Lagrangian Relaxation Algorithms for Inference in Natural Language Processing

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(based on joint work with Yin-Wen Chang, Tommi Jaakkola, Terry Koo, Roi Reichart, David Sontag)

Decoding in NLP

focus: structured prediction for natural language processing

decoding as a combinatorial optimization problem

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

where f is a scoring function and $\mathcal{Y}$ is a set of structures

for some problems, use simple combinatorial algorithms

- dynamic programming
- minimum spanning tree
- min cut

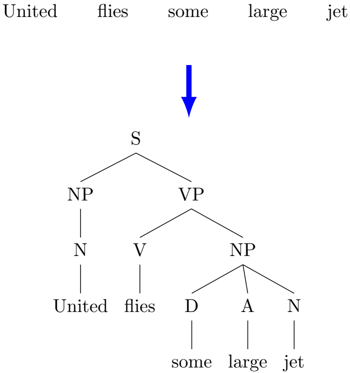
Structured prediction: Tagging

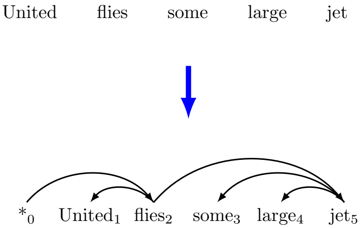
United flies some large jet

$$\Downarrow$$

$$\begin{tikzcd}
\mathrm{N} \arrow[r] \arrow[d] & \mathrm{V} \arrow[r] \arrow[d] & \mathrm{D} \arrow[r] \arrow[d] & \mathrm{A} \arrow[r] \arrow[d] & \mathrm{N} \arrow[d] \\
\text{United}_1 & \text{flies}_2 & \text{some}_3 & \text{large}_4 & \text{jet}_5
\end{tikzcd}$$

Structured prediction: Parsing





Decoding complexity

issue: simple combinatorial algorithms do not scale to richer models

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

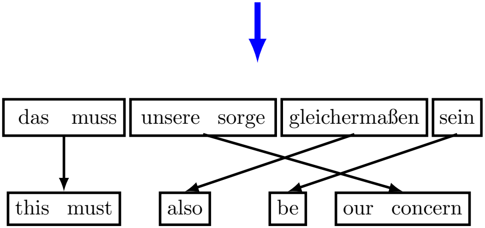
need decoding algorithms for complex natural language tasks

motivation:

- richer model structure often leads to improved accuracy
- exact decoding for complex models tends to be intractable

Structured prediction: Phrase-based translation

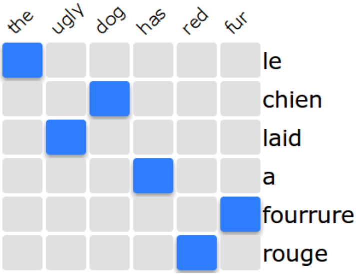
das muss unsere sorge gleichermaßen sein



Structured prediction: Word alignment

the ugly dog has red fur

le chien laid a fourrure rouge



Decoding tasks

high complexity

- combined parsing and part-of-speech tagging (Rush et al., 2010)
- "loopy" HMM part-of-speech tagging
- syntactic machine translation (Rush and Collins, 2011)

NP-Hard

- symmetric HMM alignment (DeNero and Macherey, 2011)
- phrase-based translation (Chang and Collins, 2011)
- higher-order non-projective dependency parsing (Koo et al., 2010)

in practice:

- approximate decoding methods (coarse-to-fine, beam search, cube pruning, gibbs sampling, belief propagation)
- approximate models (mean field, variational models)

Lagrangian relaxation

a general technique for constructing decoding algorithms

solve complicated models

$$y^* = \arg\max_y f(y)$$

by decomposing into smaller problems.

upshot: can utilize a toolbox of combinatorial algorithms.

- dynamic programming
- minimum spanning tree
- shortest path
- min cut
- ...

Lagrangian relaxation algorithms

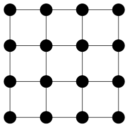
Simple - uses basic combinatorial algorithms

Efficient - faster than solving exact decoding problems

Strong guarantees

- gives a certificate of optimality when exact
- direct connections to linear programming relaxations

MAP problem in Markov random fields



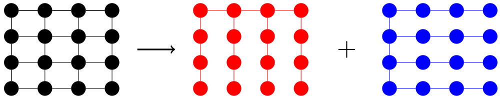
given: binary variables $x_1 \dots x_n$

goal: MAP problem

$$\arg\max_{x_1 \dots x_n} \sum_{(i,j) \in E} f_{i,j}(x_i, x_j)$$

where each $f_{i,j}(x_i, x_j)$ is a local potential for variables x_i, x_j

Dual decomposition for MRFs (Komodakis et al., 2010)



goal:

$$\arg\max_{x_1 \dots x_n} \sum_{(i,j) \in E} f_{i,j}(x_i, x_j)$$

equivalent formulation:

$$\arg\max_{x_1 \dots x_n, y_1 \dots y_n} \sum_{(i,j) \in T_1} f'_{i,j}(x_i, x_j) + \sum_{(i,j) \in T_2} f'_{i,j}(y_i, y_j)$$

such that for $i = 1 \dots n$,

$$x_i = y_i$$

Lagrangian:

$$L(u, x, y) = \sum_{(i,j) \in T_1} f'_{i,j}(x_i, x_j) + \sum_{(i,j) \in T_2} f'_{i,j}(y_i, y_j) + \sum_i u_i(x_i - y_i)$$

Related work

- belief propagation using combinatorial algorithms (Duchi et al., 2007; Smith and Eisner, 2008)
- factored A* search (Klein and Manning, 2003)

Tutorial outline

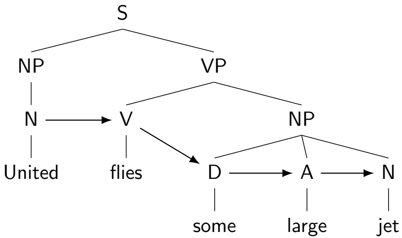
1. worked algorithm for combined parsing and tagging
2. important theorems and formal derivation
3. more examples from parsing and alignment
4. relationship to linear programming relaxations
5. practical considerations for implemention
6. further example from machine translation

1. Worked example

aim: walk through a Lagrangian relaxation algorithm for combined parsing and part-of-speech tagging

- introduce formal notation for parsing and tagging
- give assumptions necessary for decoding
- step through a run of the Lagrangian relaxation algorithm

Combined parsing and part-of-speech tagging



goal: find parse tree that optimizes

$$\textit{score}(\mathrm{S} \ \to \mathrm{NP\ VP}) + \textit{score}(\mathrm{VP} \ \to \mathrm{V\ NP}) +$$

$$\ldots + \textit{score}(\mathrm{N} \to \mathrm{V}) + \textit{score}(\mathrm{N} \to \mathrm{United}) + \ldots$$

Constituency parsing

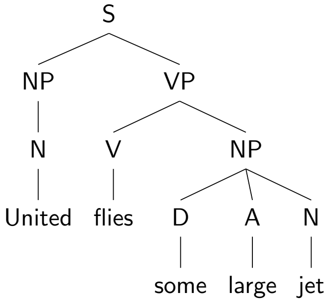
notation:

- $\mathcal{Y}$ is set of constituency parses for input
- $y \in \mathcal{Y}$ is a valid parse
- $f(y)$ scores a parse tree

goal:

$$\arg\max_{y \in \mathcal{Y}} f(y)$$

example: a context-free grammar for constituency parsing



Part-of-speech tagging

notation:

