Stanford CS223B Computer Vision, Winter 2007

Lecture 12
Tracking Motion

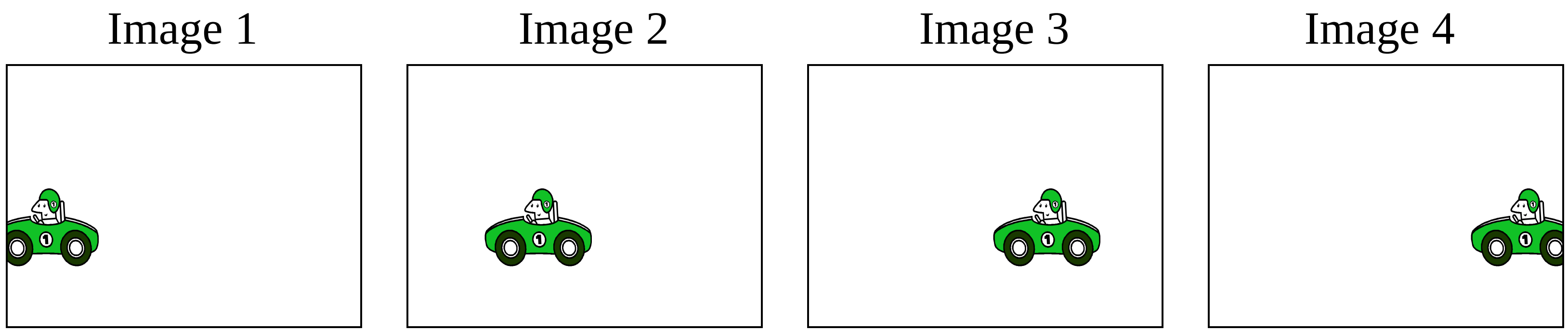
Professors Sebastian Thrun and Jana Košecká

CAs: Vaibhav Vaish and David Stavens

Overview

- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters

The Tracking Problem



- Can we estimate the position of the object?
- Can we estimate its velocity?
- Can we predict future positions?

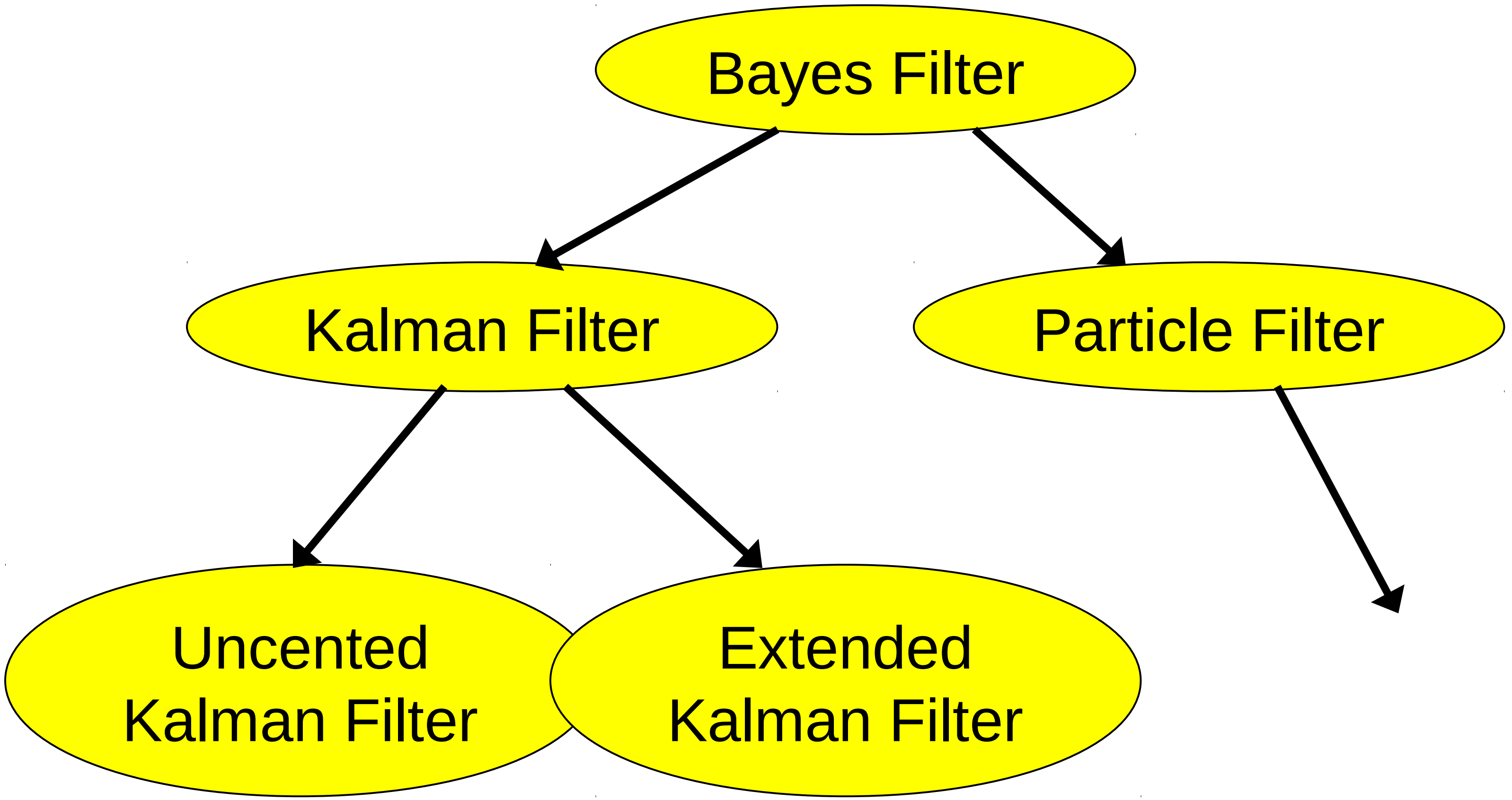
The Tracking Problem

- Given Sequence of Images
- Find center of moving object
- Camera might be moving or stationary

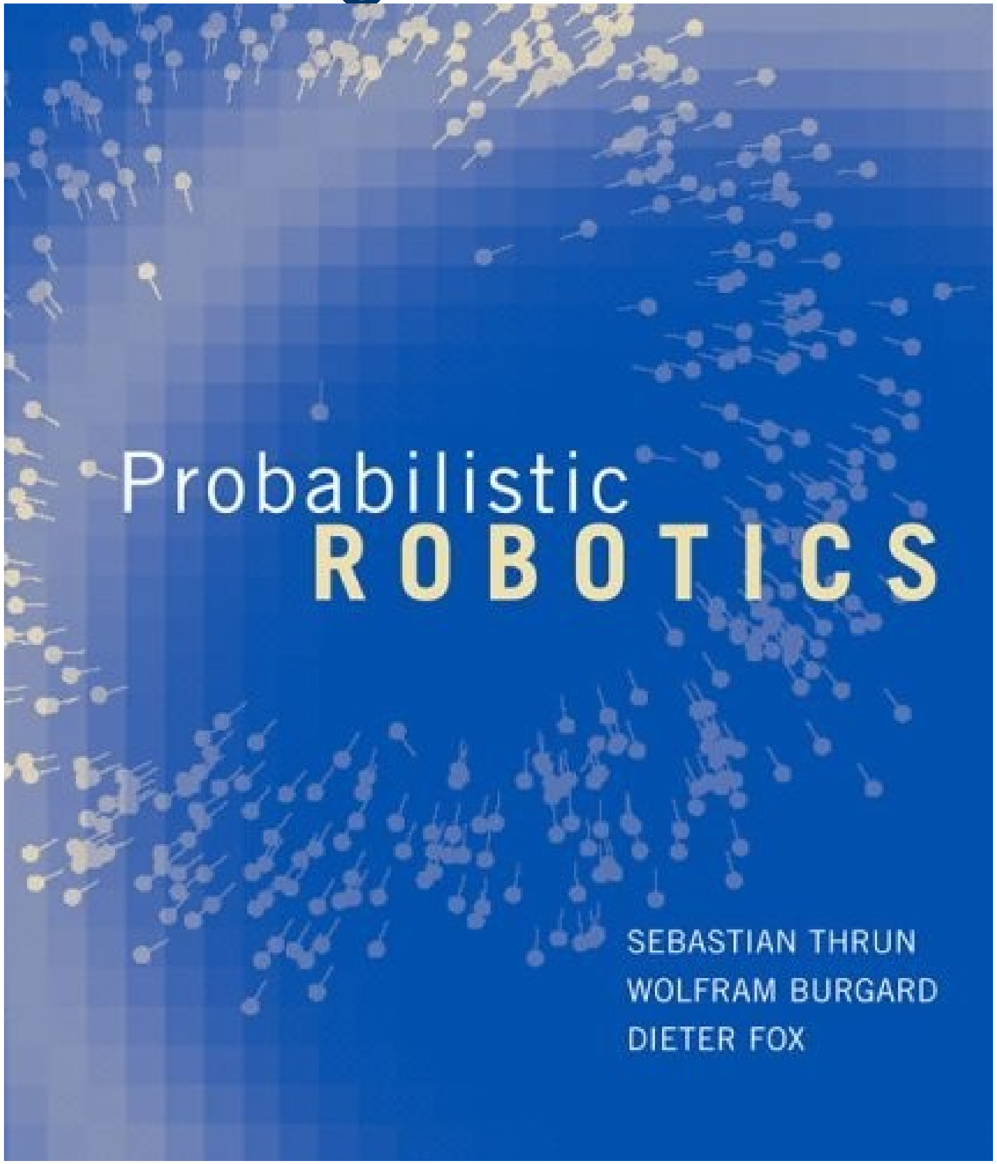
- We assume: We can find object in individual images.
- The Problem: Track across multiple images.

- Is a fundamental problem in computer vision

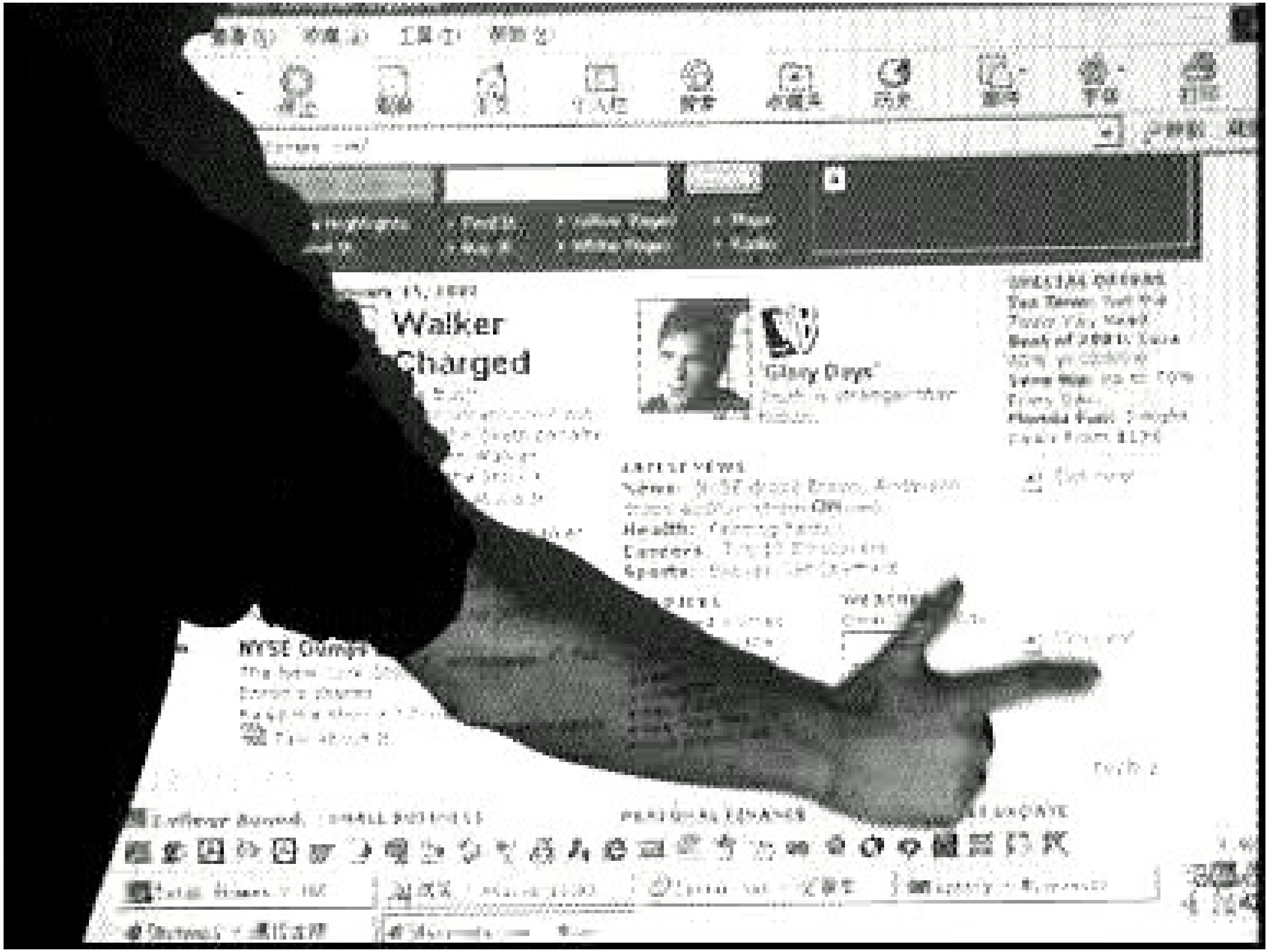
Methods


Bayes Filter
Kalman Filter
Particle Filter
Uncented
Kalman Filter
Extended
Kalman Filter

