

Homework 1

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Due Date: February 25, 2019

Turn in your solution to each problem on a separate sheet of paper, with your name on each one.

1. Given a graph G of max degree d , and a parameter ϵ , give an algorithm which has the following behavior: if G is connected, then the algorithm should pass with probability 1, and if G is ϵ -far from connected (at least $\epsilon \cdot dn$ edges must be added to connect G), then the algorithm should fail with probability at least $3/4$. Your algorithm should look at a number of edges that is independent of n , and polynomial in d, ϵ . For extra credit, try to make your algorithm as efficient as possible in terms of n, d, ϵ .

For this homework set, when proving the correctness of your algorithm, it is ok to show that if the input graph G is likely to be passed, then it is ϵ -close to a graph G' which is connected, without requiring that G' has degree at most d .

2. In class we gave an MST approximation algorithm for graphs in which the weights on each edge were integers in the set $\{1..w\}$. Show that one can get an approximation algorithm when the weights can be any value in the range $[1..w]$ (it is ok to get a slightly worse running time in terms of $w, 1/\epsilon$).
3. The diameter of an unweighted graph is the maximum distance between any pair of nodes. Give a tester for graphs with degree at most d (where d is a constant and the graph is represented in the adjacency list model) that have low diameter. The tester should have the following specific behavior:
 - (a) Graphs with diameter at most D are always accepted.
 - (b) Graphs which are ϵ -far (that is, at least ϵdn edges must be added) from having diameter $4D + 2$ are failed with probability at least $2/3$.
 - (c) The query complexity of the tester should be $O(1/\epsilon^c)$ for some constant $1 \leq c \leq \infty$.

For this homework, when proving the correctness of your algorithm, it is ok to show that if the input graph G is likely to be passed, then it is ϵ -close to a graph G' which has diameter $4D + 2$, without requiring that G' has degree at most d .

Hint: Prove that every connected graph on n nodes can be transformed into a graph of diameter at most D by adding at most $O(n/D)$ edges.