- $\mathcal{Z}$ is set of tag sequences for input
- $z \in \mathcal{Z}$ is a valid tag sequence
- $g(z)$ scores of a tag sequence

goal:

$$\arg\max_{z \in \mathcal{Z}} g(z)$$

example: an HMM for part-of speech tagging

$$\begin{tikzcd}
\mathrm{N} \arrow[r] \arrow[d] & \mathrm{V} \arrow[r] \arrow[d] & \mathrm{D} \arrow[r] \arrow[d] & \mathrm{A} \arrow[r] \arrow[d] & \mathrm{N} \arrow[d] \\
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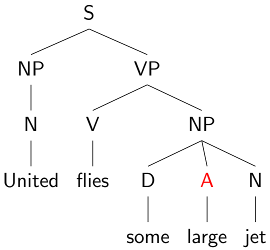
\end{tikzcd}$$

Identifying tags

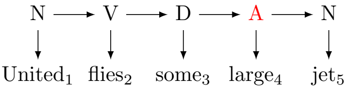
notation: identify the tag labels selected by each model

- $y(i, t) = 1$ when parse y selects tag t at position i
- $z(i, t) = 1$ when tag sequence z selects tag t at position i

example: a parse and tagging with $y(4, A) = 1$ and $z(4, A) = 1$



y



z

Combined optimization

goal:

$$\arg\max_{y \in \mathcal{Y}, z \in \mathcal{Z}} f(y) + g(z)$$

such that for all $i = 1 \ldots n$, $t \in \mathcal{T}$,

$$y(i,t) = z(i,t)$$

i.e. find the best parse and tagging pair that agree on tag labels

equivalent formulation:

$$\arg\max_{y \in \mathcal{Y}} f(y) + g(l(y))$$

where $l : \mathcal{Y} \rightarrow \mathcal{Z}$ extracts the tag sequence from a parse tree

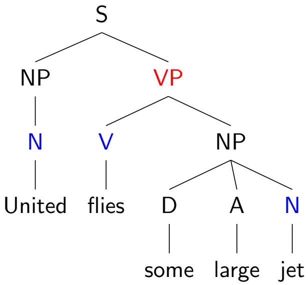
Exact method: Dynamic programming intersection

can solve by solving the product of the two models

example:

- parsing model is a context-free grammar
- tagging model is a first-order HMM
- can solve as CFG and finite-state automata intersection

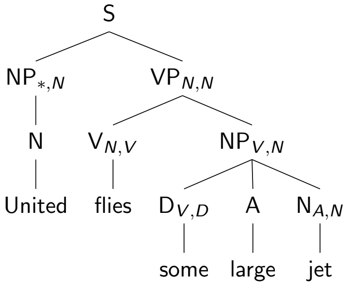
replace $\mathrm{VP} \rightarrow \mathrm{V}\ \mathrm{NP}$
with $\mathrm{VP}_{\mathrm{N,V}} \rightarrow \mathrm{V}_{\mathrm{N,V}}\ \mathrm{NP}_{\mathrm{V,N}}$



Intersected parsing and tagging complexity

let G be the number of grammar non-terminals

parsing CFG require $O(G^3 n^3)$ time with rules $\mathrm{VP} \rightarrow \mathrm{V\ NP}$



with intersection $O(G^3 n^3 |\mathcal{T}|^3)$ with rules $\mathrm{VP_{N,V}} \rightarrow \mathrm{V_{N,V}\ NP_{V,N}}$

becomes $O(G^3 n^3 |\mathcal{T}|^6)$ time for second-order HMM

Parsing assumption

assumption: optimization with u can be solved efficiently

$$\arg\max_{y \in \mathcal{Y}} f(y) + \sum_{i,t} u(i,t) y(i,t)$$

example: CFG with rule scoring function h

$$f(y) = \sum_{X \to Y\ Z \in y} h(X \to Y\ Z) + \sum_{(i,X) \in y} h(X \to w_i)$$

where

$$\begin{aligned} \arg\max_{y \in \mathcal{Y}} \quad & f(y) + \sum_{i,t} u(i,t) y(i,t) = \\ \arg\max_{y \in \mathcal{Y}} \quad & \sum_{X \to Y\ Z \in y} h(X \to Y\ Z) + \sum_{(i,X) \in y} (h(X \to w_i) + u(i,X)) \end{aligned}$$

Tagging assumption

assumption: optimization with u can be solved efficiently

$$\arg\max_{z \in \mathcal{Z}} g(z) - \sum_{i,t} u(i,t) z(i,t)$$

example: HMM with scores for transitions T and observations O

$$g(z) = \sum_{t \to t' \in z} T(t \to t') + \sum_{(i,t) \in z} O(t \to w_i)$$

where

$$\begin{aligned} \arg\max_{z \in \mathcal{Z}} \quad & g(z) - \sum_{i,t} u(i,t) z(i,t) = \\ \arg\max_{z \in \mathcal{Z}} \quad & \sum_{t \to t' \in z} T(t \to t') + \sum_{(i,t) \in z} (O(t \to w_i) - u(i,t)) \end{aligned}$$

Lagrangian relaxation algorithm

Set $u^{(1)}(i,t) = 0$ for all $i,\ t \in \mathcal{T}$

For $k = 1$ **to** K

$$y^{(k)} \leftarrow \arg\max_{y \in \mathcal{Y}} f(y) + \sum_{i,t} u^{(k)}(i,t) y(i,t) \text{ [Parsing]}$$

$$z^{(k)} \leftarrow \arg\max_{z \in \mathcal{Z}} g(z) - \sum_{i,t} u^{(k)}(i,t) z(i,t) \text{ [Tagging]}$$

If $y^{(k)}(i,t) = z^{(k)}(i,t)$ for all i,t **Return** $(y^{(k)}, z^{(k)})$

Else $u^{(k+1)}(i,t) \leftarrow u^{(k)}(i,t) - \alpha_k (y^{(k)}(i,t) - z^{(k)}(i,t))$

CKY Parsing

Penalties

$u(i, t) = 0$ for all i,t

$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,t} u(i,t) y(i,t))$$

Viterbi Decoding

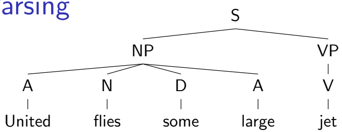
United$_1$ flies$_2$ some$_3$ large$_4$ jet$_5$

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,t} u(i,t) z(i,t))$$

Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	HMM
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Taggings
$y(i,t) = 1$	if	y contains tag t at position i			

CKY Parsing



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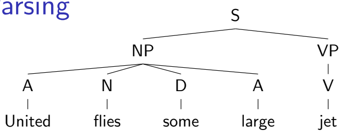

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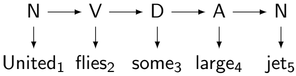

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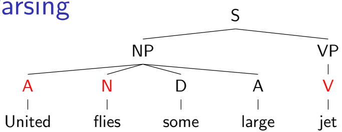

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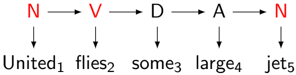

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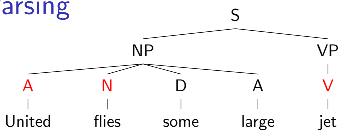

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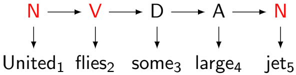

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Iteration 1	
$u(1,A)$	-1
$u(1,N)$	1
$u(2,N)$	-1
$u(2,V)$	1
$u(5,V)$	-1
$u(5,N)$	1