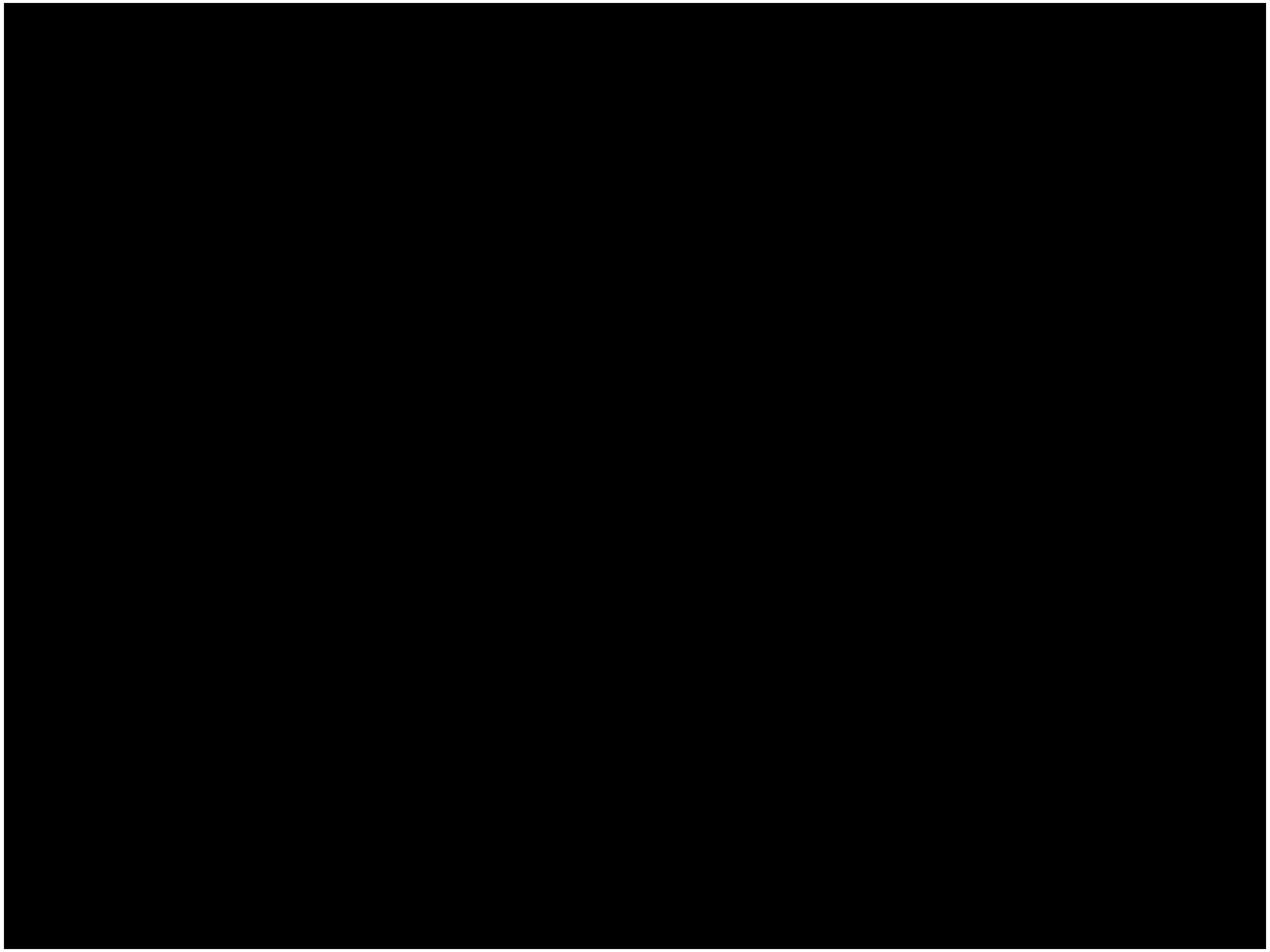
Further Reading…



Example: Moving Object

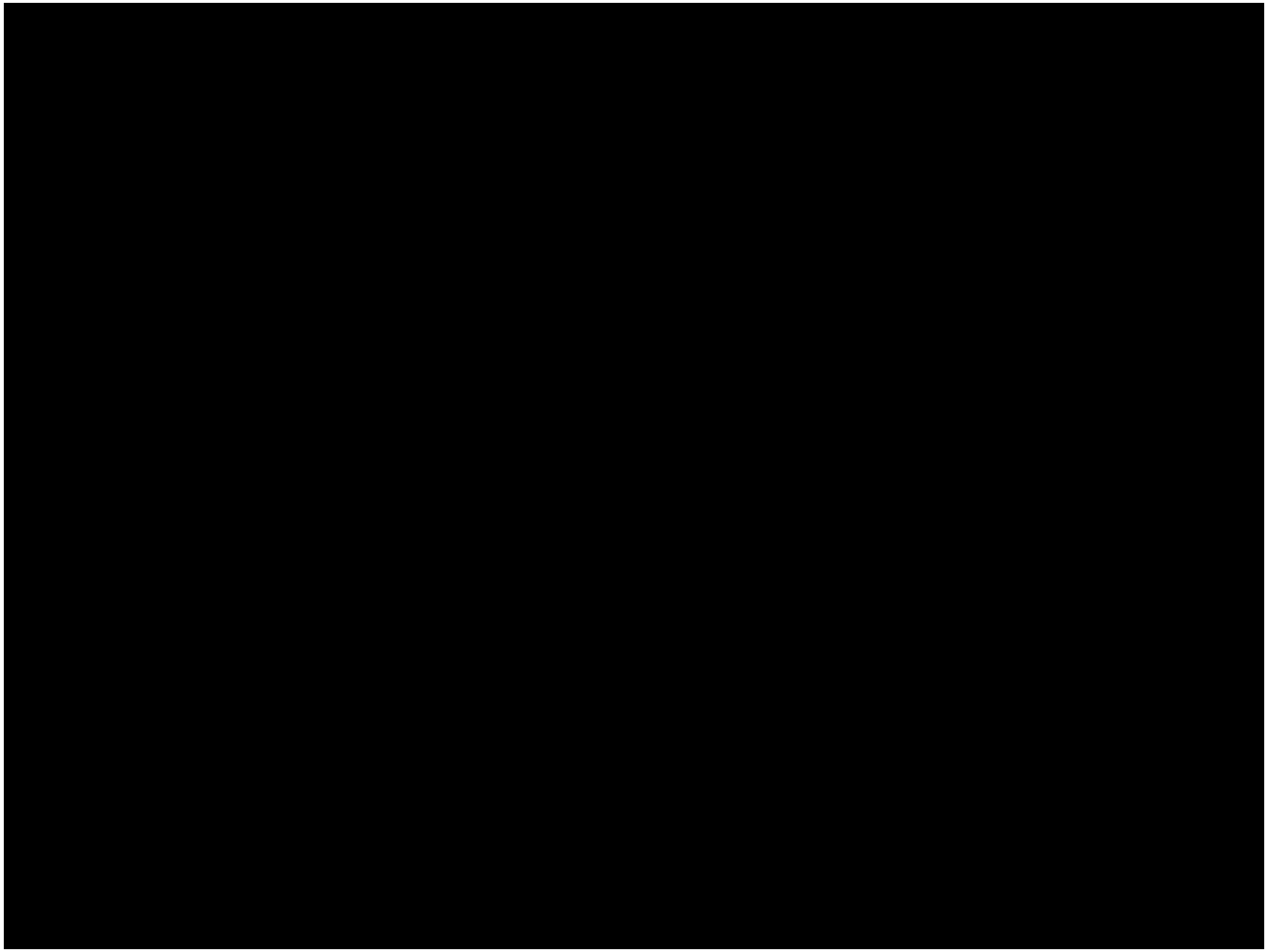


Kalman Filter Tracking



Particle Filter Tracking

Mixture of KF / PF (Unscented PF)



Overview

- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters

Example of Bayesian Inference

Cost model

Sensor model

cost(fast walk | staircase) = $1,000

p(image | staircase) = 0.7

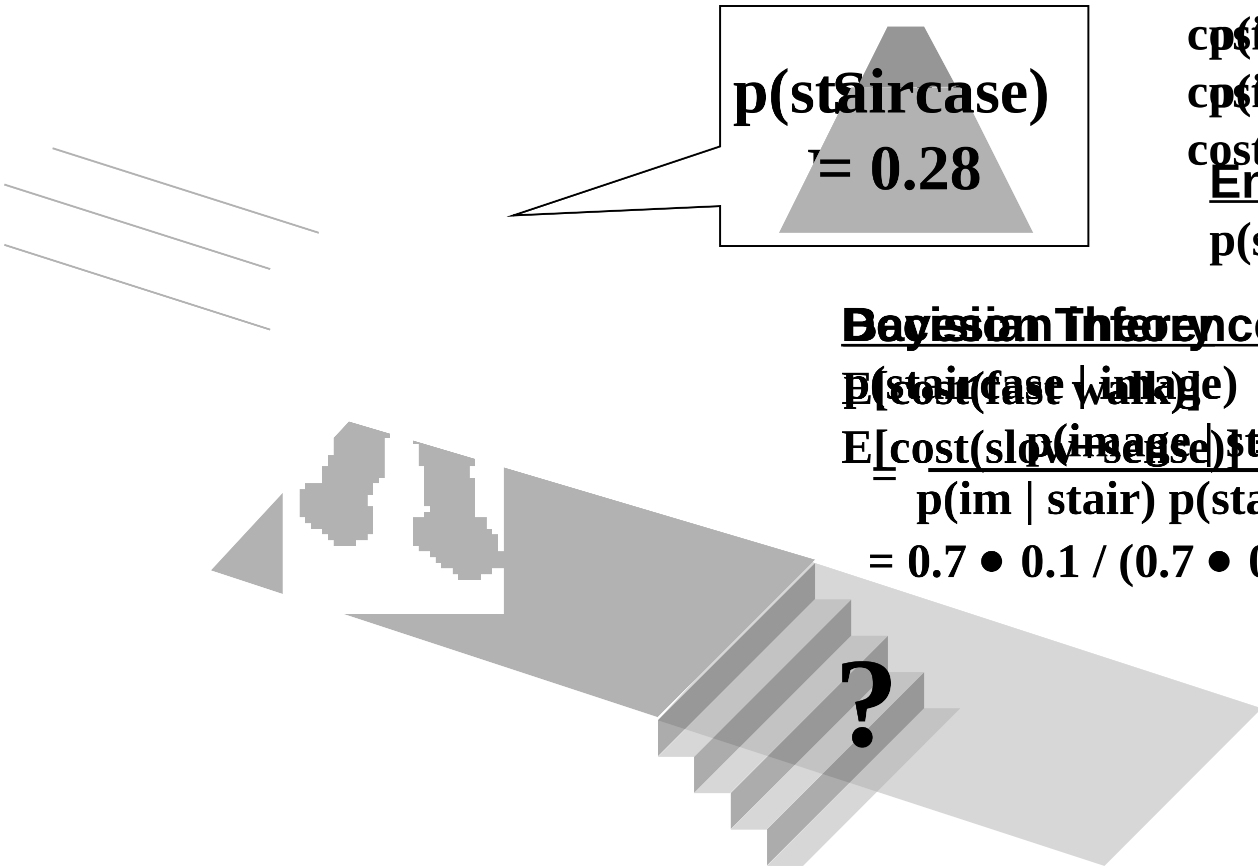
cost(fast walk | no staircase) = $0

p(image | no staircase) = 0.2

cost(slow+sense) = $1

Environment prior

p(staircase) = 0.1



Bayesian Inference

Decision Theory

$$p(\text{staircase} \mid \text{image}) = \frac{p(\text{image} \mid \text{staircase})\, p(\text{staircase})}{p(\text{im} \mid \text{stair})\, p(\text{stair}) + p(\text{im} \mid \text{no stair})\, p(\text{no stair})}$$

$$= 0.7 \bullet 0.1 / (0.7 \bullet 0.1 + 0.2 \bullet 0.9) = 0.28$$

E[cost(fast walk)] = $1,000 • 0.28 = $280

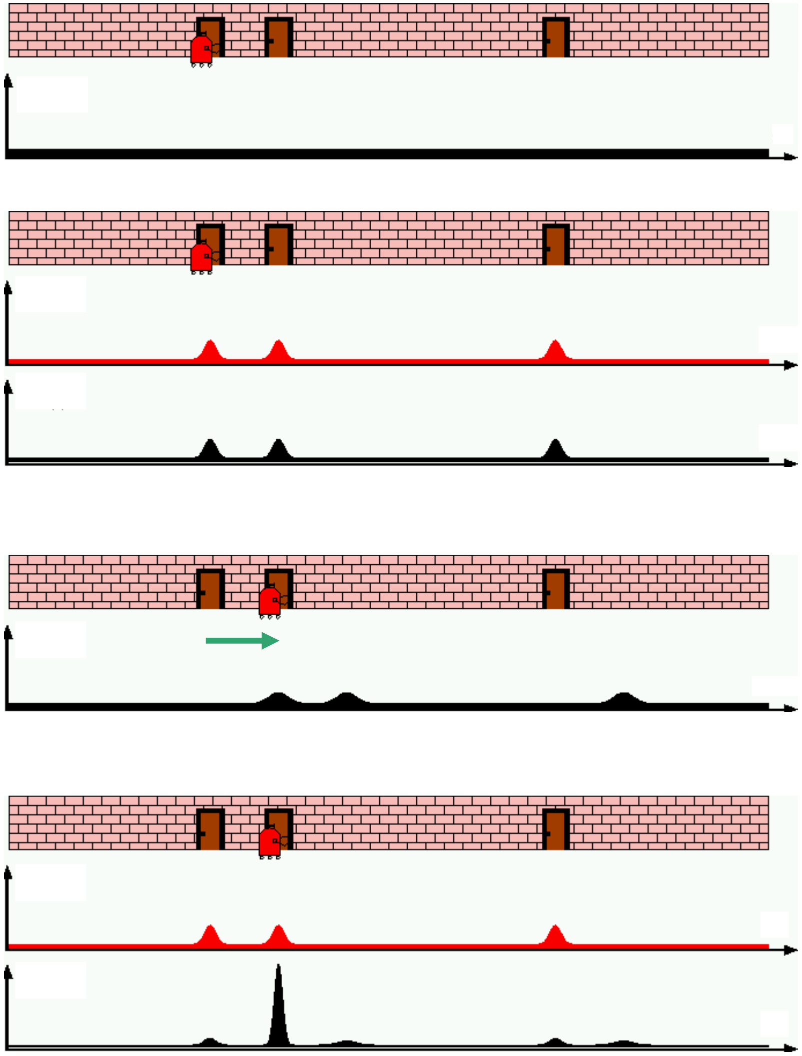
E[cost(slow+sense)] = $1

Bayes Filter Definition

- Environment state x_t
- Measurement z_t

- Can we calculate $p(x_t \mid z_1, z_2, \ldots, z_t)$?

Bayes Filters Illustrated



Bayes Filters: Essential Steps

- Belief: Bel(x_t)

- Measurement update: Bel(x_t) $\stackrel{\propto}{\leftarrow}$ Bel(x_t) p(z_t|x_t)
- Time update: Bel(x_{t+1}) $\leftarrow$ Bel(x_t) $\otimes$ $p(x_{t+1}|u_t,x_t)$

Bayes Filters

x = state
t = time
z = observation
u = action
η = constant

$$Bel\,(x_t) = p(x_t \mid z_{0\ldots t}, u_{0\ldots t})$$

$$= p(x_t \mid z_t, u_{t-1}, z_{t-1}, \ldots, u_0)$$

Bayes

$$= \eta\; p(z_t \mid x_t, u_{t-1}, z_{t-1}, \ldots, z_0)\; p(x_t \mid u_{t-1}, z_{t-1}, \ldots, z_0)$$

Markov

$$= \eta\; p(z_t \mid x_t)\; p(x_t \mid u_{t-1}, z_{t-1}, \ldots, z_0)$$

$$= \eta\; p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1}, \ldots, z_0)\; p(x_{t-1} \mid u_{t-1}, \ldots, z_0)\; dx_{t-1}$$

Markov

$$= \eta\; p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\; p(x_{t-1} \mid z_{t-1}, u_{t-2} \ldots, u_0)\; dx_{t-1}$$

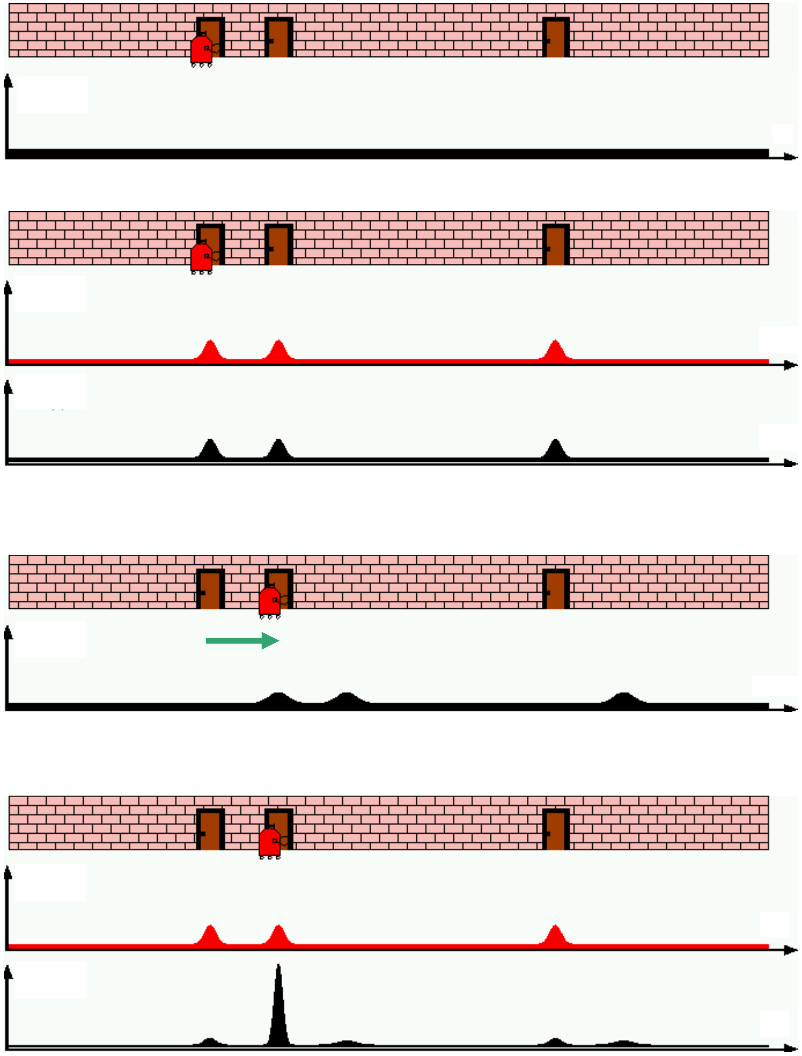
$$= \eta\; p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\; Bel\,(x_{t-1})\; dx_{t-1}$$

