Sublinear Time Algorithms September 6, 2022

Homework 0

Lecturer: Ronitt Rubinfeld *Due Date: September 7, 2022*

Homework guidelines: The following problems are for your understanding. Do not turn in your solutions, but make sure you can solve it.

1. You are given an algorithm A for a decision problem (i.e., answer for each input is either 0 or 1), that runs in time $T(n)$ on inputs of size n, with probability of error 1/4. Show how to convert it into a new algorithm B that runs in time $O(T(n) \log 1/\beta)$ with probability of error at most β. (Hint: run A $O(\log 1/\beta)$ times and take the "majority", i.e., the most common, answer. Use Chernoff bounds to show that the correct answer is highly likely to be the output.)

2. Let f be a function which maps inputs of size n to a number. You are given an approximation scheme $\mathcal{A}$ for f such that $Pr[\frac{f(x)}{1+\epsilon} \leq \mathcal{A}(x) \leq f(x)(1+\epsilon)] \geq 3/4$, and $\mathcal{A}$ runs in time polynomial in $1/\epsilon, |x|$. Construct an approximation scheme $\mathcal{B}$ for f such that $Pr[\frac{f(x)}{1+\epsilon} \leq \mathcal{B}(x) \leq f(x)(1+\epsilon)] \geq 1-\delta$, and $\mathcal{B}$ runs in time polynomial in $\frac{1}{\epsilon}, |x|, \log \frac{1}{\delta}$.

3. (Coupon Collector Problem). Given a die with n sides. What is the expected number of times you need to roll the die in order to see each of the n sides? (Hint: Given that you saw i sides, how many times do you need to roll the die to see the $(i+1)^{st}$ side? Then use linearity of expectation.)