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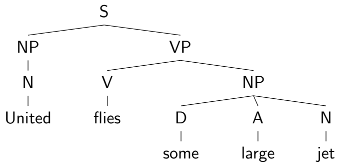

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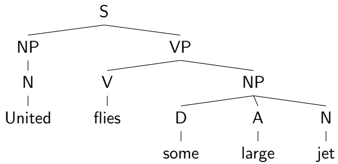

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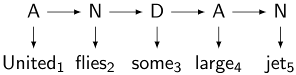

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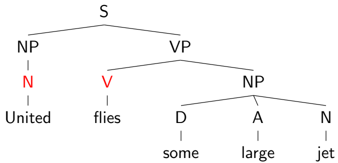

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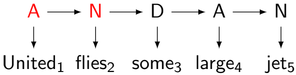

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$u(2,V)$	1
$u(5,V)$	-1
$u(5,N)$	1

Iteration 2	
$u(5,V)$	-1
$u(5,N)$	1

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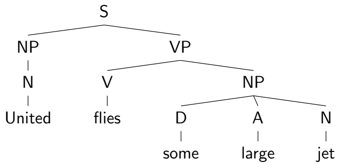
CKY Parsing



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,t} u(i,t) y(i,t))$$

Viterbi Decoding

United$_1$ flies$_2$ some$_3$ large$_4$ jet$_5$

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,t} u(i,t) z(i,t))$$

Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	HMM
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Taggings
$y(i,t) = 1$	if	y contains tag t at position i			

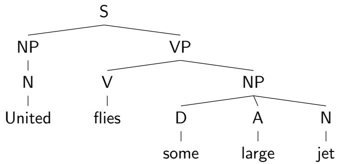
Penalties

$u(i,t) = 0$ for all i,t

Iteration 1	
$u(1,A)$	-1
$u(1,N)$	1
$u(2,N)$	-1
$u(2,V)$	1
$u(5,V)$	-1
$u(5,N)$	1

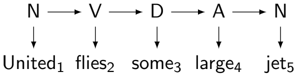
Iteration 2	
$u(5,V)$	-1
$u(5,N)$	1

CKY Parsing



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,t} u(i,t) y(i,t))$$

Viterbi Decoding



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,t} u(i,t) z(i,t))$$

Penalties

$u(i,t) = 0$ for all i,t

Iteration 1	
$u(1, A)$	-1
$u(1, N)$	1
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$u(2, V)$	1
$u(5, V)$	-1
$u(5, N)$	1

Iteration 2	
$u(5, V)$	-1
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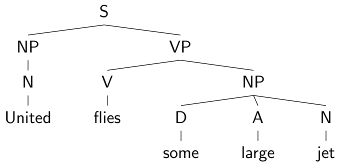
Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	HMM
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Taggings
$y(i,t) = 1$	if	y contains tag t at position i			

CKY Parsing



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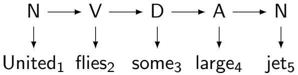
Viterbi Decoding



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,t} u(i,t) z(i,t))$$

Penalties

$u(i,t) = 0$ for all i,t

Iteration 1	
$u(1, A)$	-1
$u(1, N)$	1
$u(2, N)$	-1
$u(2, V)$	1
$u(5, V)$	-1
$u(5, N)$	1

Iteration 2	
$u(5, V)$	-1
$u(5, N)$	1

Converged

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y) + g(y)$$

Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	HMM
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Taggings
$y(i,t) = 1$	if	y contains tag t at position i			

Main theorem

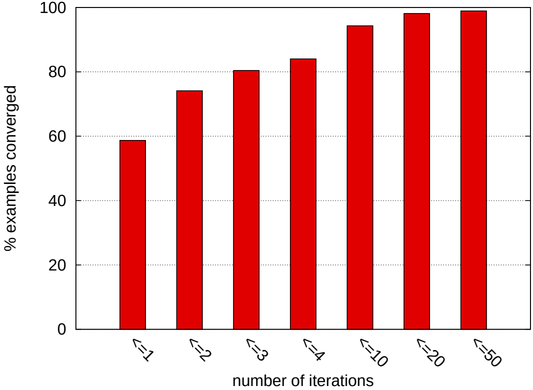
theorem: if at any iteration, for all i, $t \in \mathcal{T}$

$$y^{(k)}(i,t) = z^{(k)}(i,t)$$

then $(y^{(k)}, z^{(k)})$ is the global optimum

proof: focus of the next section

Convergence



2. Formal properties

aim: formal derivation of the algorithm given in the previous section

- derive Lagrangian dual
- prove three properties
  - upper bound
  - convergence
  - optimality
- describe subgradient method

Lagrangian

goal:

$$\arg \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} f(y) + g(z) \text{ such that } y(i,t) = z(i,t)$$

Lagrangian:

$$L(u, y, z) = f(y) + g(z) + \sum_{i,t} u(i,t) \left(y(i,t) - z(i,t)\right)$$

redistribute terms

$$L(u, y, z) \quad = \quad \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right)$$

Lagrangian dual

Lagrangian:

$$L(u, y, z) = \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right)$$

Lagrangian dual:

$$\begin{aligned} L(u) &= \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} L(u, y, z) \\ &= \max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \\ &\quad \max_{z \in \mathcal{Z}} \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right) \end{aligned}$$

Theorem 1. Upper bound

define:

- y^*, z^* is the optimal combined parsing and tagging solution with $y^*(i,t) = z^*(i,t)$ for all i, t

theorem: for any value of u

$$L(u) \geq f(y^*) + g(z^*)$$

$L(u)$ provides an upper bound on the score of the optimal solution

note: upper bound may be useful as input to branch and bound or A* search

Theorem 1. Upper bound (proof)

theorem: for any value of u, $L(u) \geq f(y^*) + g(z^*)$

proof:

$$\begin{aligned} L(u) &= \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} L(u, y, z) && (1) \\ &\geq \max_{y \in \mathcal{Y}, z \in \mathcal{Z} : y = z} L(u, y, z) && (2) \\ &= \max_{y \in \mathcal{Y}, z \in \mathcal{Z} : y = z} f(y) + g(z) && (3) \\ &= f(y^*) + g(z^*) && (4) \end{aligned}$$

Formal algorithm (reminder)

Set $u^{(1)}(i,t) = 0$ for all $i,\ t \in \mathcal{T}$

For $k = 1$ **to** K

$$y^{(k)} \leftarrow \arg\max_{y \in \mathcal{Y}} f(y) + \sum_{i,t} u^{(k)}(i,t) y(i,t) \text{ [Parsing]}$$

$$z^{(k)} \leftarrow \arg\max_{z \in \mathcal{Z}} g(z) - \sum_{i,t} u^{(k)}(i,t) z(i,t) \text{ [Tagging]}$$

If $y^{(k)}(i,t) = z^{(k)}(i,t)$ for all i,t **Return** $(y^{(k)}, z^{(k)})$

Else $u^{(k+1)}(i,t) \leftarrow u^{(k)}(i,t) - \alpha_k (y^{(k)}(i,t) - z^{(k)}(i,t))$

Theorem 2. Convergence

notation:

- $u^{(k+1)}(i,t) \leftarrow u^{(k)}(i,t) + \alpha_k(y^{(k)}(i,t) - z^{(k)}(i,t))$ is update
- $u^{(k)}$ is the penalty vector at iteration k
- $\alpha_k > 0$ is the update rate at iteration k

theorem: for any sequence $\alpha^1, \alpha^2, \alpha^3, \ldots$ such that

$$\lim_{t \to \infty} \alpha^t = 0 \quad \text{and} \quad \sum_{t=1}^{\infty} \alpha^t = \infty,$$

we have

$$\lim_{t \to \infty} L(u^t) = \min_u L(u)$$

i.e. the algorithm converges to the tightest possible upper bound

proof: by subgradient convergence (next section)