Bayes Filters

x = state
t = time
z = observation
u = action

$$Bel(x_t) = \eta \ p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1}$$

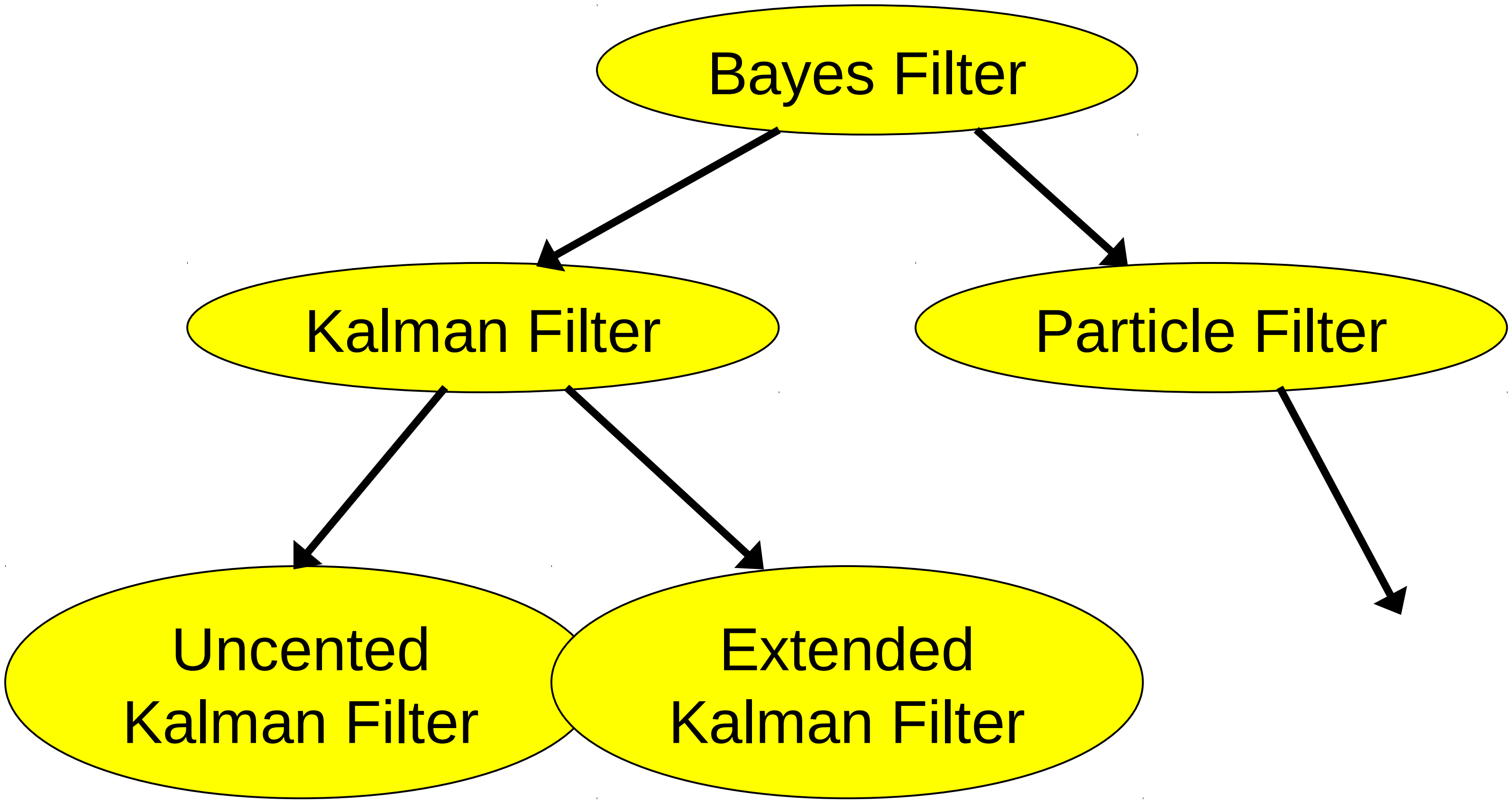
Bayes Filters Illustrated



Bayes Filters

- Initial Estimate of State
- Iterate
  - Receive measurement, update your belief (uncertainty shrinks)
  - Predict, update your belief (uncertainty grows)

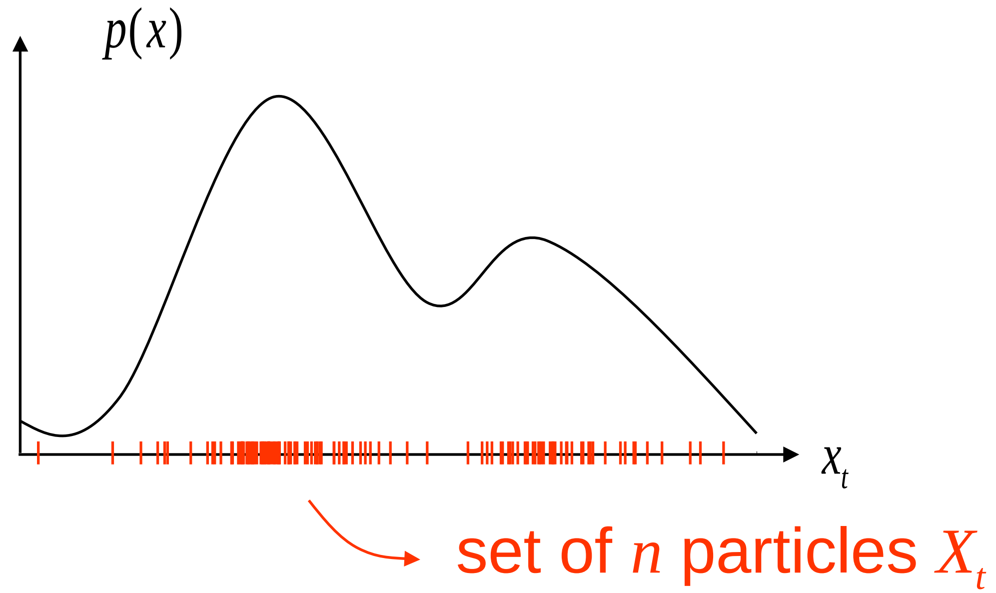
Methods


Bayes Filter
Kalman Filter
Particle Filter
Uncented
Kalman Filter
Extended
Kalman Filter

Overview

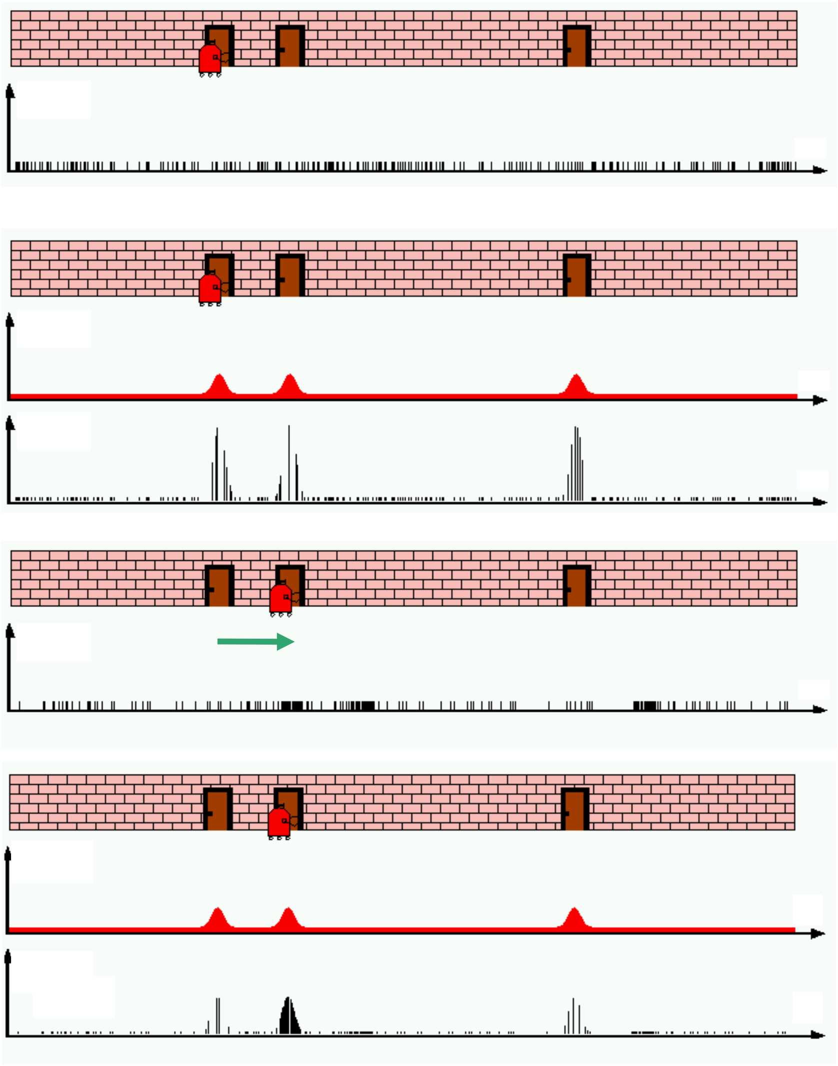
- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters

Particle Filters: Basic Idea



$$p(x_t \in X_t) \approx p(x_t \mid z_{1\ldots t}) \qquad \text{(equality for } n \uparrow \infty\text{)}$$

Particle Filter Explained



Basic Particle Filter Algorithm

Initialization:

$X_0 \leftarrow n$ particles $x_0^{[i]} \sim p(x_0)$

particleFilters(X_{t-1}){

for i=1 to n

$x_t^{[i]} \sim \mathrm{p}(x_t \mid x_{t-1}^{[i]})$ (prediction)

$w_t^{[i]} = \mathrm{p}(z_t \mid x_t^{[i]})$ (importance weights)

endfor

for i=1 to n

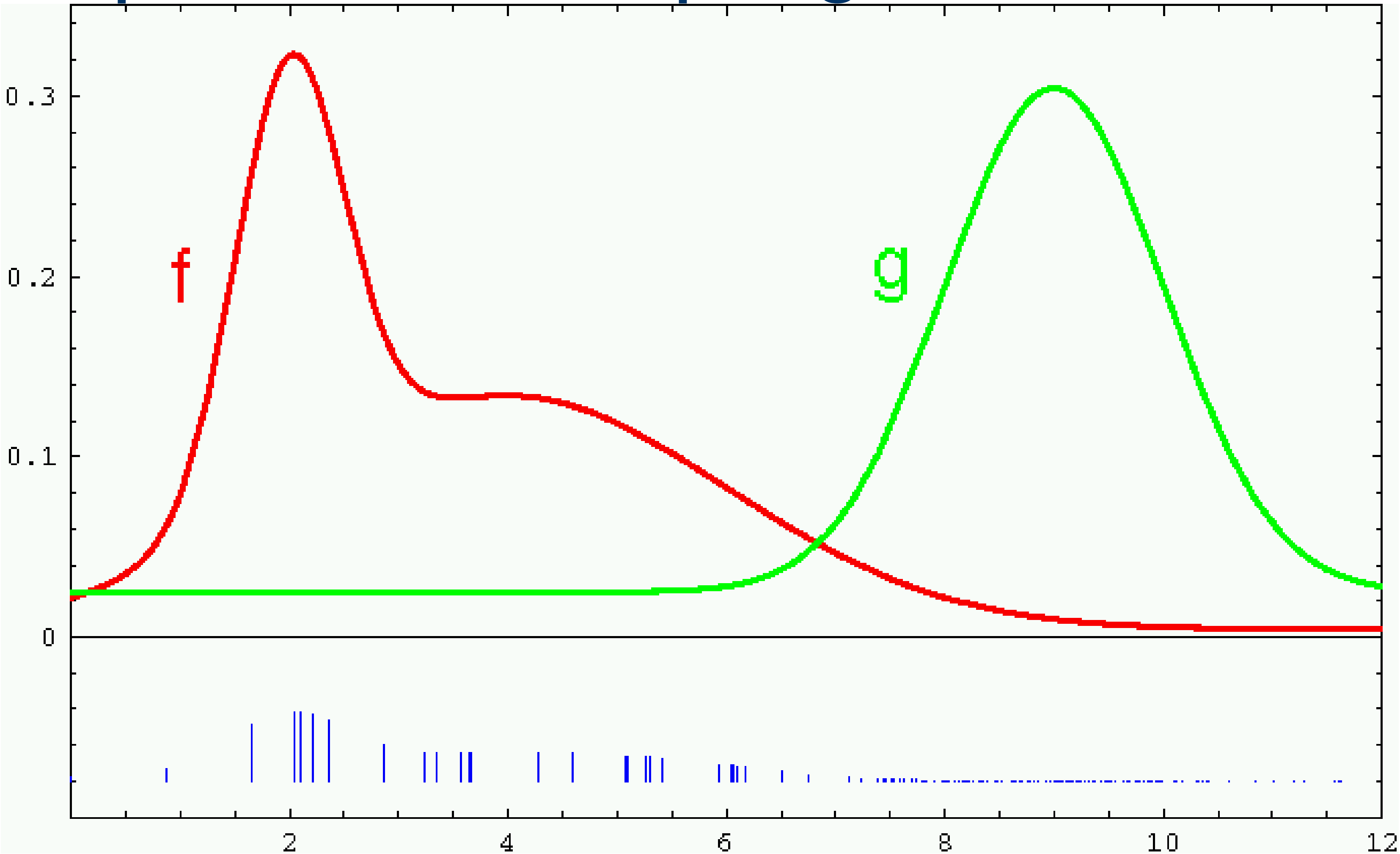
include $x_t^{[i]}$ in X_t with probability $\propto w_t^{[i]}$ (resampling)

}

$$p(x_t \mid z_{1\ldots t}, u_{1\ldots t}) = \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1} \mid z_{1\ldots t-1}, u_{1\ldots t-1})\, dx_{t-1}$$

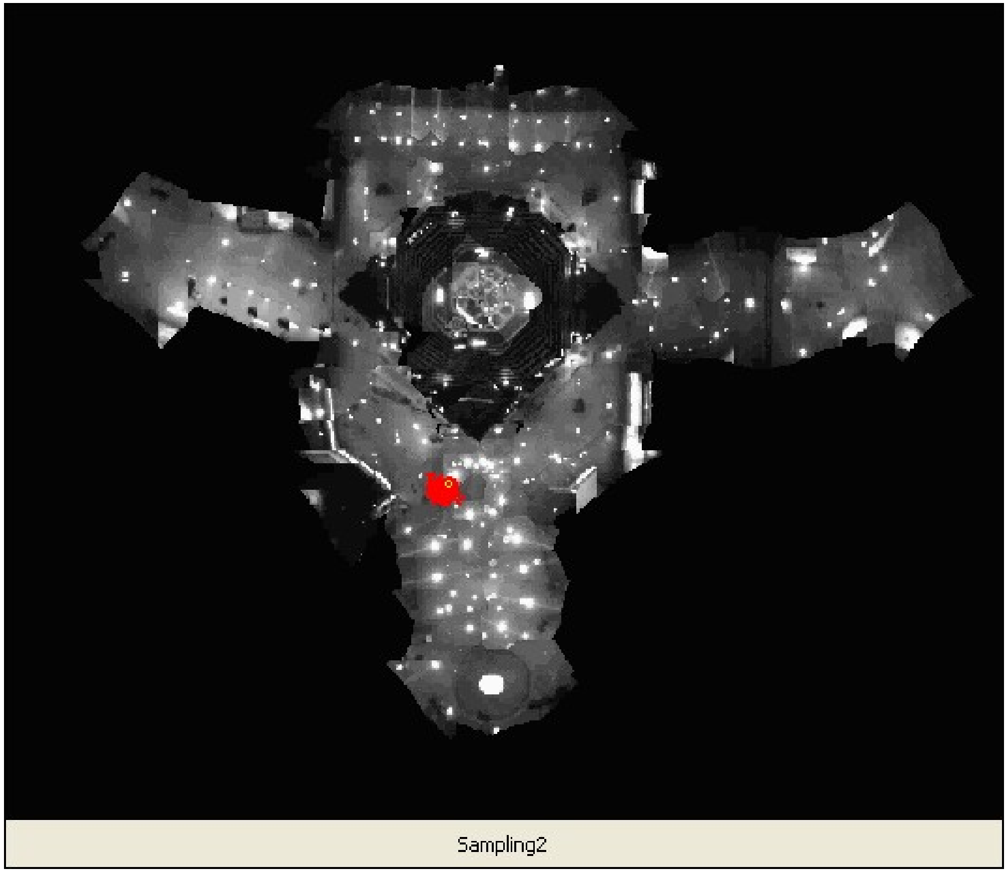
$$p(x_t \in X_t) \approx p(x_t \mid z_{1\ldots t}, u_{1\ldots t})$$

Importance Sampling



Weight samples: $w = f / g$

Particle Filter



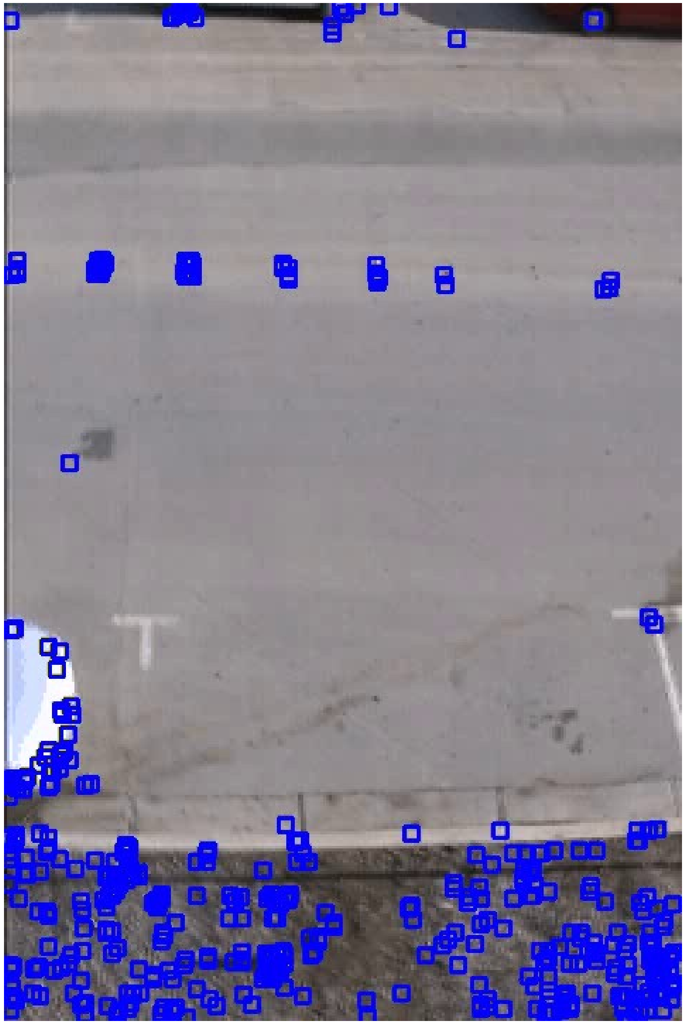
By Frank Dellaert

Case Study:
Track moving objects from Helicopter

1. Harris Corners
2. Optical Flow (with clustering)
3. Motion likelihood function
4. Particle Filter
5. Centroid Extraction

