipedia, the free encyclopedia". The article text begins with "The names **Alice and Bob** are commonly used [placeholder names](#) are used for convenience; for example, "Alice sends a message to Party B encrypted by Party B's public key within these fields—helping technical topics to be explained". The article also mentions "In [cryptography](#) and [computer security](#), there are a number of [various protocols](#). The names are conventional, somewhat".

## Independent Invention of Public-Key Revealed



In 1999 GCHQ announced that James Ellis, Clifford Cocks, and Malcolm Williamson had invented public-key cryptography, the “RSA” algorithm, and “Diffie-Hellman key exchange” in the 1970’s, before their invention outside.

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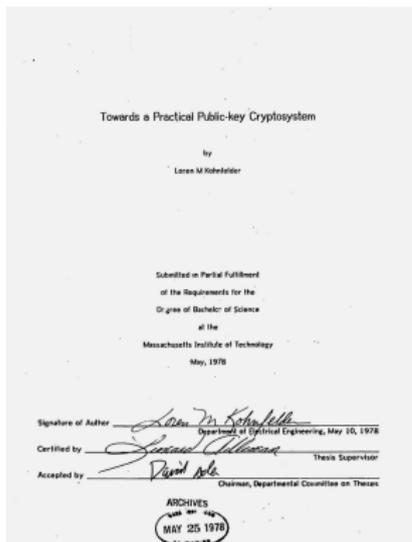
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# Loren Kohnfelder – Invention of Digital Certificates



- ▶ Loren Kohnfelder's B.S. thesis (MIT 1978, supervised by Len Adleman), proposed notion of *digital certificate*—a digitally signed message attesting to another party's public key.

# IACR—International Assn. for Cryptologic Research

- ▶ Established 1982 by David Chaum, myself, and others, to promote academic research in cryptology.
- ▶ Sponsors three major conferences/year (Crypto, Eurocrypt, Asiacrypt) and four workshops; about 200 papers/year, plus another 600/year posted on web. Publishes J. Cryptography
- ▶ Around 1600 members, (25% students), from 74 countries, 27 Fellows.



# Theoretical Foundations of Security



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- ▶ “A Digital Signature Scheme Secure Against Adaptive Chosen Message Attacks” Goldwasser, Micali, Rivest (1988) (Uses well-defined *game* to define security objective.)

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- ▶ Extremely simple and fast: uses array  $S[0..255]$  to keep a permutation of  $0..255$ , initialized using secret key, and uses two pointers  $i, j$  into  $S$ .

To output a pseudo-random byte:

```
i = (i + 1) mod 256
```

```
j = (j + S[i]) mod 256
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swap S[i] and S[j]
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Output S[(S[i] + S[j]) mod 256]
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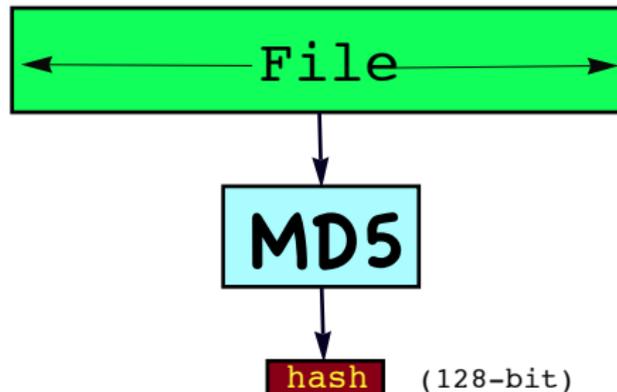
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swap  $S[i]$  and  $S[j]$

Output  $S[(S[i] + S[j]) \bmod 256]$

- ▶ Used in: WEP, BitTorrent, SSL, Kerberos, PDF, Skype, ...

## MD5 Cryptographic Hash Function (Rivest, 1991)



- ▶ MD5 proposed as pseudo-random function mapping files to 128-bit fingerprints. (variant of earlier MD4)
- ▶ Collision-resistance was a design goal – it should be infeasible to find two files with the same fingerprint.