Dual solutions

define:

- for any value of u

$$y_u = \arg\max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right)$$

and

$$z_u = \arg\max_{z \in \mathcal{Z}} \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right)$$

- y_u and z_u are the dual solutions for a given u

Theorem 3. Optimality

theorem: if there exists u such that

$$y_u(i,t) = z_u(i,t)$$

for all i, t then

$$f(y_u) + g(z_u) = f(y^*) + g(z^*)$$

i.e. if the dual solutions agree, we have an optimal solution

$$(y_u, z_u)$$

Theorem 3. Optimality (proof)

theorem: if u such that $y_u(i,t) = z_u(i,t)$ for all i, t then

$$f(y_u) + g(z_u) = f(y^*) + g(z^*)$$

proof: by the definitions of y_u and z_u

$$\begin{aligned} L(u) &= f(y_u) + g(z_u) + \sum_{i,t} u(i,t)(y_u(i,t) - z_u(i,t)) \\ &= f(y_u) + g(z_u) \end{aligned}$$

since $L(u) \geq f(y^*) + g(z^*)$ for all values of u

$$f(y_u) + g(z_u) \geq f(y^*) + g(z^*)$$

but y^* and z^* are optimal

$$f(y_u) + g(z_u) \leq f(y^*) + g(z^*)$$

Dual optimization

Lagrangian dual:

$$
\begin{aligned}
L(u) &= \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} L(u, y, z) \\
&= \max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \\
&\quad \max_{z \in \mathcal{Z}} \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right)
\end{aligned}
$$

goal: dual problem is to find the tightest upper bound

$$
\min_u L(u)
$$

Dual subgradient

$$L(u) \quad = \quad \max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \max_{z \in \mathcal{Z}} \left(g(z) - \sum_{i,t} u(i,t) z(i,t) \right)$$

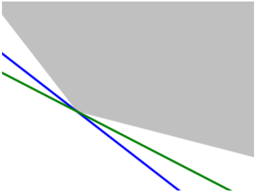
properties:

- L(u) is convex in u (no local minima)
- L(u) is not differentiable (because of max operator)

handle non-differentiability by using subgradient descent

define: a subgradient of $L(u)$ at u is a vector g_u such that for all v

$$L(v) \geq L(u) + g_u \cdot (v - u)$$



Subgradient algorithm

$$L(u) \quad = \quad \max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \max_{z \in \mathcal{Z}} \left(g(z) - \sum_{i,j} u(i,t) z(i,t) \right)$$

recall, y_u and z_u are the argmax's of the two terms

subgradient:

$$g_u(i,t) \quad = \quad y_u(i,t) - z_u(i,t)$$

subgradient descent: move along the subgradient

$$u'(i,t) = u(i,t) - \alpha \left(y_u(i,t) - z_u(i,t) \right)$$

guaranteed to find a minimum with conditions given earlier for α

3. More examples

aim: demonstrate similar algorithms that can be applied to other decoding applications

- context-free parsing combined with dependency parsing
- combined translation alignment

Combined constituency and dependency parsing

(Rush et al., 2010)

setup: assume separate models trained for constituency and dependency parsing

problem: find constituency parse that maximizes the sum of the two models

example:

- combine lexicalized CFG with second-order dependency parser

Lexicalized constituency parsing

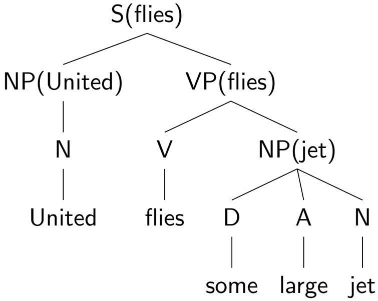
notation:

- $\mathcal{Y}$ is set of lexicalized constituency parses for input
- $y \in \mathcal{Y}$ is a valid parse
- $f(y)$ scores a parse tree

goal:

$$\arg\max_{y \in \mathcal{Y}} f(y)$$

example: a lexicalized context-free grammar

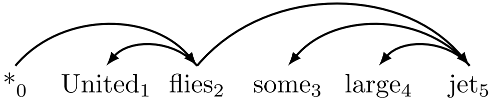


Dependency parsing

define:

- $\mathcal{Z}$ is set of dependency parses for input
- $z \in \mathcal{Z}$ is a valid dependency parse
- $g(z)$ scores a dependency parse

example:

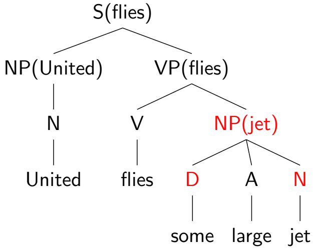


Identifying dependencies

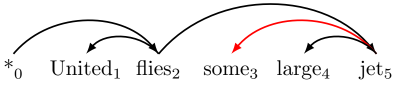
notation: identify the dependencies selected by each model

- $y(i, j) = 1$ when word i modifies of word j in constituency parse y
- $z(i, j) = 1$ when word i modifies of word j in dependency parse z

example: a constituency and dependency parse with $y(3, 5) = 1$ and $z(3, 5) = 1$



y



z

Combined optimization

goal:

$$\arg \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} f(y) + g(z)$$

such that for all $i = 1 \ldots n$, $j = 0 \ldots n$,

$$y(i, j) = z(i, j)$$

CKY Parsing

Penalties

$u(i,j) = 0$ for all i,j

$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

Dependency Parsing

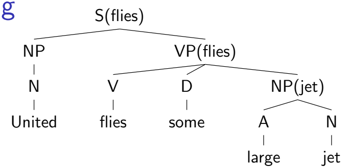
$*_0$ United$_1$ flies$_2$ some$_3$ large$_4$ jet$_5$

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	Dependency Model
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Dependency Trees
$y(i,j) = 1$	if	y contains dependency i,j			

CKY Parsing



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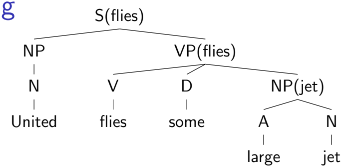

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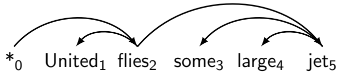
Dependency Parsing



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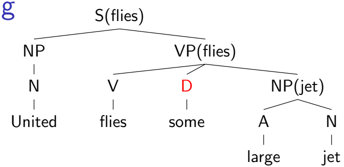

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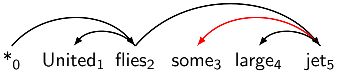
Dependency Parsing



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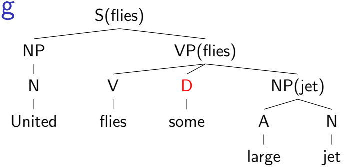

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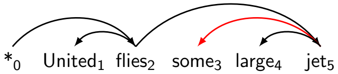
Penalties

$u(i,j) = 0$ for all i,j

Iteration 1	
$u(2,3)$	-1
$u(5,3)$	1

Dependency Parsing



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Dependency Parsing

$*_0$ United$_1$ flies$_2$ some$_3$ large$_4$ jet$_5$

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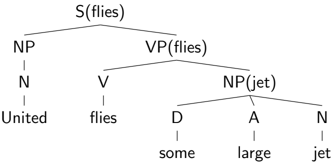