David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

1. Harris Corner Extraction

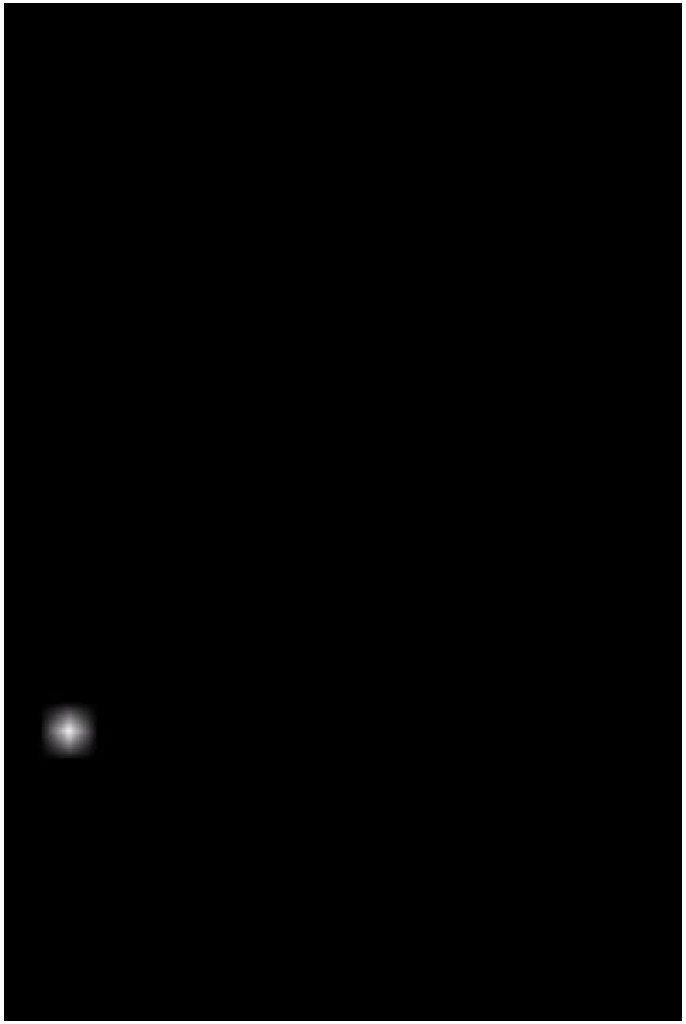


David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

2. Optical Flow + Motion Detection

David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

3. Motion Likelihood Function



David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

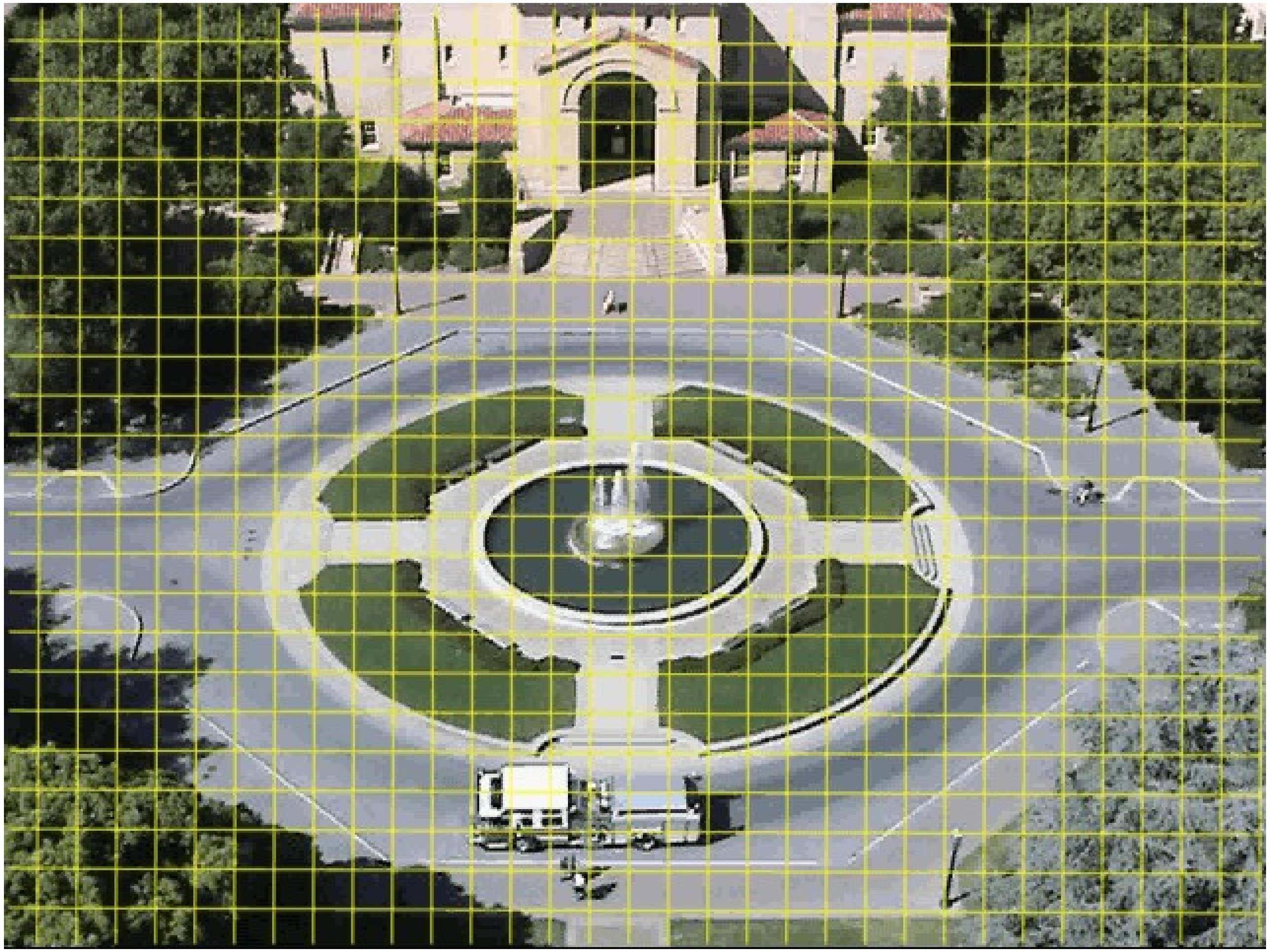
4. Particle Filters

David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

5. Extract Centroid

David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

More Particle Filter Tracking



David Stavens, Andrew Lookingbill, David Lieb, CS223b Winter 2004

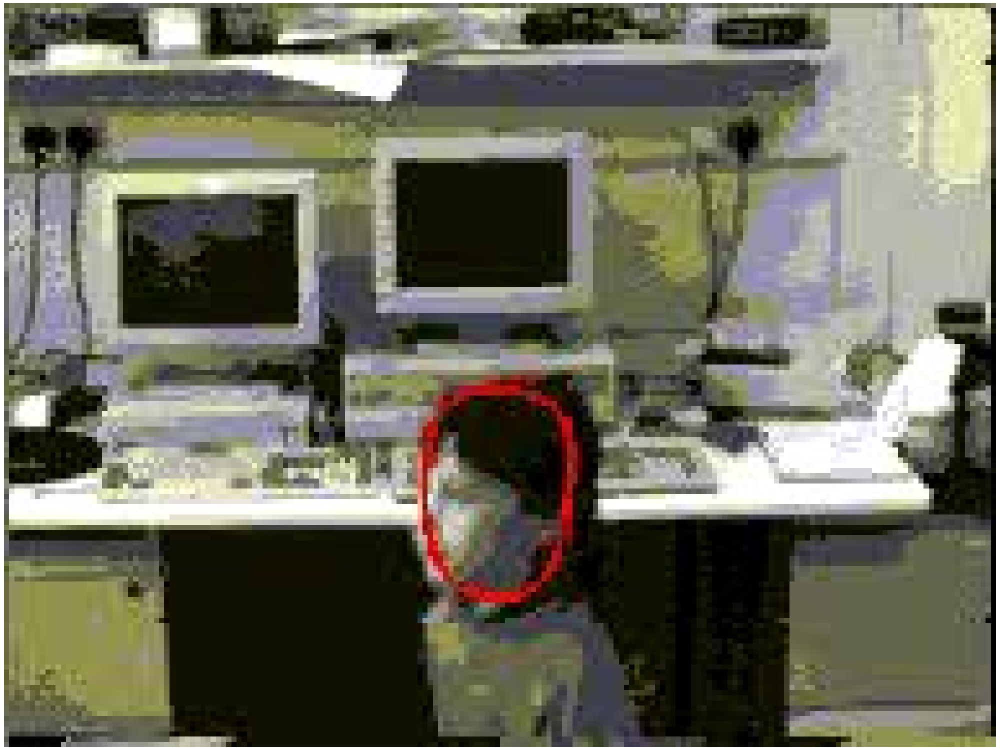
Some Robotics Examples

- Tracking Hands, People
- Mobile Robot localization
- People localization
- Car localization
- Mapping

