6.842 Randomness and Computation	February 27, 2008

Homework 2

Lecturer: Ronitt Rubinfeld — *Due Date: March 10, 2008*

Homework guidelines: Same as for homework 1.

1. For function $f : \{1,-1\}^n \to \{1,-1\}$, the NAE test chooses $x, y, z \in \{1,-1\}^n$ by choosing, independently for each i, the triple (x_i, y_i, z_i) uniformly from the set of "not all equal" triples (that is, all 3-tuples from $1, -1$ except for $(1,1,1)$ and $(-1,-1,-1)$). Then the test accepts iff the three outcomes $(f(x), f(y), f(z))$ are not all equal. Show that the probability that the NAE test passes a function f is
$$\frac{3}{4} - \frac{3}{4} \sum_{S \subseteq [n]} \left(\frac{-1}{3}\right)^{|S|} \hat{f}(S)^2$$

2. Show that if there is a PAC learning algorithm for a class C then there is a PAC learning algorithm for C with sample complexity dependence on δ that is only $\log 1/\delta$. The sample complexity dependence on the other parameters should not go up. (It is ok to assume that the learning algorithm is over the uniform distribution on inputs, although the claim is true in general.)

3. Show that for any monotone function $f : \{+1,-1\}^n \to \{+1,-1\}$, the influence of the i^{th} variable is equal to the value of the Fourier coefficient of $\{i\}$, that is $\inf_i(f) = \hat{f}(\{i\})$.

4. Show that the majority function $f(x) = \mathrm{sign}(\sum_i x_i)$ maximizes the total influence among n-variable monotone functions mapping $\{+1,-1\}^n$ to $\{+1,-1\}$, for n odd.

Useful definitions:

1. For $x = (x_1, \ldots, x_n) \in \{+1,-1\}^n$, $x^{\oplus i}$ is x with the i-th bit flipped, that is,
$$x^{\oplus i} = (x_1, \ldots, x_{i-1}, -x_i, x_{i+1}, \ldots, x_n).$$
The *influence of the i-th variable on* $f : \{+1,-1\}^n \to \{+1,-1\}$ is
$$\mathrm{Inf}_i(f) = \Pr_x\left[f(x) \neq f\left(x^{\oplus i}\right)\right].$$
The *total influence* of f is
$$\mathrm{Inf}(f) = \sum_{i=1}^{n} \mathrm{Inf}_i(f).$$

2. A function $f : \{+1,-1\}^n \to \{+1,-1\}$ is *monotone* if for all $x, y \in \{+1,-1\}^n$ such that $x_i \leq y_i$ for each i, $f(x) \leq f(y)$. Assume that $-1 < +1$.