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Dependency Parsing

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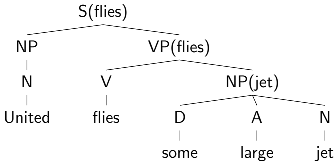
CKY Parsing



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Penalties

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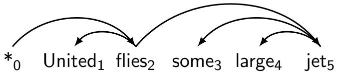
Dependency Parsing



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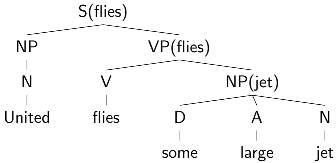
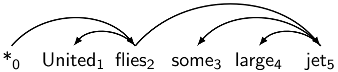
CKY Parsing



$$y^* = \arg\max_{y \in \mathcal{Y}} \Big(f(y) + \sum_{i,j} u(i,j) y(i,j) \Big)$$

Dependency Parsing



$$z^* = \arg\max_{z \in \mathcal{Z}} \Big(g(z) - \sum_{i,j} u(i,j) z(i,j) \Big)$$

Key

$f(y)$	$\Leftarrow$	CFG	$g(z)$	$\Leftarrow$	Dependency Model
$\mathcal{Y}$	$\Leftarrow$	Parse Trees	$\mathcal{Z}$	$\Leftarrow$	Dependency Trees
$y(i,j) = 1$	if	y contains dependency i,j			

Penalties

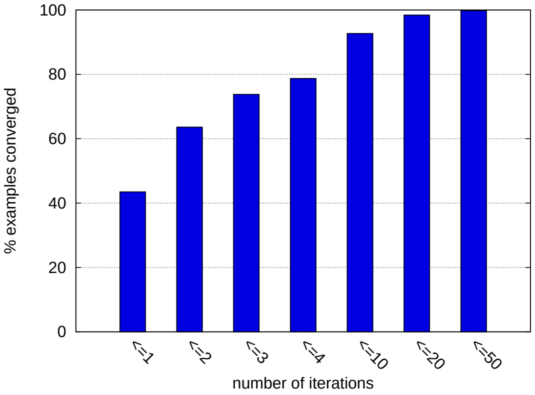
$u(i,j) = 0$ for all i,j

Iteration 1	
$u(2,3)$	-1
$u(5,3)$	1

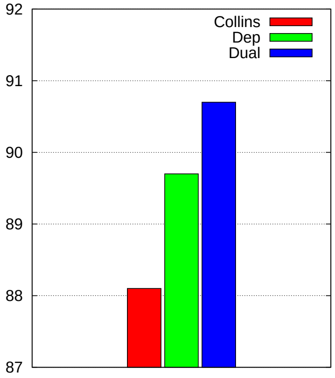
Converged

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y) + g(y)$$

Convergence



Integrated Constituency and Dependency Parsing: Accuracy



F_1 Score

- Collins (1997) Model 1
- Fixed, First-best Dependencies from Koo (2008)
- Dual Decomposition

Combined alignment (DeNero and Macherey, 2011)

setup: assume separate models trained for English-to-French and French-to-English alignment

problem: find an alignment that maximizes the score of both models

example:

- HMM models for both directional alignments (assume correct alignment is one-to-one for simplicity)

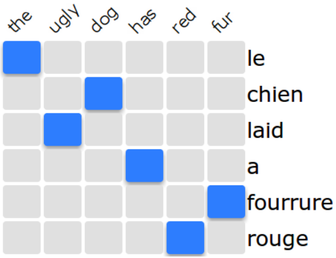
English-to-French alignment

define:

- $\mathcal{Y}$ is set of all possible English-to-French alignments
- $y \in \mathcal{Y}$ is a valid alignment
- $f(y)$ scores of the alignment

example: HMM alignment

$$\begin{tikzcd}
\text{Le}_1 \arrow[r] \arrow[d] & \text{laid}_3 \arrow[r] \arrow[d] & \text{chien}_2 \arrow[r] \arrow[d] & \text{a}_4 \arrow[r] \arrow[d] & \text{rouge}_6 \arrow[r] \arrow[d] & \text{fourrure}_5 \arrow[d] \\
\text{The}_1 & \text{ugly}_2 & \text{dog}_3 & \text{has}_4 & \text{red}_5 & \text{fur}_6
\end{tikzcd}$$



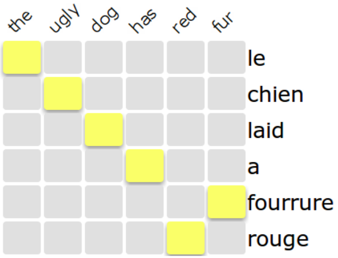
French-to-English alignment

define:

- $\mathcal{Z}$ is set of all possible French-to-English alignments
- $z \in \mathcal{Z}$ is a valid alignment
- $g(z)$ scores of an alignment

example: HMM alignment

$$\begin{tikzcd}
\text{The}_1 \arrow[r] \arrow[d] & \text{ugly}_2 \arrow[r] \arrow[d] & \text{dog}_3 \arrow[r] \arrow[d] & \text{has}_4 \arrow[r] \arrow[d] & \text{fur}_6 \arrow[r] \arrow[d] & \text{red}_5 \arrow[d] \\
\text{Le}_1 & \text{chien}_2 & \text{laid}_3 & \text{a}_4 & \text{fourrure}_5 & \text{rouge}_6
\end{tikzcd}$$

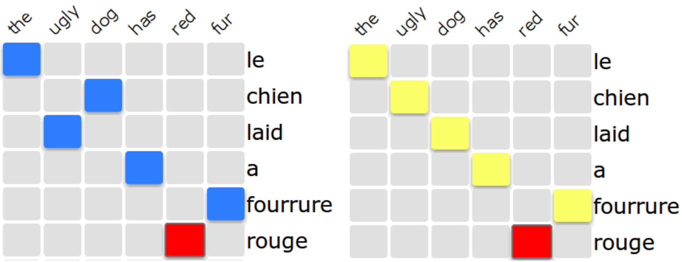


Identifying word alignments

notation: identify the tag labels selected by each model

- $y(i,j) = 1$ when e-to-f alignment y selects French word i to align with English word j
- $z(i,j) = 1$ when f-to-e alignment z selects French word i to align with English word j

example: two HMM alignment models with $y(6,5) = 1$ and $z(6,5) = 1$



Combined optimization

goal:

$$\arg\max_{y \in \mathcal{Y}, z \in \mathcal{Z}} f(y) + g(z)$$

such that for all $i = 1 \dots n$, $j = 1 \dots n$,

$$y(i,j) = z(i,j)$$

English-to-French

Penalties

$u(i,j) = 0$ for all i,j

$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

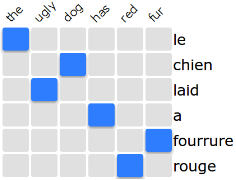
French-to-English

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Key

$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
$y(i,j) = 1$	if	French word i aligns to English word j			

English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

Penalties

$$u(i,j) = 0 \text{ for all } i,j$$

French-to-English

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Key

$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
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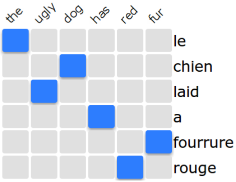
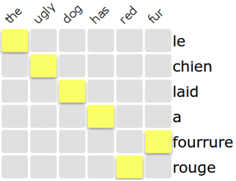
English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