Examples Particle Filter

Siu Chi Chan McGill University

Another Example



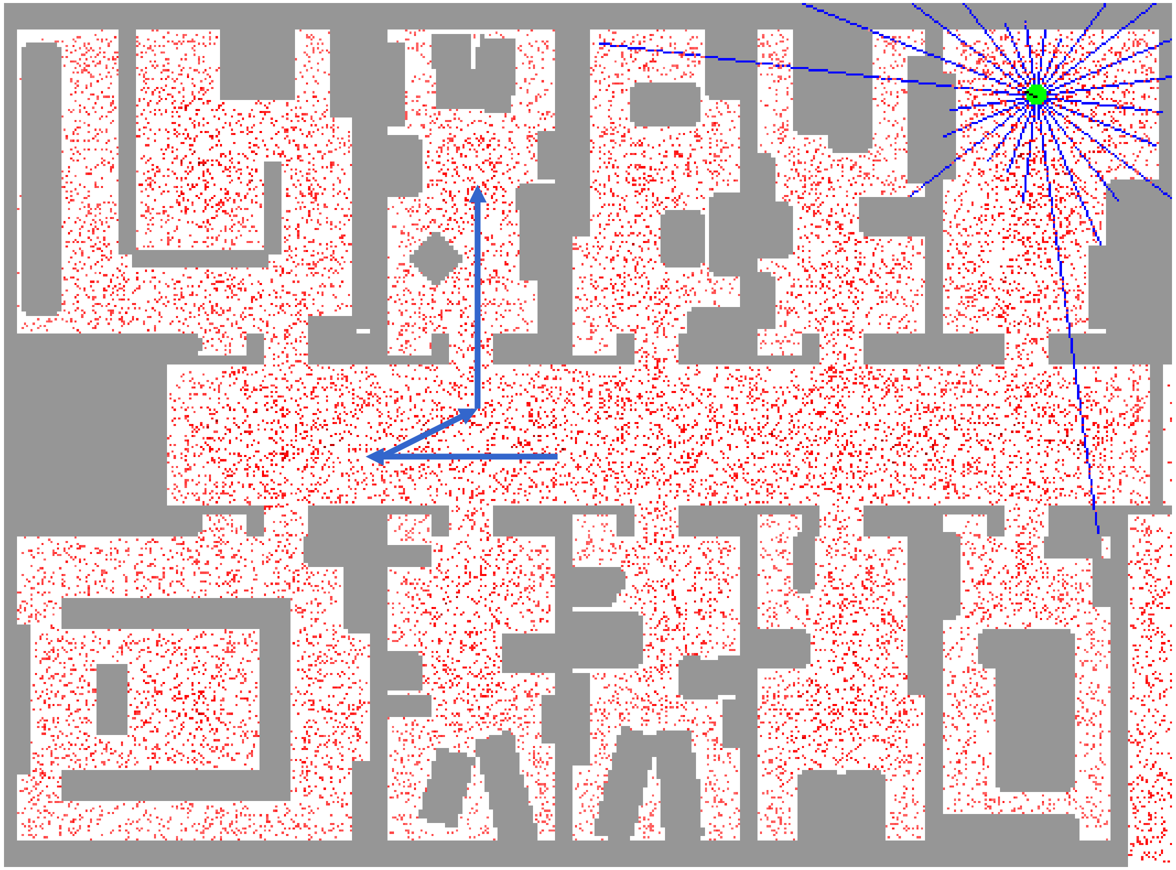
Mike Isard and Andrew Blake

Tracking Fast moving Objects

K. Toyama, A.Blake

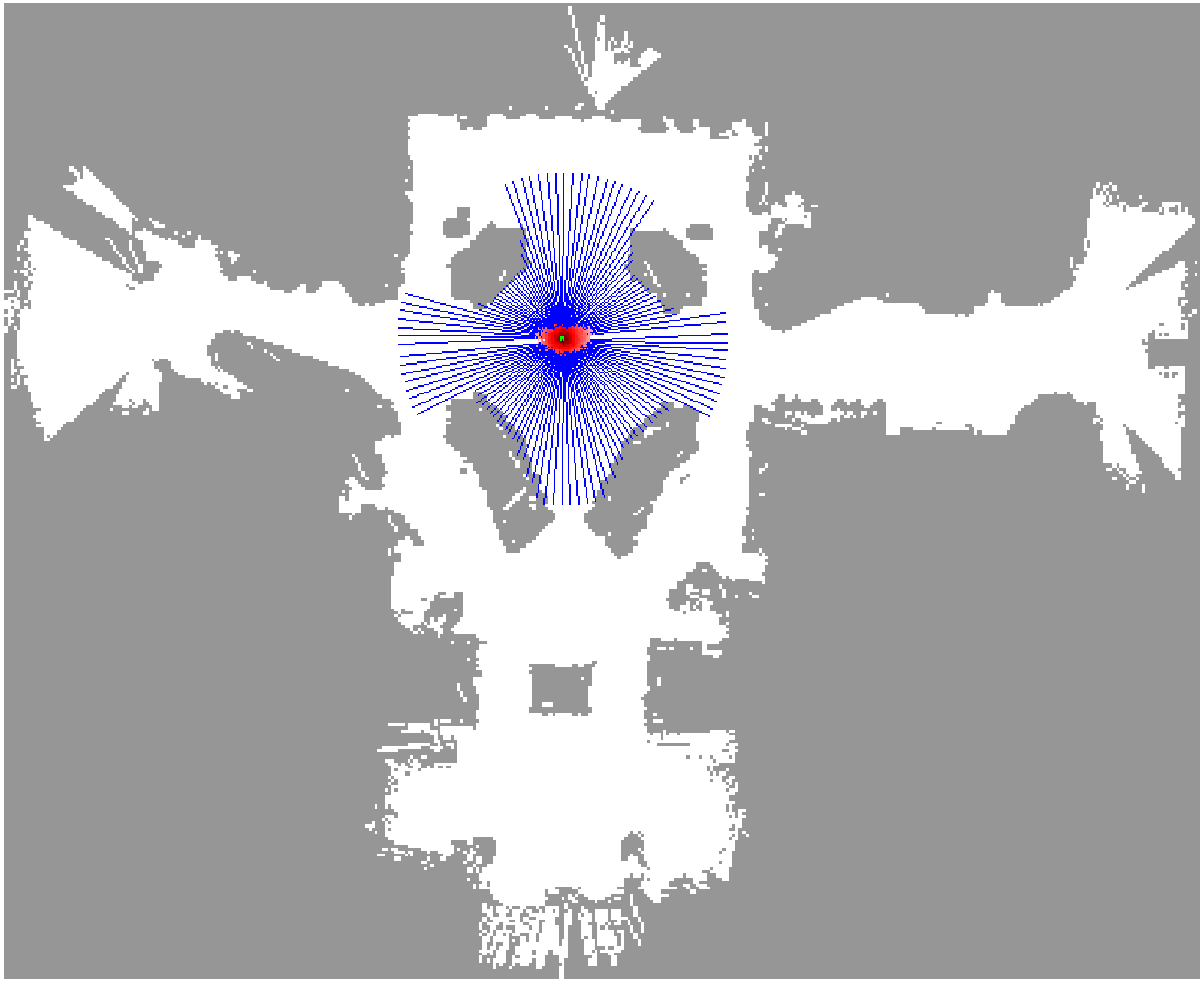
Particle Filters: Illustration

With: Wolfram Burgard, Dieter Fox, Frank Dellaert

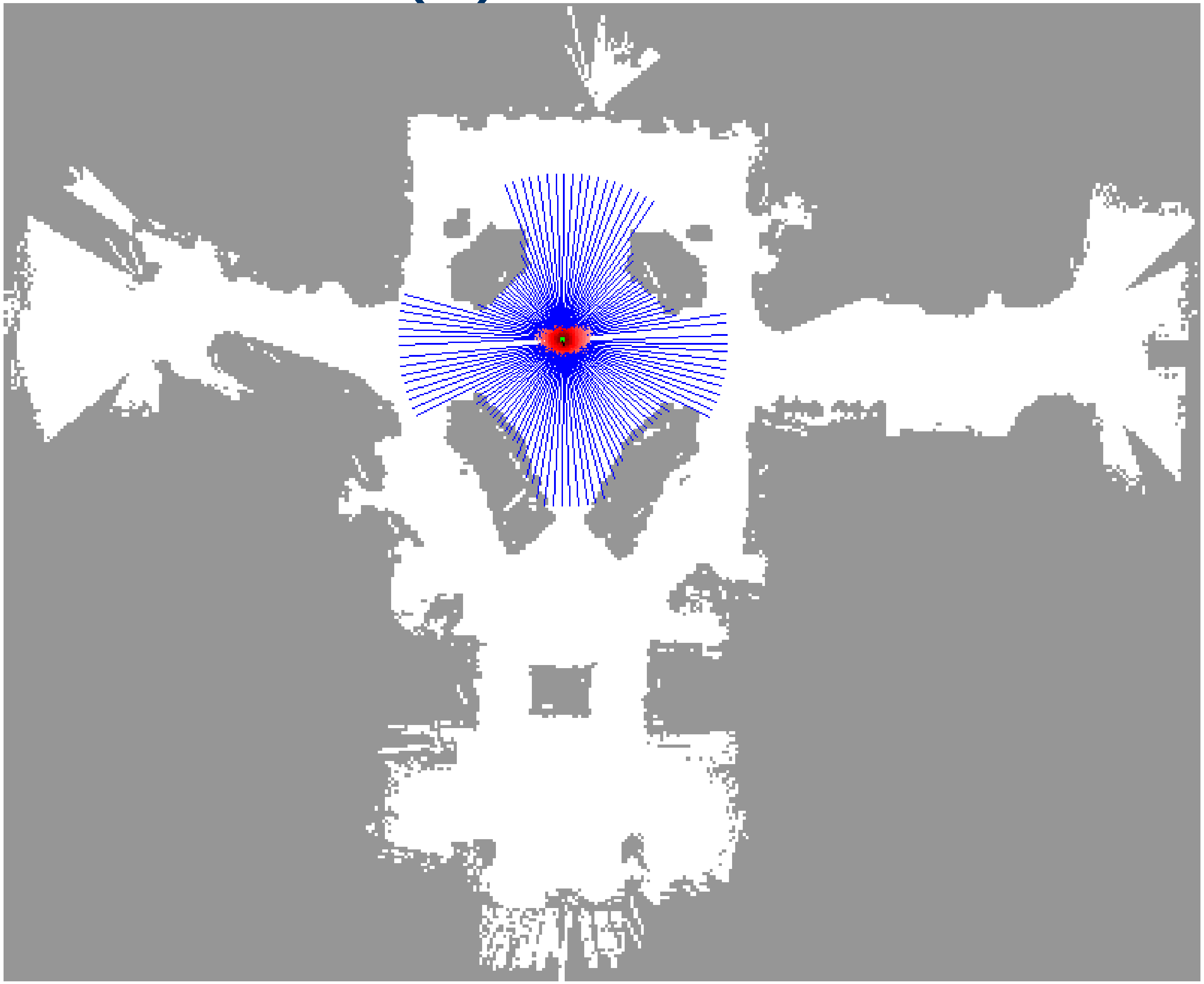


$$p(x_t \in X_t) \approx p(x_t \mid z_{1\ldots t}, u_{1\ldots t})$$

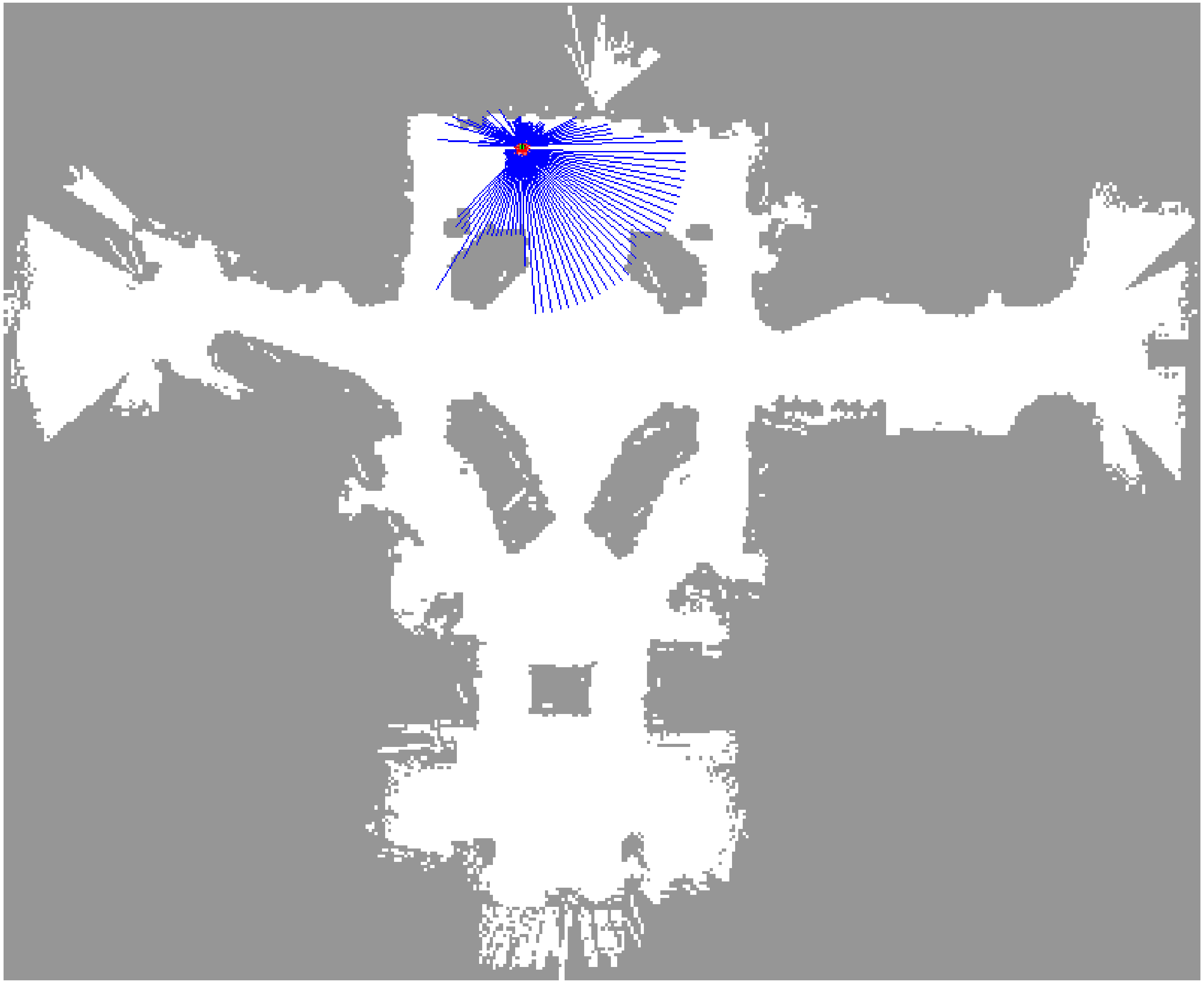
Particle Filters (1)



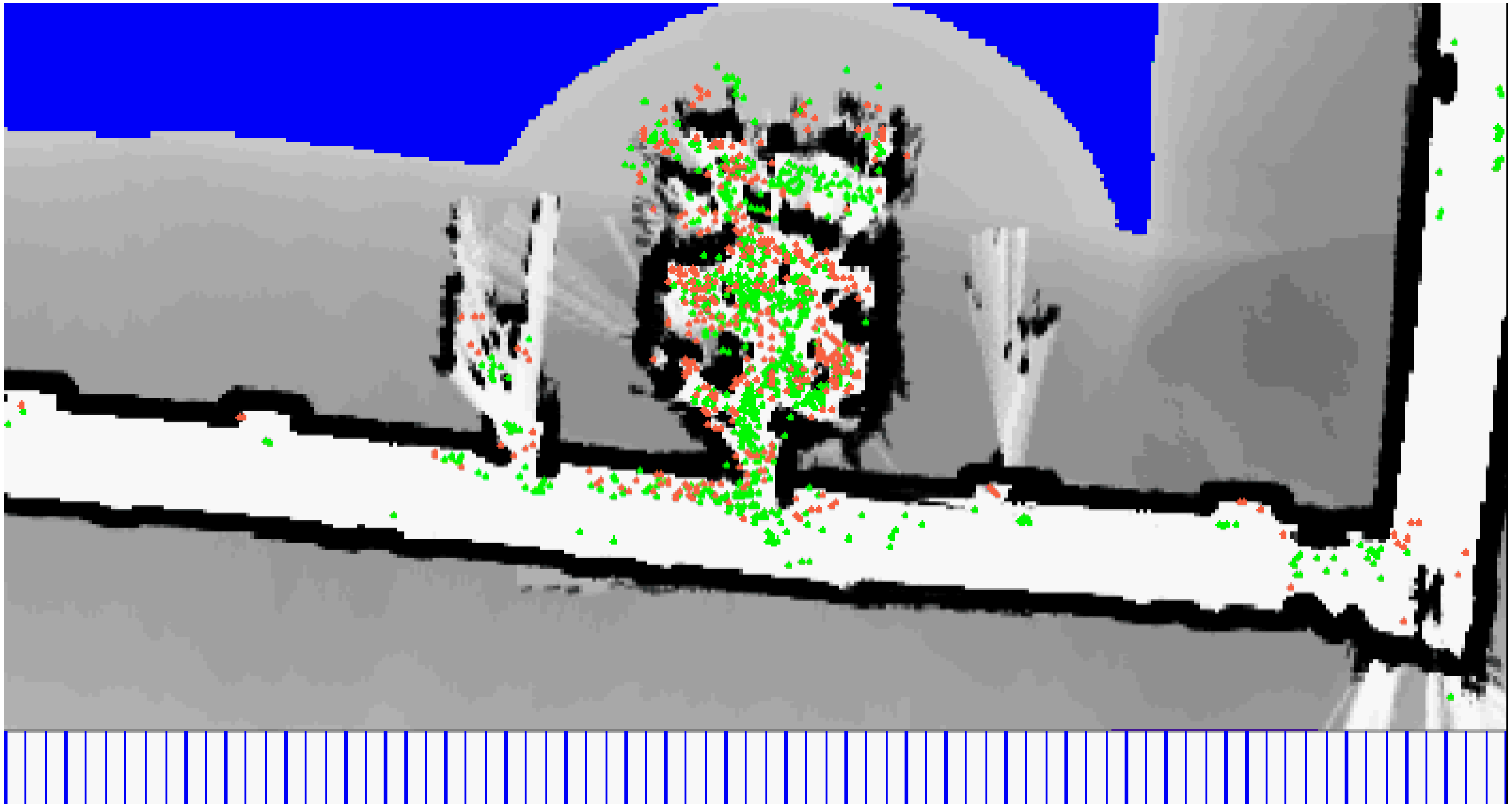
Particle Filters (2)



Particles = Robustness



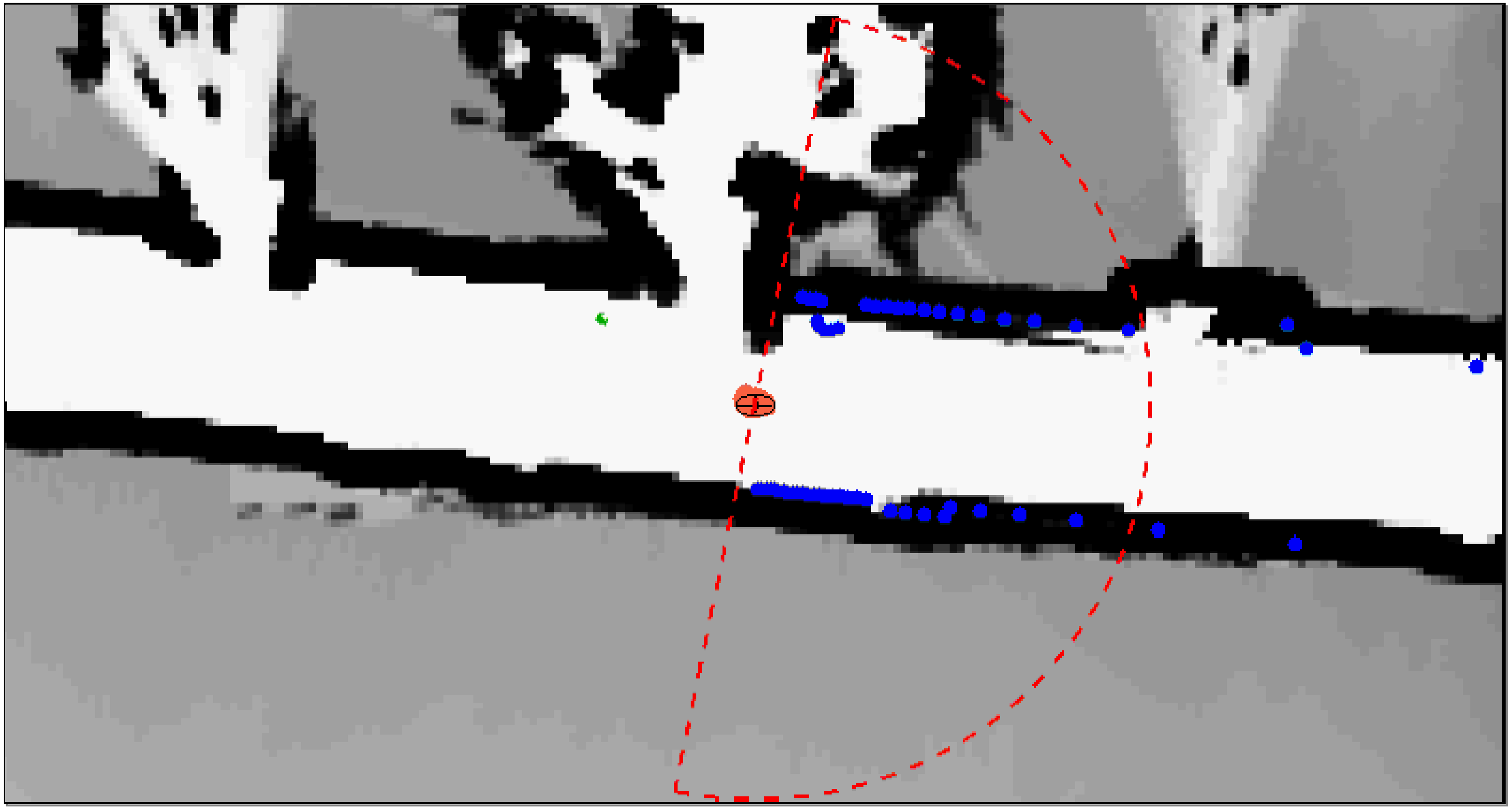
Tracking People from Moving Platform



- robot location (particles)
- people location (particles)
- laser measurements (wall)