- ▶ Many, many uses (e.g. in digital signatures) – very widely used, and a model for many other later hash function designs.

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# U.S. Patent 4,405,829

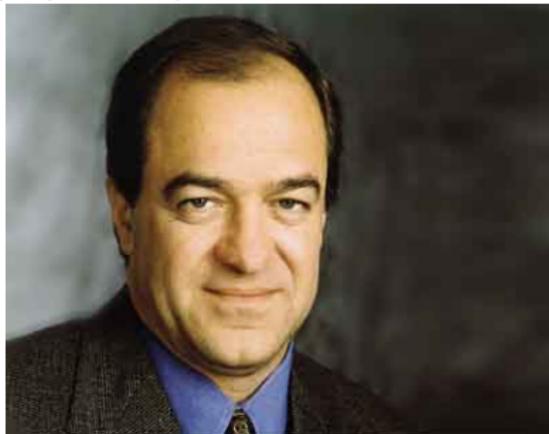
<b>United States Patent</b> [19]		[11]	<b>4,405,829</b>
<b>Rivest et al.</b>		[45]	<b>Sep. 20, 1983</b>
<b>[54] CRYPTOGRAPHIC COMMUNICATIONS SYSTEM AND METHOD</b>			
<b>[75] Inventors:</b> Ronald L. Rivest, Belmont; Adi Shamir, Cambridge; Leonard M. Adleman, Arlington, all of Mass.			
<b>[73] Assignee:</b> Massachusetts Institute of Technology, Cambridge, Mass.			
<b>[21] Appl. No.:</b> 860,586			
<b>[22] Filed:</b> Dec. 14, 1977			
<b>[51] Int. Cl.:</b> H04K 1/00; H04L 9/04			
<b>[52] U.S. Cl.:</b> 178/22.1; 178/22.11			
<b>[54] Field of Search:</b> 178/22, 22.1, 22.11, 178/22.14, 22.15			
<b>[56] References Cited</b>			
U.S. PATENT DOCUMENTS			
1,657,476 4/1972 Aiken ..... 178/22			
OTHER PUBLICATIONS			
"New Directions in Cryptography", Diffie et al., <i>IEEE Transactions on Information Theory</i> , vol. IT-22, No. 6, Nov. 1976, pp. 644-654.			
"Theory of Numbers" Stewart, MacMillan Co., 1952, pp. 133-135.			
"Diffie et al., Multi-User Cryptographic Techniques", AFIPS, Conference Proceedings, vol. 45, pp. 109-112, Jun. 8, 1976.			
<i>Primary Examiner</i> —Sal Cangialosi <i>Attorney, Agent, or Firm</i> —Arthur A. Smith, Jr.; Robert J. Horn, Jr.			
<b>ABSTRACT</b>			
<b>[57]</b> A cryptographic communications system and method. The system includes a communications channel coupled to at least one terminal having an encoding device and to at least one terminal having a decoding device. A message-to-be-transferred is encrypted to ciphertext at the encoding terminal by first encoding the message as a number M in a predetermined set, and then raising that number to a first predetermined power (associated with the intended receiver) and finally computing the remainder, or residue, C, when the exponentiated number is divided by the product of two predetermined prime numbers (associated with the intended receiver). The residue C is the ciphertext. The ciphertext is deciphered to the original message at the decoding terminal in a similar manner by raising the ciphertext to a second predetermined power (associated with the intended receiver), and then computing the residue, M', when the exponentiated ciphertext is divided by the product of the two predetermined prime numbers associated with the intended receiver. The residue M' corresponds to the original encoded message M.			
40 Claims, 7 Drawing Figures			

```
graph TD
    M((M)) --> Enc[ENCODING]
    Enc --> CA[CA_s]
    CA --> CC[COMMUNICATIONS CHANNEL]
    CC --> CD[CD_s]
    CD --> Dec[DECODING]
    Dec --> M_prime((M))
```

Filed December 1977 (MIT TLO)  
Issued September 1983

RSA the company (1983)

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- ▶ Jim Bidzos joined in 1986

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  - 65 billion DNS requests/day (DNSSEC coming)

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- ▶ RSA acquired by Security Dynamics in 1996, now part of EMC.