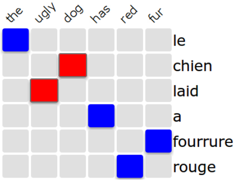
Penalties

$$u(i,j) = 0 \text{ for all } i,j$$

Key

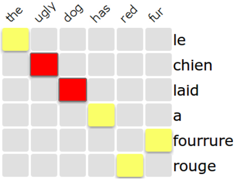
$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
$y(i,j) = 1$	if	French word i aligns to English word j			

English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

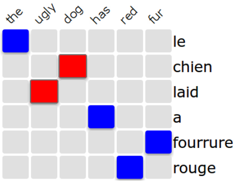
Penalties

$u(i,j) = 0$ for all i,j

Key

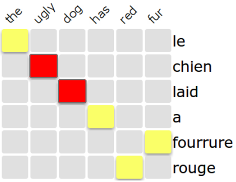
$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
$y(i,j) = 1$	if	French word i aligns to English word j			

English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Penalties

$u(i,j) = 0$ for all i,j

	Iteration 1
$u(3,2)$	-1
$u(2,2)$	1
$u(2,3)$	-1
$u(3,3)$	1

Key

$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
$y(i,j) = 1$	if	French word i aligns to English word j			

English-to-French

$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Penalties

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Key

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$\mathcal{Y}$	$\Leftarrow$	English-to-French model
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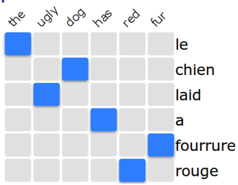
$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Z}$	$\Leftarrow$	French-to-English model

English-to-French



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French-to-English

$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

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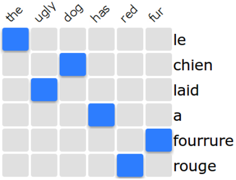
Key

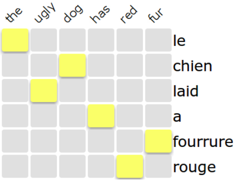
$f(y)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model
$y(i,j) = 1$	if	French word i aligns to English word j
$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Z}$	$\Leftarrow$	French-to-English model

English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Penalties

$u(i,j) = 0$ for all i,j

	Iteration 1
$u(3,2)$	-1
$u(2,2)$	1
$u(2,3)$	-1
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Key

$f(y)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model
$y(i,j) = 1$	if	French word i aligns to English word j
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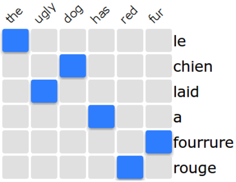
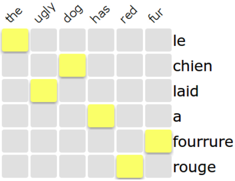
English-to-French



$$y^* = \arg\max_{y \in \mathcal{Y}} (f(y) + \sum_{i,j} u(i,j) y(i,j))$$

French-to-English



$$z^* = \arg\max_{z \in \mathcal{Z}} (g(z) - \sum_{i,j} u(i,j) z(i,j))$$

Penalties

$u(i,j) = 0$ for all i,j

	Iteration 1
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$u(3,3)$	1

Key

$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
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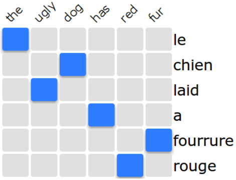
English-to-French



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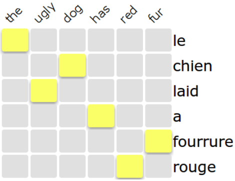
French-to-English



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Penalties

$u(i,j) = 0$ for all i,j

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Key

$f(y)$	$\Leftarrow$	HMM Alignment	$g(z)$	$\Leftarrow$	HMM Alignment
$\mathcal{Y}$	$\Leftarrow$	English-to-French model	$\mathcal{Z}$	$\Leftarrow$	French-to-English model
$y(i,j) = 1$	if	French word i aligns to English word j			

4. Linear programming

aim: explore the connections between Lagrangian relaxation and linear programming

- basic optimization over the simplex
- formal properties of linear programming
- full example with fractional optimal solutions

Simplex

define:

- $\Delta_y \subset \mathcal{R}^{|\mathcal{Y}|}$ is the simplex over $\mathcal{Y}$ where $\alpha \in \Delta_y$ implies

$$\alpha_y \geq 0 \text{ and } \sum_y \alpha_y = 1$$

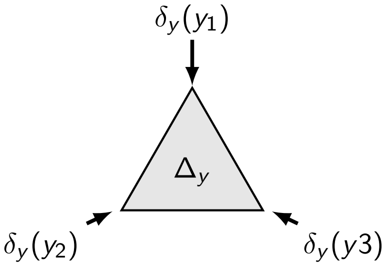
- α is distribution over $\mathcal{Y}$
- Δ_z is the simplex over $\mathcal{Z}$
- $\delta_y : \mathcal{Y} \to \Delta_y$ maps elements to the simplex

example:

$$\mathcal{Y} = \{y_1, y_2, y_3\}$$

vertices

- $\delta_y(y_1) = (1, 0, 0)$
- $\delta_y(y_2) = (0, 1, 0)$
- $\delta_y(y_3) = (0, 0, 1)$



Theorem 1. Simplex linear program

optimize over the simplex Δ_{y} instead of the discrete sets $\mathcal{Y}$

goal: optimize linear program

$$\max_{\alpha \in \Delta_y} \sum_y \alpha_y f(y)$$

theorem:

$$\max_{y \in \mathcal{Y}} f(y) = \max_{\alpha \in \Delta_y} \sum_y \alpha_y f(y)$$

proof: points in $\mathcal{Y}$ correspond to the exteme points of simplex

$$\{\delta_y(y) : y \in \mathcal{Y}\}$$

linear program has optimum at extreme point

proof shows that best distribution chooses a single parse

Combined linear program

optimize over the simplices Δ_y and Δ_z instead of the discrete sets $\mathcal{Y}$ and $\mathcal{Z}$

goal: optimize linear program

$$\max_{\alpha \in \Delta_y, \beta \in \Delta_z} \sum_y \alpha_y f(y) + \sum_z \beta_z g(z)$$

such that for all i, t

$$\sum_y \alpha_y y(i,t) = \sum_z \beta_z z(i,t)$$

note: the two distributions must match in expectation of POS tags

the best distributions α^*,β^* are possibly no longer a single parse tree or tag sequence

Lagrangian

Lagrangian:

$$
\begin{aligned}
M(u,\alpha,\beta) &= \sum_y \alpha_y f(y) + \sum_z \beta_z g(z) + \sum_{i,t} u(i,t)\left(\sum_y \alpha_y y(i,t) - \sum_z \beta_z z(i,t)\right) \\
&= \left(\sum_y \alpha_y f(y) + \sum_{i,t} u(i,t) \sum_y \alpha_y y(i,t)\right) + \\
&\quad \left(\sum_z \beta_z g(z) - \sum_{i,t} u(i,t) \sum_z \beta_z z(i,t)\right)
\end{aligned}
$$

Lagrangian dual:

$$
M(u) = \max_{\alpha \in \Delta_y, \beta \in \Delta_z} M(u,\alpha,\beta)
$$

Theorem 2. Strong duality

define:

- α^*, β^* is the optimal assignment to α, β in the linear program

theorem:

$$\min_u M(u) = \sum_y \alpha^*_y f(y) + \sum_z \beta^*_z g(z)$$

proof: by linear programming duality

Theorem 3. Dual relationship

theorem: for any value of u,

$$M(u) = L(u)$$

note: solving the original Lagrangian dual also solves dual of the linear program