With Michael Montemerlo

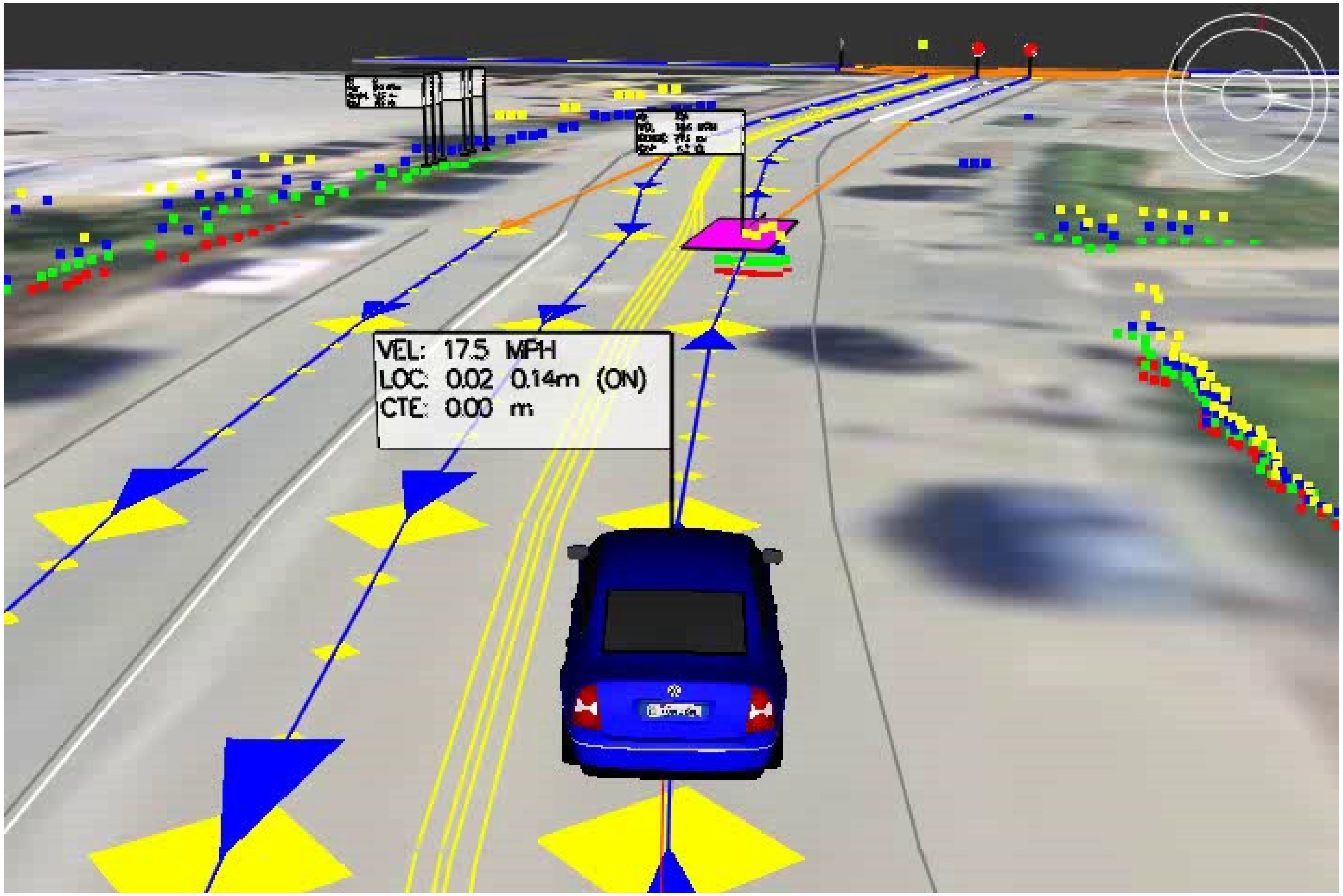
Tracking People from Moving Platform



- **robot location (particles)**
- **people location (particles)**
- **laser measurements (wall)**

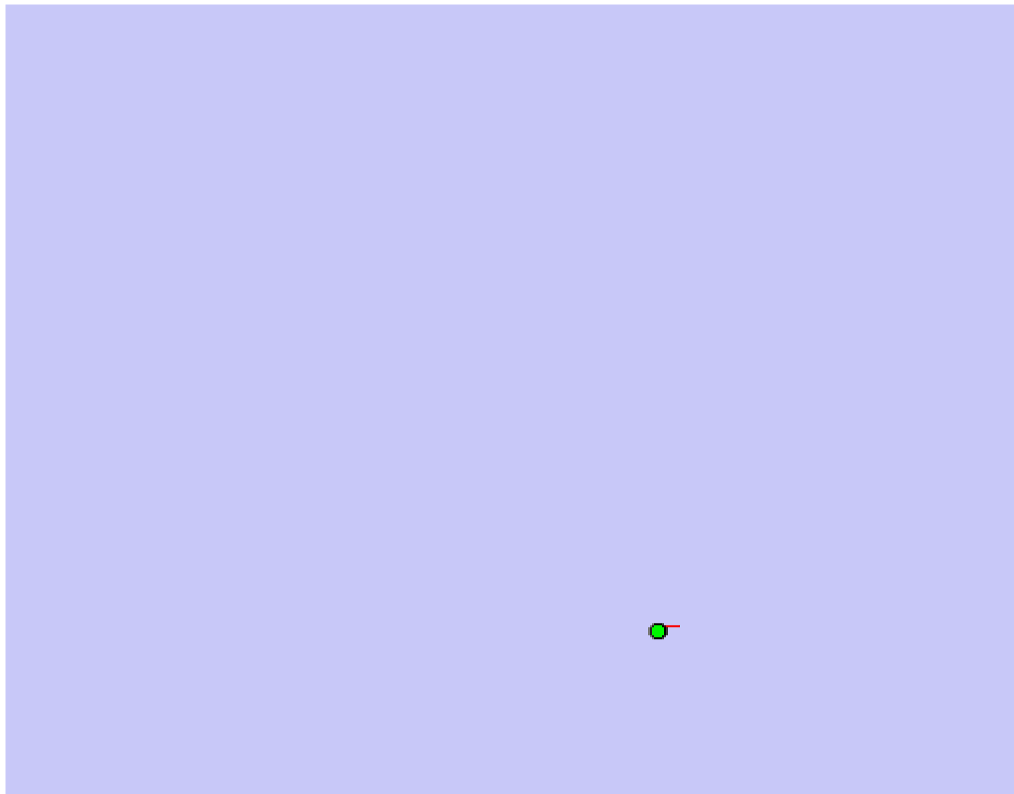
With Michael Montemerlo

Particle Filters for Tracking Cars



With Anya Petrovskaya

Mapping Environments (SFM)



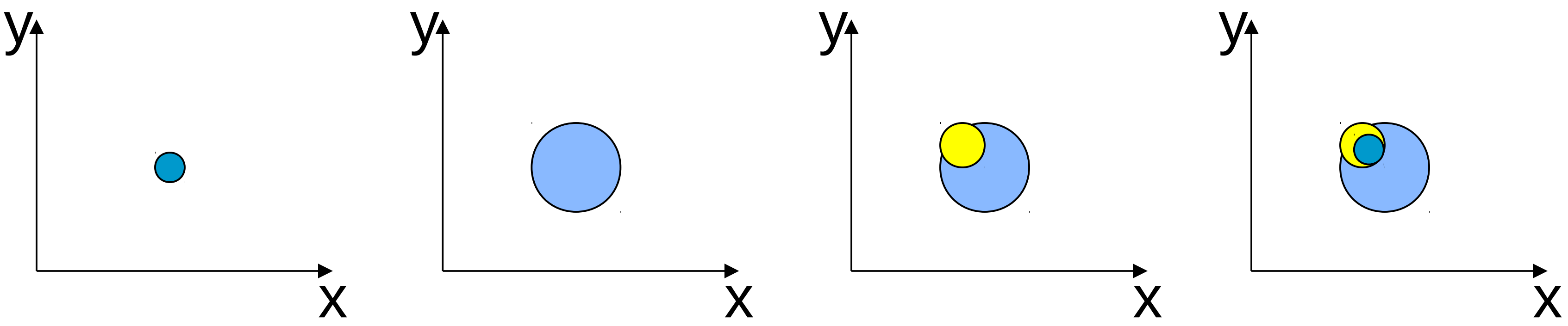
With Dirk Haehnel

Overview

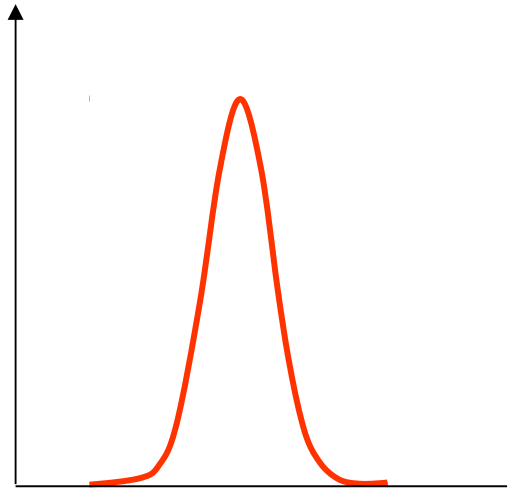
- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters

Tracking with KFs: Gaussians!

initial estimte prediction measurement update



Kalman Filters

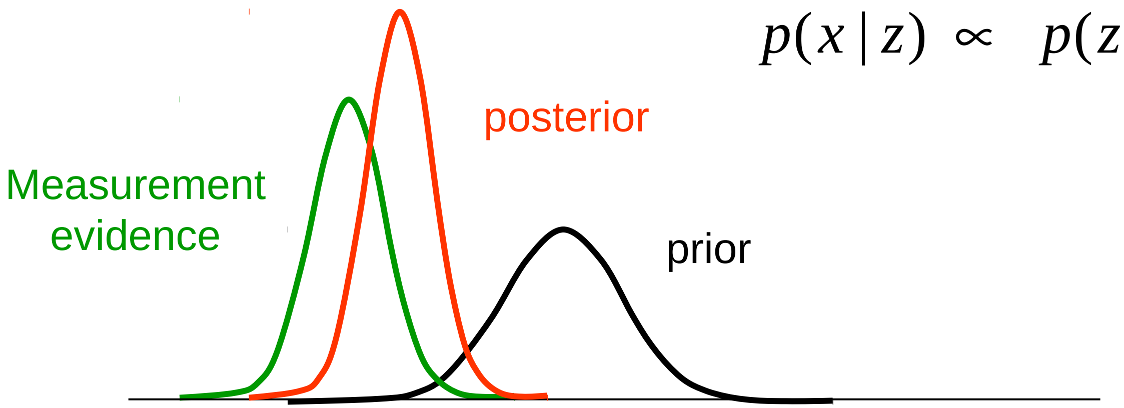


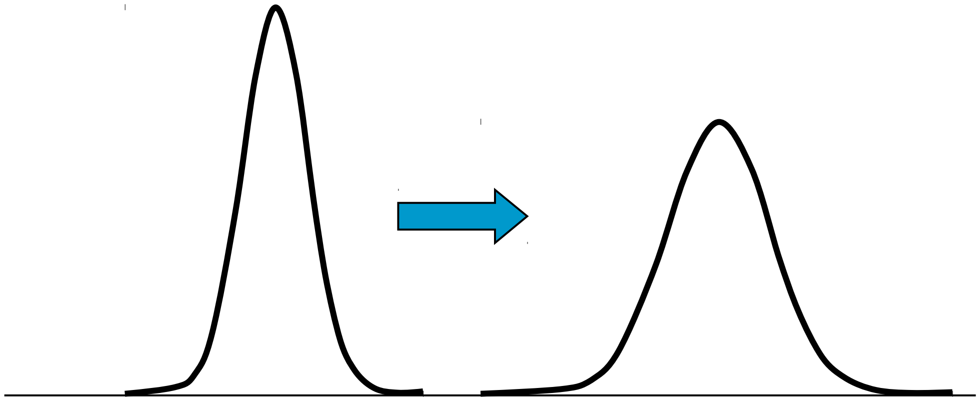
$$p(x) \sim N(\mu, \Sigma)$$

$$p(x) \propto \exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\}$$

Kalman Filters

$$p(x \mid z) \propto p(z \mid x)\, p(x)$$

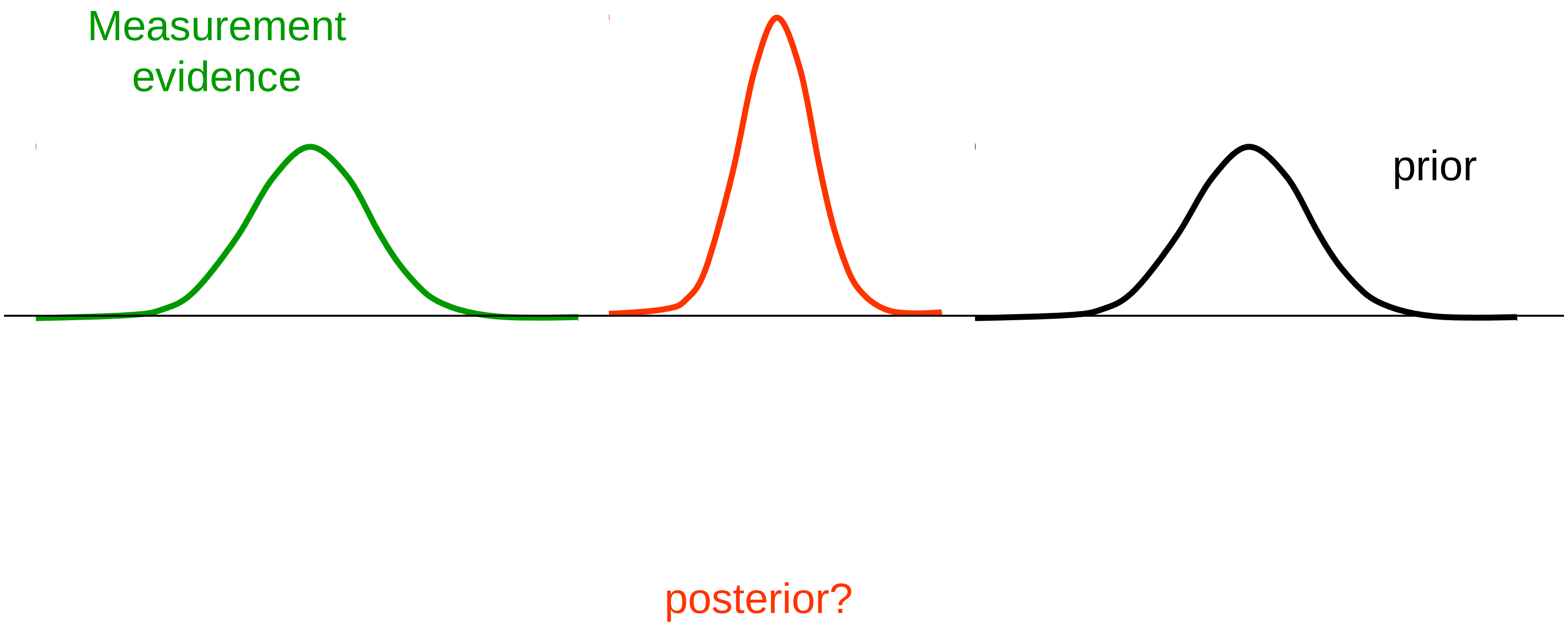




$$p(x') = \int p(x' \mid x)\ p(x)\ dx$$

A Quiz

$$p(x \mid z) \propto p(z \mid x)\, p(x)$$

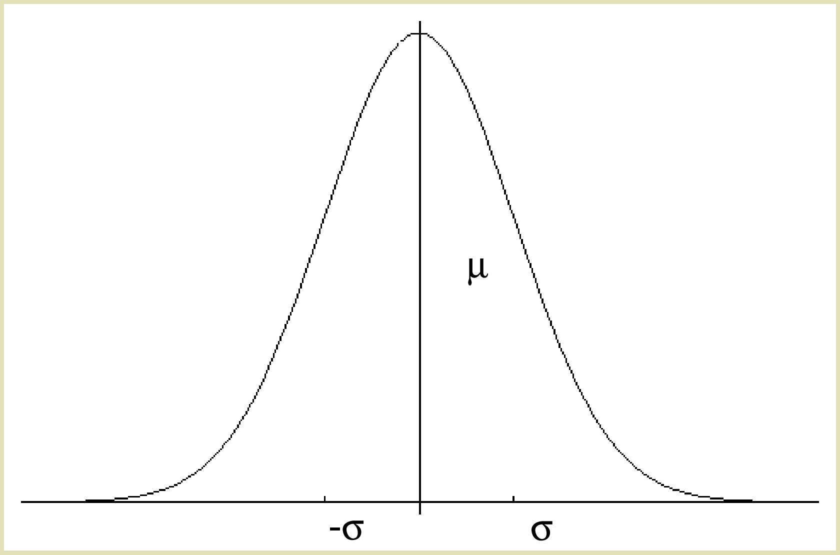


Gaussians

Univariate

$$p(x) \sim N(\mu, \sigma^2):$$

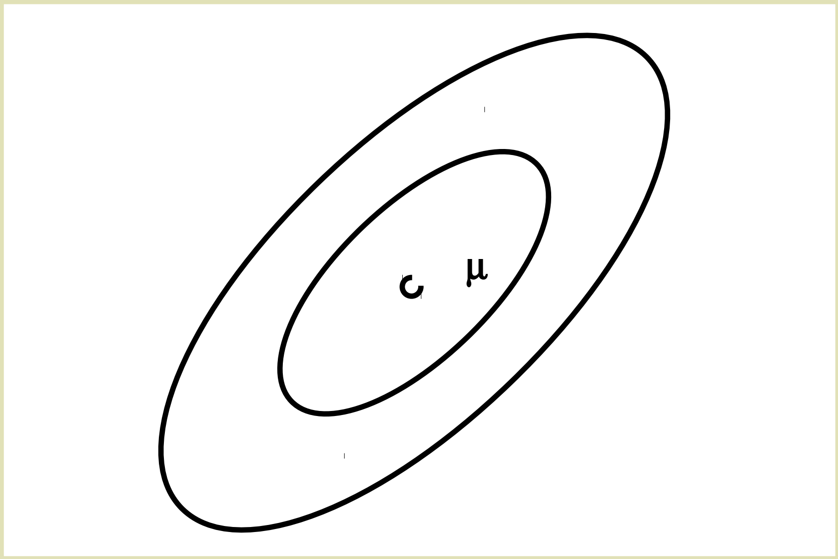
$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



Multivariate

$$p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}):$$

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2} |\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})}$$