## World Wide Web (Sir Tim Berners-Lee, 1990)



- ▶ Just as radio did, this new communication medium, the World-Wide Web, drove demand for cryptography to new heights.
- ▶ Cemented transition of cryptography from primarily military to primarily commercial.

# Outline

Some pre-1976 context

Invention of Public-Key Crypto and RSA

Early steps

The cryptography business

**Crypto policy**

Attacks

More New Directions

What Next?

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# U.S. cryptography policy evolves

- ▶ U.S. government initially tried to control and limit public-sector research and use of cryptography
- ▶ Attempt to chill research via ITAR (1977)
- ▶ MIT “Changing Nature of Information” Committee (1981; Dertouzos, Low, Rosenblith, Deutch, Rivest,...)

## MIT Committee Seeks Cryptography Policy

*Questions of who should do research on cryptography and how results should be disseminated are the first order of business*

Within the next 10 years, networks consisting of tens of thousands of computers will connect businesses, corporations and homes in ways that make

communications for individuals and for society if computers continue to be connected, as they are now, according to local deci-

isions. It is not difficult to send computer programs between connected machines and to instruct a program to search for, select,

*Science, 13 Mar 1981*

## U.S. cryptography policy evolves

- ▶ U.S. government tried to mandate availability of all encryption keys via “key escrow” and/or “Clipper Chip” (1993)

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- ▶ Today, US policy leans toward strong cybersecurity, including strong cryptography, for all information systems as a matter of national security.

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## Factorization of RSA-129 (April 1994)

▶ RSA-129 =

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11438162575788886766923577997614661201021829  
67212423625625618429357069352457338978305971  
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- ▶ Derek Atkins, Michael Graff, Arjen Lenstra,  
Paul Leyland: RSA-129 =

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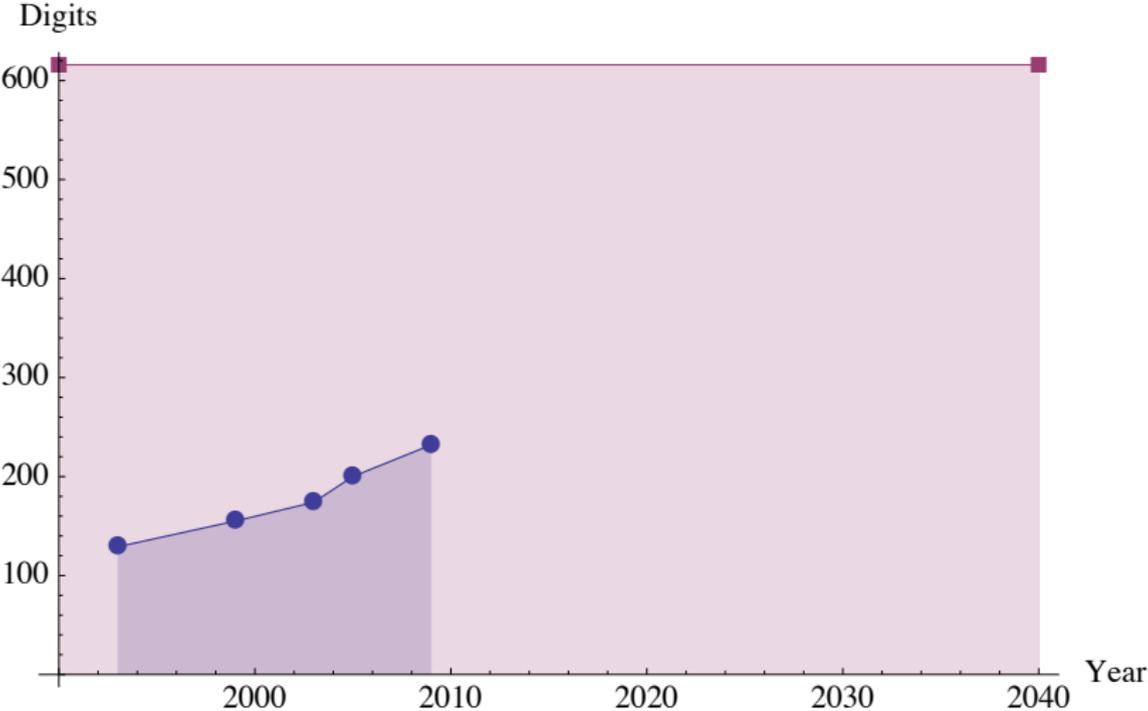
- ▶ secret message:

The Magic Words Are Squeamish Ossifrage



<b>BayBank</b> For Solving the Scientific American RSA Challenge		0254643
Massachusetts	53-235 113	Official Bank Check
Date		April 22, 1994
PAY	The sum of 100 dollars 00 cts	\$ *****100.00*****
		AMOUNTS IN EXCESS OF \$100,000.00 REQUIRE TWO SIGNATURES
To the order of	**Derek Atkins or Michael Graff or Arjen Lenstra or Paul Leyland**	 Authorized Signature
		Authorized Signature