Theorem 3. Dual relationship (proof sketch)

focus on $\mathcal{Y}$ term in Lagrangian

$$\begin{aligned}
L(u) &= \max_{y \in \mathcal{Y}} \left(f(y) + \sum_{i,t} u(i,t) y(i,t) \right) + \ldots \\
M(u) &= \max_{\alpha \in \Delta_y} \left(\sum_{y} \alpha_y f(y) + \sum_{i,t} u(i,t) \sum_{y} \alpha_y y(i,t) \right) + \ldots
\end{aligned}$$

by theorem 1. optimization over $\mathcal{Y}$ and Δ_y have the same max

similar argument for $\mathcal{Z}$ gives $L(u) = M(u)$

Summary

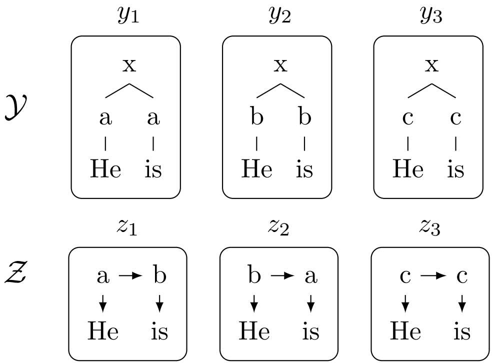
$f(y) + g(z)$	original primal objective
$L(u)$	original dual
$\sum_y \alpha_y f(y) + \sum_z \beta_z g(z)$	LP primal objective
$M(u)$	LP dual

relationship between LP dual, original dual, and LP primal objective

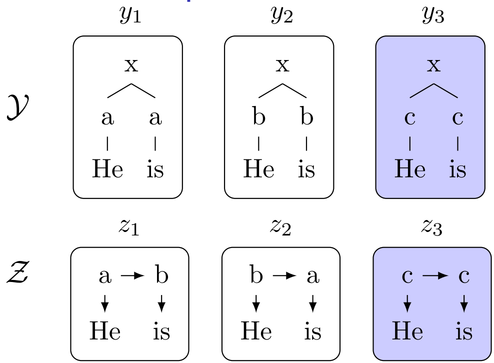
$$\min_u M(u) = \min_u L(u) = \sum_y \alpha^*_y f(y) + \sum_z \beta^*_z g(z)$$

Concrete example

- $\mathcal{Y} = \{y_1, y_2, y_3\}$
- $\mathcal{Z} = \{z_1, z_2, z_3\}$
- $\Delta_y \subset \mathbb{R}^3$, $\Delta_z \subset \mathbb{R}^3$



Simple solution



choose:

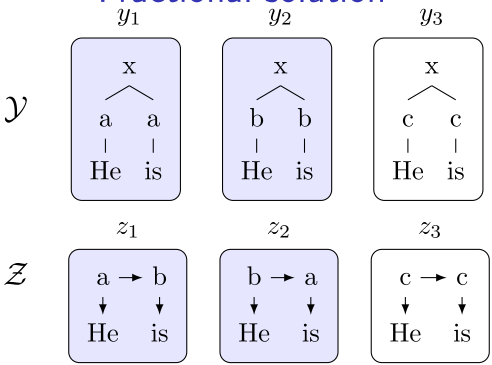
- $\alpha^{(1)} = (0, 0, 1) \in \Delta_y$ is representation of y_3
- $\beta^{(1)} = (0, 0, 1) \in \Delta_z$ is representation of z_3

confirm:

$$\sum_y \alpha_y^{(1)} y(i,t) = \sum_z \beta_z^{(1)} z(i,t)$$

$\alpha^{(1)}$ and $\beta^{(1)}$ satisfy agreement constraint

Fractional solution



choose:

- $\alpha^{(2)} = (0.5, 0.5, 0) \in \Delta_y$ is combination of y_1 and y_2
- $\beta^{(2)} = (0.5, 0.5, 0) \in \Delta_z$ is combination of z_1 and z_2

confirm:

$$\sum_y \alpha_y^{(2)} y(i,t) = \sum_z \beta_z^{(2)} z(i,t)$$

$\alpha^{(2)}$ and $\beta^{(2)}$ satisfy agreement constraint, but not integral

Optimal solution

weights:

- the choice of f and g determines the optimal solution
- if (f, g) favors $(\alpha^{(2)}, \beta^{(2)})$, the optimal solution is fractional

example: $f = [1\ 1\ 2]$ and $g = [1\ 1\ -2]$

- $f \cdot \alpha^{(1)} + g \cdot \beta^{(1)} = 0$ vs $f \cdot \alpha^{(2)} + g \cdot \beta^{(2)} = 2$
- $\alpha^{(2)}, \beta^{(2)}$ is optimal, even though it is fractional

summary: dual and LP primal optimal:

$$\min_u M(u) = \min_u L(u) = \sum_y \alpha_y^{(2)} f(y) + \sum_z \beta_z^{(2)} g(z) = 2$$

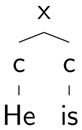
original primal optimal:

$$f(y^*) + g(z^*) = 0$$

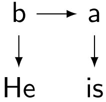
round 1

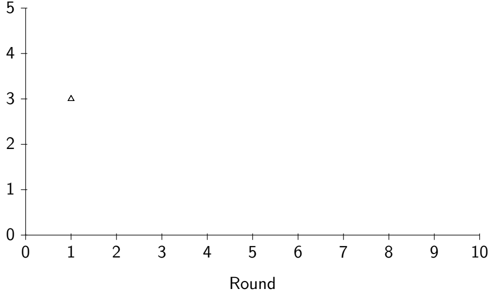
dual solutions:

y_3



z_2





dual values:

$y^{(1)}$	2.00
$z^{(1)}$	1.00
$L(u^{(1)})$	3.00

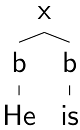
previous solutions:

y_3 z_2

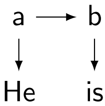
round 2

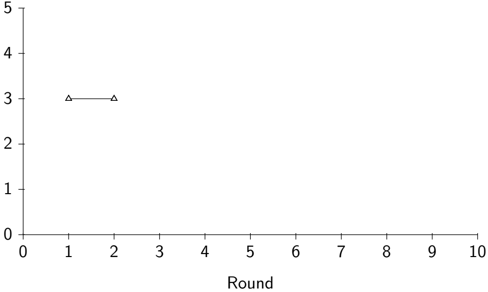
dual solutions:

y_2



z_1





dual values:

$y^{(2)}$	2.00
$z^{(2)}$	1.00
$L(u^{(2)})$	3.00

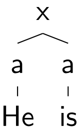
previous solutions:

y_3	z_2
y_2	z_1

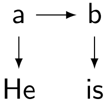
round 3

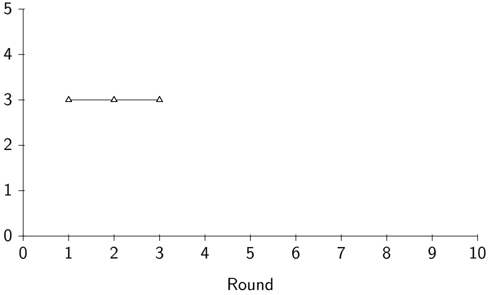
dual solutions:

y_1



z_1





dual values:

$y^{(3)}$	2.50
$z^{(3)}$	0.50
$L(u^{(3)})$	3.00

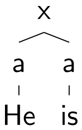
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1