Properties of Univariate Gaussians

$$\left.\begin{array}{l} X \sim N(\mu, \sigma^2) \\ Y = aX + b \end{array}\right\} \Rightarrow \quad Y \sim N(a\mu + b, a^2\sigma^2)$$

$$\left.\begin{array}{l} X_1 \sim N(\mu_1, \sigma_1{}^2) \\ X_2 \sim N(\mu_2, \sigma_2{}^2) \end{array}\right\} \Rightarrow p(X_1) \cdot p(X_2) \sim N\left(\frac{\sigma_2{}^2}{\sigma_1{}^2 + \sigma_2{}^2}\mu_1 + \frac{\sigma_1{}^2}{\sigma_1{}^2 + \sigma_2{}^2}\mu_2, \quad \frac{1}{\sigma_1{}^{-2} + \sigma_2{}^{-2}} \right)$$

Measurement Update Derived

$$\left.\begin{array}{l} X_1 \sim N(\mu_1, \sigma_1^{\,2}) \\ X_2 \sim N(\mu_2, \sigma_2^{\,2}) \end{array}\right\} \Rightarrow p(X_1)\cdot p(X_2) = \text{const.}\cdot \exp\left\{-\frac{1}{2}\frac{(x-\mu_1)^2}{\sigma_1^{\,2}} - \frac{1}{2}\frac{(x-\mu_2)^2}{\sigma_2^{\,2}}\right\}$$

$$\frac{\partial}{\partial x}\left\{-\frac{1}{2}\frac{(x-\mu_1)^2}{\sigma_1^{\,2}} - \frac{1}{2}\frac{(x-\mu_2)^2}{\sigma_2^{\,2}}\right\} = \frac{x-\mu_1}{\sigma_1^{\,2}} + \frac{x-\mu_2}{\sigma_2^{\,2}} = 0 \ \text{(for new } \mu)$$

$$(\mu-\mu_1)\sigma_2^{\,2} + (\mu-\mu_2)\sigma_1^{\,2} = 0$$

$$\mu(\sigma_1^{\,2}+\sigma_2^{\,2}) = \mu_1\sigma_2^{\,2} + \mu_2\sigma_1^{\,2}$$

$$\mu = \frac{\mu_1\sigma_2^{\,2} + \mu_2\sigma_1^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}$$

$$\frac{\partial^2}{\partial x^2}\left\{-\frac{1}{2}\frac{(x-\mu_1)^2}{\sigma_1^{\,2}} - \frac{1}{2}\frac{(x-\mu_2)^2}{\sigma_2^{\,2}}\right\} = \sigma_1^{\,-2} + \sigma_2^{\,-2}$$

$$\sigma = \frac{1}{\sigma_1^{\,-2} + \sigma_2^{\,-2}}$$

Properties Multivariate Gaussians

Essentially the same as in the 1-D case, but with more general notation

$$\left.\begin{array}{r} X \sim N(\mu, \Sigma) \\ Y = AX + B \end{array}\right\} \Rightarrow \quad Y \sim N(A\mu + B, A\Sigma A^T)$$

$$\left.\begin{array}{r} X_1 \sim N(\mu_1, \Sigma_1) \\ X_2 \sim N(\mu_2, \Sigma_2) \end{array}\right\} \Rightarrow p(X_1) \cdot p(X_2) \sim N\left((\Sigma_1 + \Sigma_2)^{-1} \Sigma_2 \mu_1 + (\Sigma_1 + \Sigma_2)^{-1} \Sigma_1 \mu_2, \quad (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1} \right)$$

- We stay in the “Gaussian world” as long as we start with Gaussians and perform only linear transformations.

Linear Kalman Filter

Estimates the state x of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

with a measurement

$$z_t = C_t x_t + \delta_t$$

Components of a Kalman Filter

A_t Matrix (n $\times$ n) that describes how the state evolves from t to $t+1$ without controls or noise.

B_t Matrix (n $\times$ i) that describes how the control u_t changes the state from t to $t+1$.

C_t Matrix (k $\times$ n) that describes how to map the state x_t to an observation z_t.

ε_t

δ_t

Random variables representing the process and measurement noise that are assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

Kalman Filter Algorithm

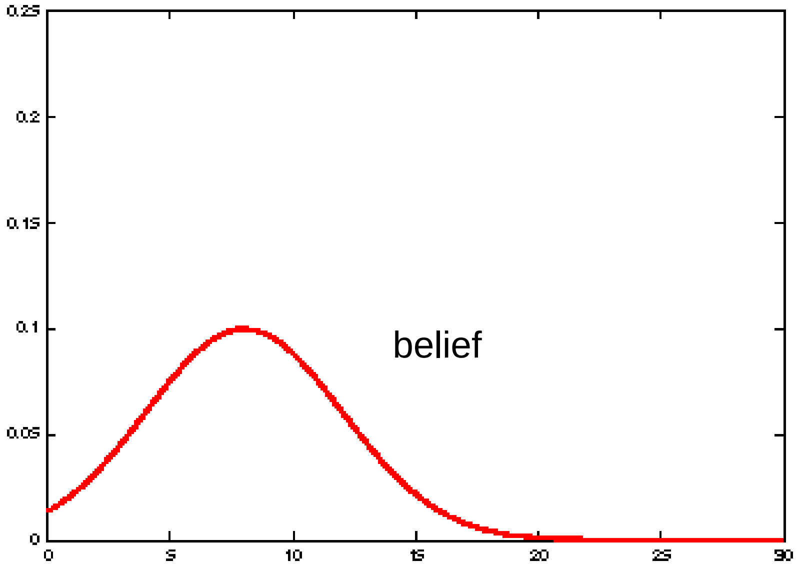
1. Algorithm **Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

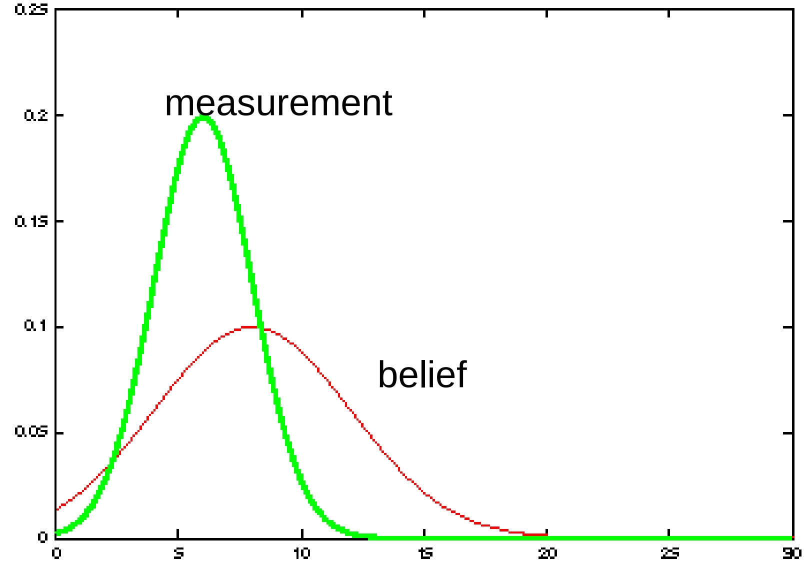
2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

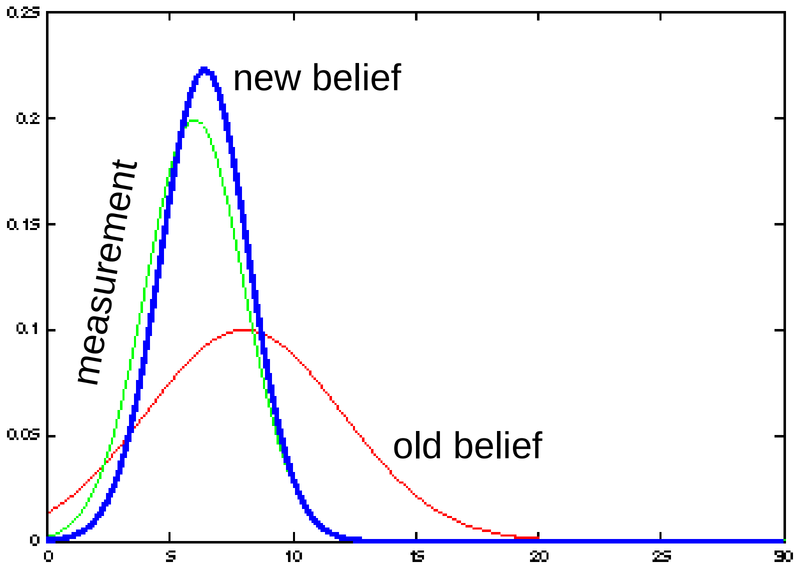
5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

Kalman Filter Updates in 1D



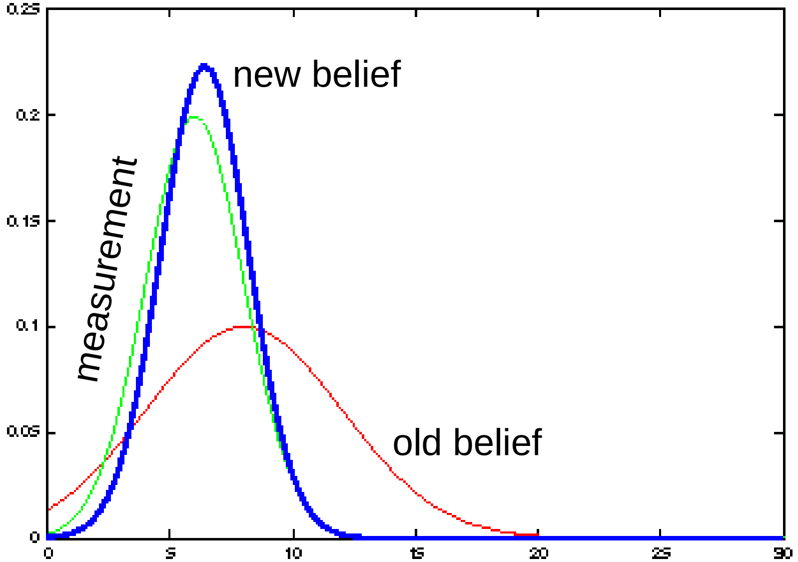




Kalman Filter Updates in 1D

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t (z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases} \qquad \text{with} \quad K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

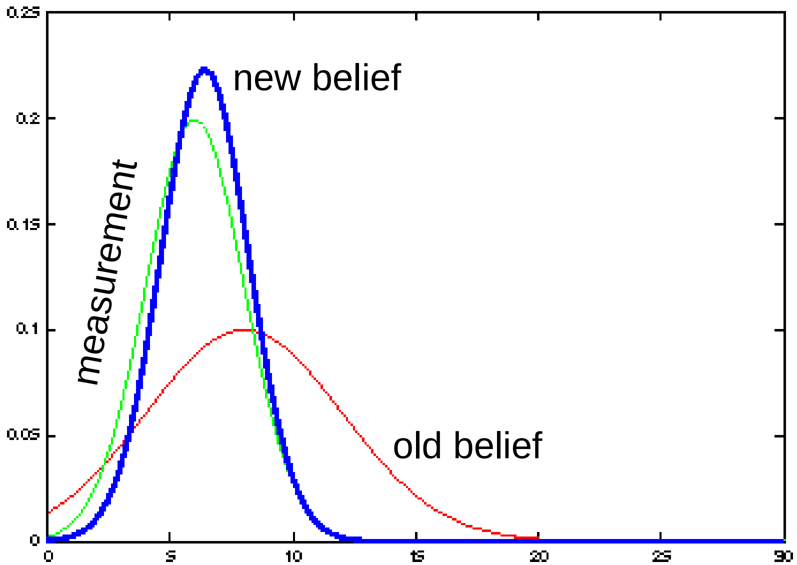
$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases} \qquad \text{with} \quad K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

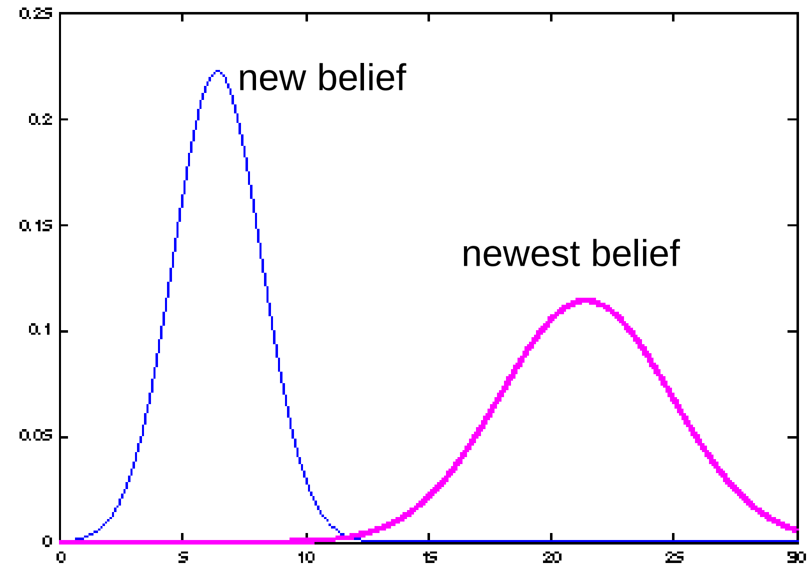


Kalman Filter Updates in 1D

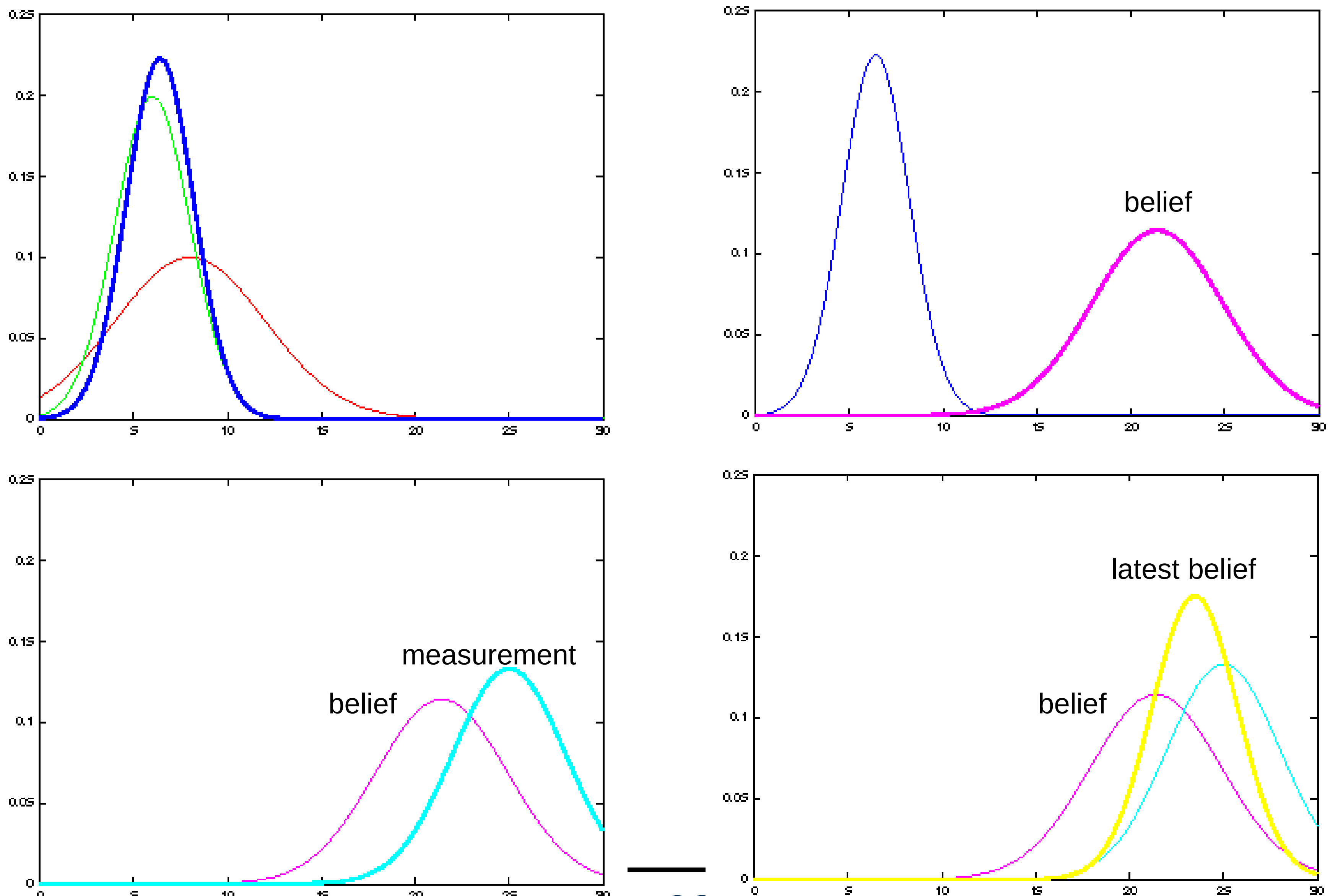
$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