⑆0254643⑆ ⑆011302357⑆ ⑆117 83321⑆		

# Factoring Records



# Factoring on a Quantum Computer?



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$|\alpha|^2 + |\beta|^2 = 1$$

The diagram shows a quantum state  $|\psi\rangle$  represented by a circle with a diagonal arrow. This state is equal to the sum of two basis states:  $\alpha|0\rangle$  (circle with a vertical arrow pointing up) and  $\beta|1\rangle$  (circle with a vertical arrow pointing down). The corresponding column vectors are  $\alpha|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\beta|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

In 1994, Peter Shor invented a fast factorization algorithm that runs on a (hypothetical) *quantum computer* and works by determining multiplicative period of elements mod  $n$ .

# Factoring on a Quantum Computer?



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$|\alpha|^2 + |\beta|^2 = 1$$

The diagram shows a circle with a diagonal arrow pointing from the bottom-left to the top-right. This is equal to the sum of two circles: one with a vertical arrow pointing up and one with a vertical arrow pointing down.

In 1994, Peter Shor invented a fast factorization algorithm that runs on a (hypothetical) *quantum computer* and works by determining multiplicative period of elements mod  $n$ .

- ▶ In 2001, researchers at IBM used this algorithm on a (real) quantum computer to factor  $15 = 3 \times 5$ .

# Factoring on a Quantum Computer?



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
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In 1994, Peter Shor invented a fast factorization algorithm that runs on a (hypothetical) *quantum computer* and works by determining multiplicative period of elements mod  $n$ .

- ▶ In 2001, researchers at IBM used this algorithm on a (real) quantum computer to factor  $15 = 3 \times 5$ .
- ▶ Dark clouds on horizon for RSA?

# Hash Function Attacks



- ▶ In 2004 Xiaoyun Wang and colleagues found a way to produce *collisions* for MD5:

$$\text{MD5}(\textit{file1}) = \text{MD5}(\textit{file2}) \quad !!!$$

Also for SHA-1 and many other hash functions.  
Major break!!

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- ▶ NIST now running competition for new hash function standard (SHA-3).

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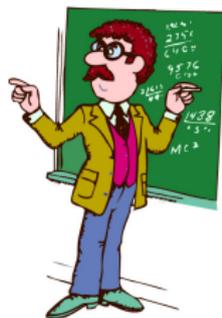
## Many new research problems and directions

- ▶ secret-sharing
- ▶ anonymity
- ▶ commitments
- ▶ multi-party protocols
- ▶ elliptic curves
- ▶ crypto hardware
- ▶ key leakage
- ▶ proxy encryption
- ▶ crypto for smart cards
- ▶ password-based keys
- ▶ random oracles
- ▶ oblivious transfer
- ▶ ...
- ▶ zero-knowledge proofs
- ▶ payment systems
- ▶ voting systems
- ▶ homomorphic encryption
- ▶ lattice-based crypto
- ▶ private information retrieval
- ▶ public-key infrastructure
- ▶ concurrent protocols
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- ▶ tweakable encryption
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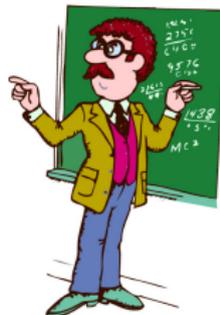
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# Zero-Knowledge Proofs

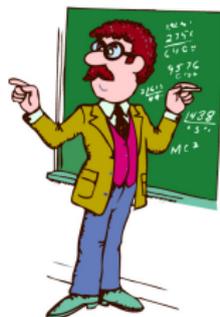


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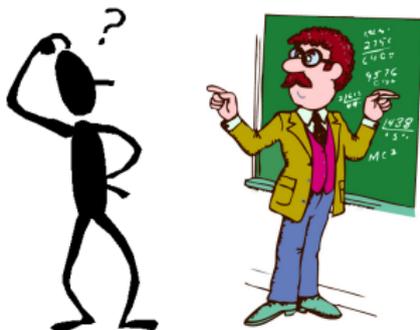
*I can convince you*

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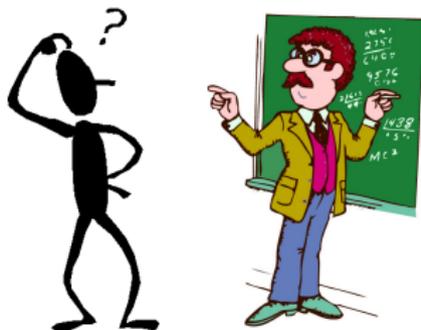