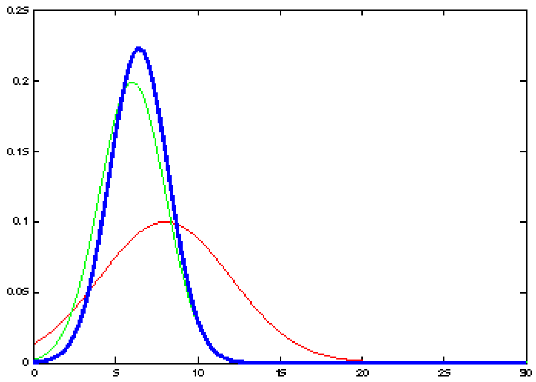




Kalman Filter Updates

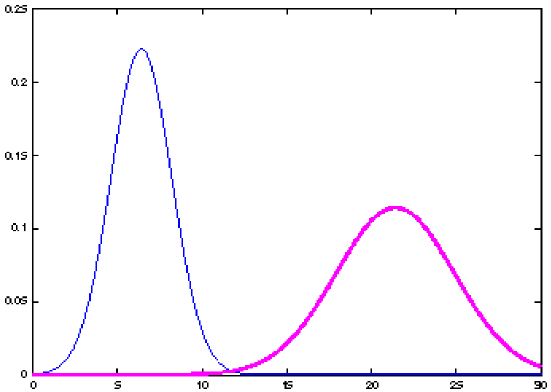


The Prediction-Correction-Cycle

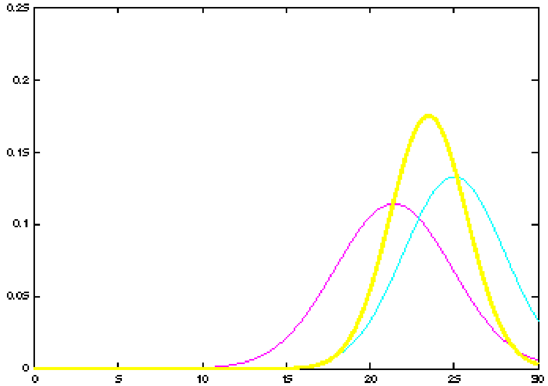


$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

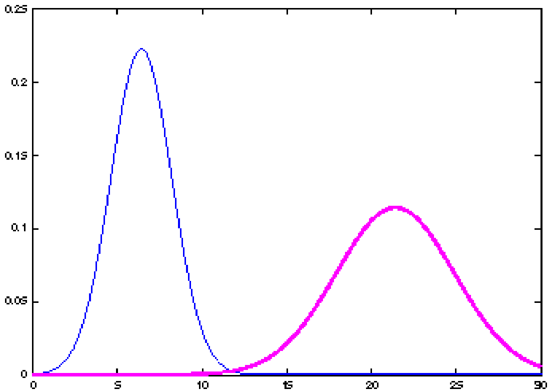


The Prediction-Correction-Cycle



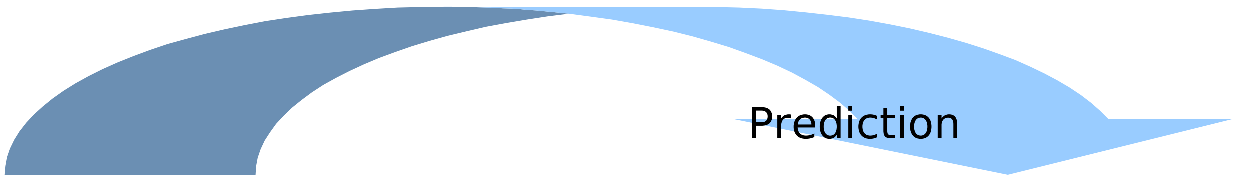
$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$



Correction

The Prediction-Correction-Cycle

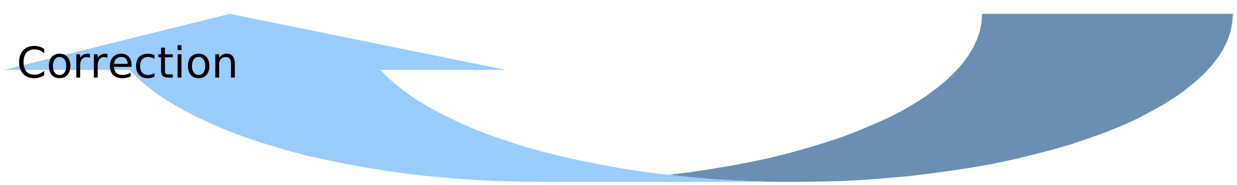


$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$



Kalman Filter Summary

- *Highly efficient*: Polynomial in measurement dimensionality k and state dimensionality n:
  $$O(k^{2.376} + n^2)$$

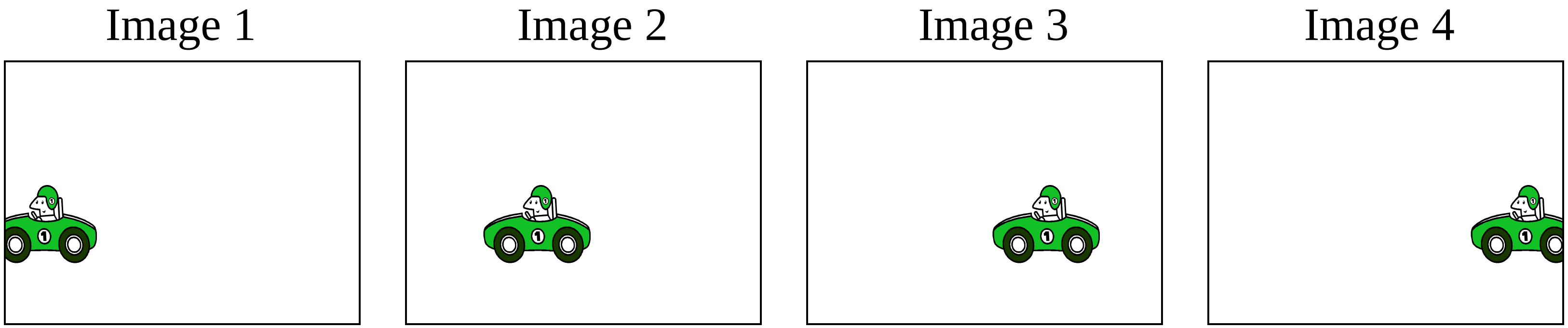
- *Optimal for linear Gaussian systems!*

- Most robotics systems are *nonlinear!*

Overview

- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters

Let’s Apply KFs to Tracking Problem



Kalman Filter with 2-Dim Linear Model

- Linear Change (Motion)

$$x' = Ax + B + N(0, R)$$

What is C, D, R?

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad R = \begin{pmatrix} \rho^2 & 0 \\ 0 & \rho^2 \end{pmatrix}$$

- Linear Measurement Model

$$z = Cx + D + N(0, Q)$$

What is A, B, Q?

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad Q = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$$

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad R = \begin{pmatrix} \rho^2 & 0 \\ 0 & \rho^2 \end{pmatrix}$$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t - D)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad Q = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$$

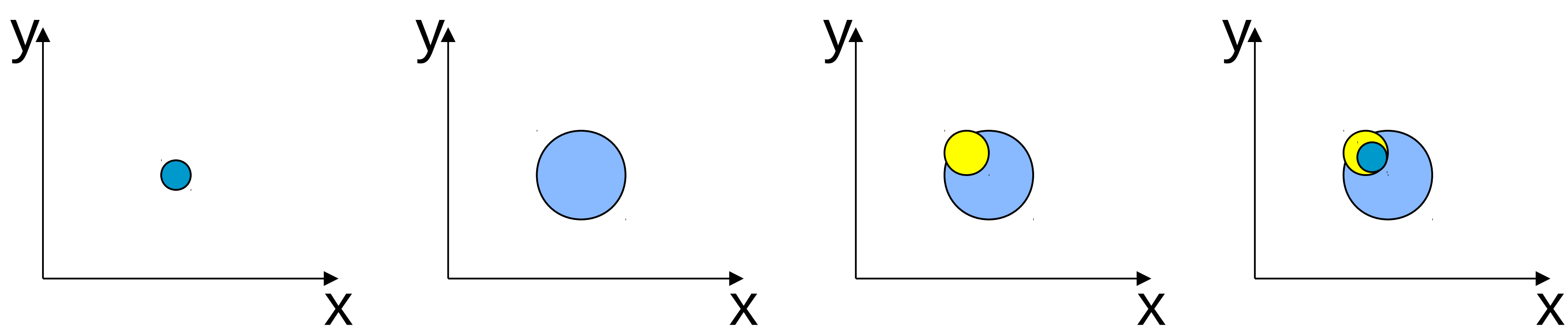
9. Return μ_t, Σ_t

Kalman Filter in Detail

- Measurements $z = x + N(0, R)$
- Change $x' = x + N(0, Q)$
- Prediction $\Sigma' = \Sigma + Q \qquad \mu' = \mu$
- Measurement Update $\Sigma'' = \left(\Sigma'^{-1} + R^{-1}\right)^{-1}$

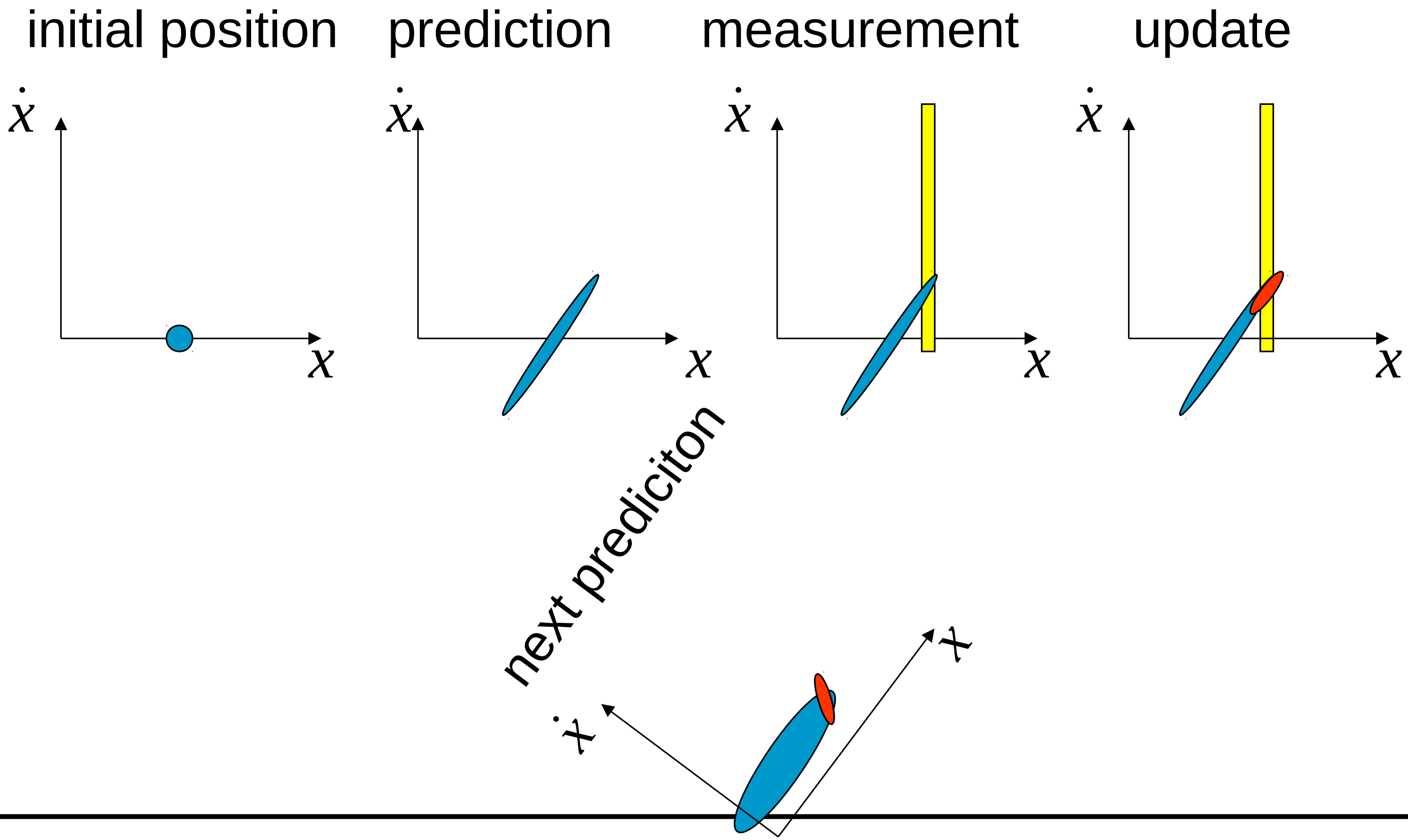
$$\mu'' = \mu' + \Sigma'' R^{-1}(z - \mu')$$

initial position prediction measurement update

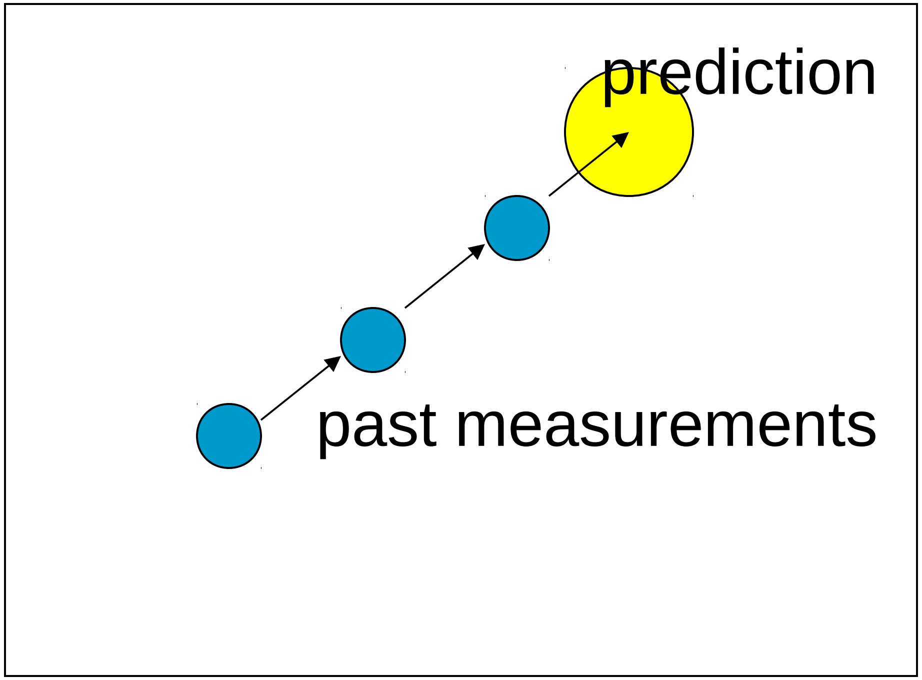


Can We Do Better?

Kalman, Better!



We Can Estimate Velocity!



Kalman Filter For 2D Tracking

- Linear Measurement model (now with 4 state variables)

$$\begin{pmatrix} x_{obs} \\ y_{obs} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} + N(0, R) \qquad R = \begin{pmatrix} r^2 & 0 \\ 0 & r^2 \end{pmatrix}$$

- Linear Change

$$\begin{pmatrix} x' \\ y' \\ \dot{x}' \\ \dot{y}' \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} + N(0, Q) \qquad Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q^2 & 0 \\ 0 & 0 & 0 & q^2 \end{pmatrix}$$

Putting It Together Again

- Measurements

$$\vec{z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \vec{x} + N(0, R)$$

- Change

$$\vec{x}' = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \vec{x} + N(0, Q)$$

- Prediction

$$\Sigma' = \Sigma + CQC^T$$
$$\mu' = C\mu$$

- Measurement Update

$$\Sigma' = \left(\Sigma^{-1} + A^T R^{-1} A\right)^{-1}$$
$$\mu' = \mu + \Sigma' A^T R^{-1} (z - A\mu)$$

Summary Kalman Filter

- Estimates state of a system
  - Position
  - Velocity
  - Many other continuous state variables possible
- KF maintains
  - Mean vector for the state
  - Covariance matrix of state uncertainty
- Implements
  - Time update = prediction
  - Measurement update
- Standard Kalman filter is linear-Gaussian
  - Linear system dynamics, linear sensor model
  - Additive Gaussian noise (independent)
  - Nonlinear extensions: extended KF, unscented KF: linearize
- More info:
  - CS226
  - Probabilistic Robotics (Thrun/Burgard/Fox, MIT Press)

Summary ~~Kalman~~ Particle Filter

- Estimates state of a system
  - Position
  - Velocity
  - Many other continuous and discrete state variables possible
- KF maintains
  - ~~Mean vector for the state~~
  - ~~Covariance matrix of state uncertainty~~

  set of particles (example states)
- Implements
  - Time update = prediction = predictive sampling
  - Measurement update = resampling, importance weights
- ~~Standard Kalman filter is linear-Gaussian~~
  - ~~Linear system dynamics, linear sensor model~~
  - ~~Additive Gaussian noise (independent)~~
  - ~~Nonlinear extensions: extended KF, unscented KF: linearize~~

  fully nonlinear

  easy to implement
- More info:
  - CS226
  - Probabilistic Robotics (Thrun/Burgard/Fox, MIT Press)

Summary

- The Tracking Problem
- Bayes Filters
- Particle Filters
- Kalman Filters
- Using Kalman Filters