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*I can convince you  
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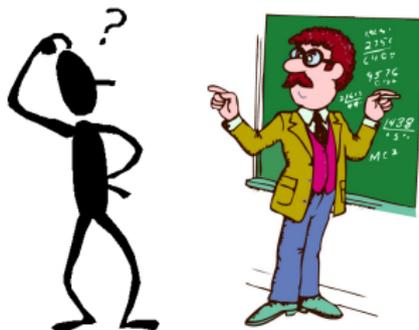
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*Goldwasser, Micali, Rackoff (1985)  
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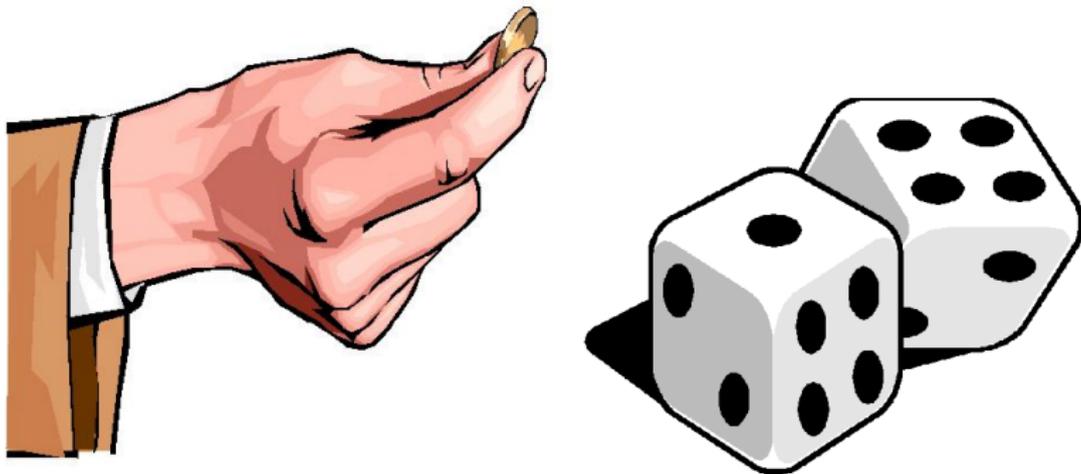


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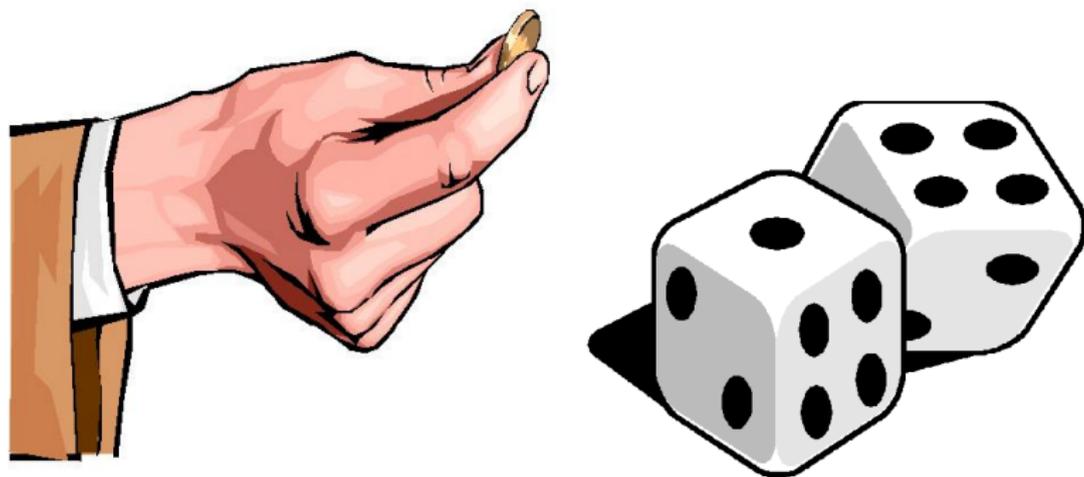
An enormously useful capability!

# Probabilistic MicroPayment System



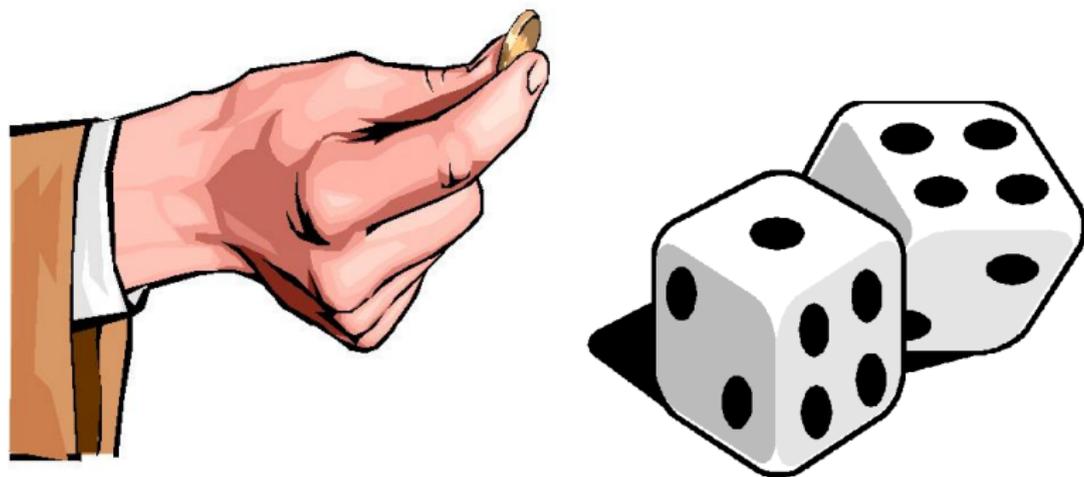
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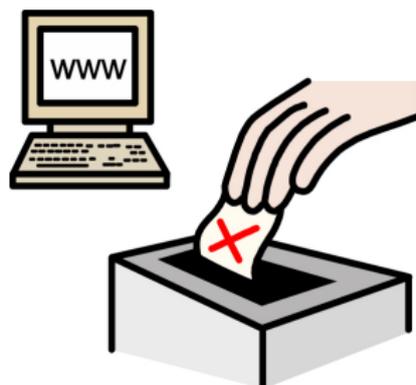
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- ▶ Peppercoin Company founded 2001, sold in 2007.

# Voting Systems



New “end-to-end” cryptographic voting systems (Chaum, Neff, Benaloh, Ryan, Rivest, Adida, ...):

- ▶ all ballots posted on web (encrypted)
- ▶ voters verify their votes are correct (while preventing vote-selling and coercion)
- ▶ anyone can verify final tally
- ▶ may be done with paper ballots

Cryptography *increases* transparency and verifiability!

# Fully Homomorphic Encryption



- ▶ In 1978, Rivest, Adleman, and Dertouzos asked, *“Can one compute on encrypted data, while keeping it encrypted?”*

# Fully Homomorphic Encryption



?



!

- ▶ In 1978, Rivest, Adleman, and Dertouzos asked, *“Can one compute on encrypted data, while keeping it encrypted?”*
- ▶ In 2009, Craig Gentry (Stanford, IBM) gave solution based on use of lattices. If efficiency can be greatly improved, could be huge implications (e.g. for cloud computing).

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# Challenges

- ▶ Make more crypto theory results practical
- ▶ Is factoring really hard?
- ▶ Minimize assumptions
- ▶ Show  $P \neq NP$  !
- ▶ Is quantum computing practical?
- ▶ Give Alice and Bob smartphones!
- ▶ Ground crypto practice better in vulnerable computer systems

## Stata Center Time Capsule Crypto Puzzle



“Time Capsule”  
(really, a large lead bag)  
sealed at LCS/AI 35th Celebration  
April 1999

- ▶ Contains many historic LCS/AI artifacts
- ▶ There is an associated cryptographic puzzle I designed to be solvable around 2034, given predicted advances in computational power.
- ▶ When puzzle is solved, Time Capsule will be opened.

## Conclusions

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- ▶ While we have accomplished a lot in a few decades, much remains to be done.
- ▶ Like Alice and Bob, cryptography is here to stay.
- ▶ Cryptography is fun!

# Acknowledgments

L. Adleman, R. Anderson, D. Bailey, M. Bellare, J. Benaloh, J. Bidzos, M. Bond, K. Bowers, M. Burmeister, J. Brainard, R. Canetti, D. Chaum, J. Clulow, M. Dertouzos, S. Devadas, Y. Dodis, W. Diffie, C. Ellison, M. Gardner, O. Goldreich, S. Goldwasser, M. Hellman, P. Herson, M. Jakobsson, A. Juels, L. Knudsen, B. Lampson, R. Lee, R. Merkle, S. Micali, A. Oprea, M. Rabin, T. Rabin, L. Reyzin, V. Rijmen, M. Robshaw, P. Ryan, A. Shamir, A. Sherman, R. Silverman, M. Sudan, D. Sutherland, M. Szydlo, Y. Tauman, L. Trevisan, E. Tromer, S. Vadhan, M. Van Dijk, D. Wagner, P. Vora, H. Wee, T. Ylonen, M. Yung

Students: B. Adida, S. Boyack, V. Boyko, M. Burnside, R. Carback, B. Chor, D. Chau, J. Clark, D. Clarke, C. Crutchfield, J. Elien, A. Essex, K. Fleming, M. Fredette, K. Fu, B. Gassend, J. Herzog, S. Hohenberger, A. Khan, B. Kaliski, T. Kotwal, J. Krishnamurthy, A. Lin, A. Lysyanskaya, M. Liskov, A. Maywah, S. Popoveniuc, Z. Ramzan, E. Shen, R. Sidney, J. Sukha, S. Weis, Y. Yin

Be Blackburn

Mom and Dad, Gail, Alex, Chris.

Thank You!