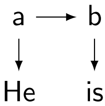
round 4

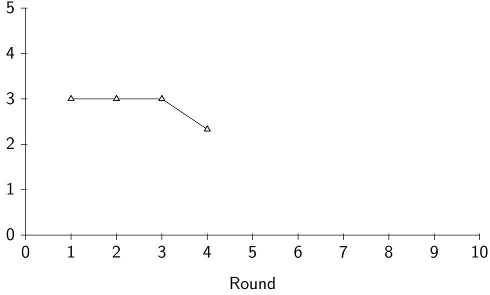
dual solutions:

y_1



z_1





dual values:

$y^{(4)}$	2.17
$z^{(4)}$	0.17
$L(u^{(4)})$	2.33

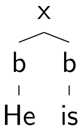
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1

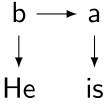
round 5

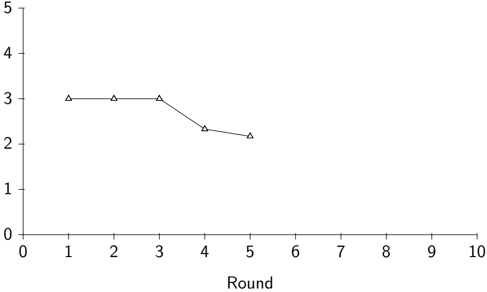
dual solutions:

y_2



z_2





dual values:

$y^{(5)}$	2.08
$z^{(5)}$	0.08
$L(u^{(5)})$	2.17

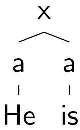
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1
y_2	z_2

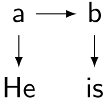
round 6

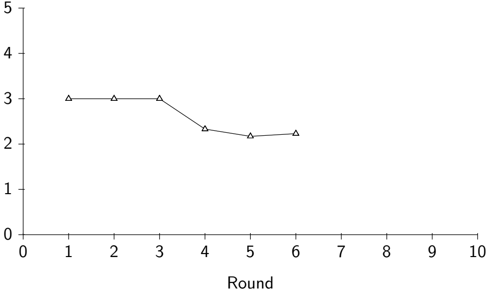
dual solutions:

y_1



z_1





dual values:

$y^{(6)}$	2.12
$z^{(6)}$	0.12
$L(u^{(6)})$	2.23

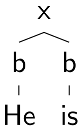
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1
y_2	z_2
y_1	z_1

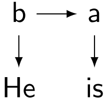
round 7

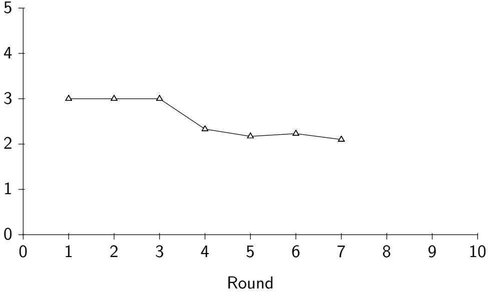
dual solutions:

y_2



z_2





dual values:

$y^{(7)}$	2.05
$z^{(7)}$	0.05
$L(u^{(7)})$	2.10

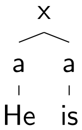
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1
y_2	z_2
y_1	z_1
y_2	z_2

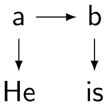
round 8

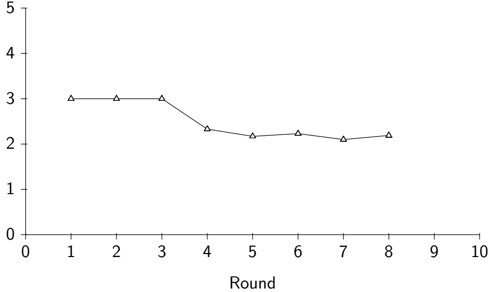
dual solutions:

y_1



z_1





dual values:

$y^{(8)}$	2.09
$z^{(8)}$	0.09
$L(u^{(8)})$	2.19

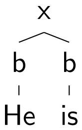
previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1
y_2	z_2
y_1	z_1
y_2	z_2
y_1	z_1

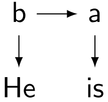
round 9

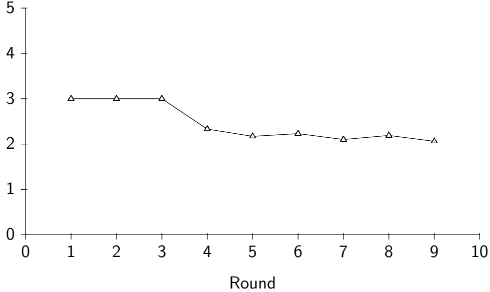
dual solutions:

y_2



z_2





dual values:

$y^{(9)}$	2.03
$z^{(9)}$	0.03
$L(u^{(9)})$	2.06

previous solutions:

y_3	z_2
y_2	z_1
y_1	z_1
y_1	z_1
y_2	z_2
y_1	z_1
y_2	z_2
y_1	z_1
y_2	z_2

5. Practical issues

tracking the progress of the algorithm

- know current dual value and (possibly) primal value

choice of update rate α_k

- various strategies; success with rate based on dual progress

lazy update of dual solutions

- if updates are sparse, can avoid dynamically update soltuions

extracting solutions if algorithm does not converge

- best primal feasible solution; average solutions

Phrase-Based Translation

define:

- source-language sentence words $x_1, \ldots, x_N$
- phrase translation $p = (s, e, t)$
- translation derivation $y = p_1, \ldots, p_L$

example:

x_1	x_2	x_3	x_4	x_5	x_6
das	muss	unsere	sorge	gleichermaßen	sein

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- source-language sentence words $x_1, \dots, x_N$
- phrase translation $p = (s, e, t)$
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example:

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \text{das} & \text{muss} & \text{unsere} & \text{sorge} & \text{gleichermaßen} & \text{sein} \end{array}$$

$$\boxed{\text{das} \quad \text{muss}} \longrightarrow \underbrace{\boxed{\text{this} \quad \text{must}}}_{p_1}$$

$y = \{(1, 2, \text{this must}),$

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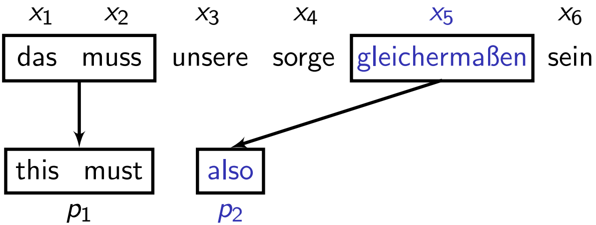

$y = \{(1, 2, \text{this must}), (5, 5, \text{also}),$

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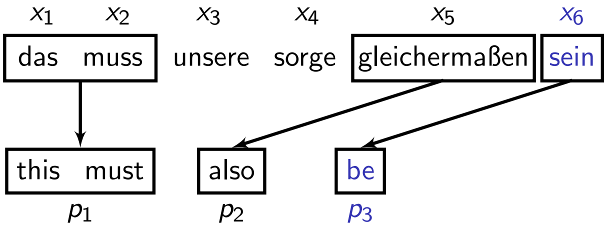

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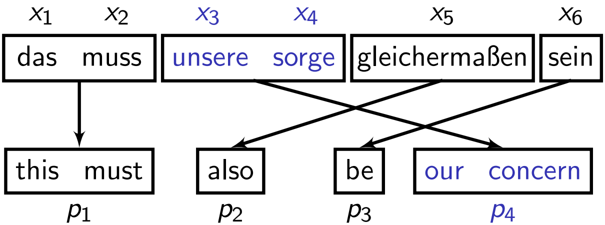

$y = \{(1, 2, \text{this must}), (5, 5, \text{also}), (6, 6, \text{be}), (3, 4, \text{our concern})\}$

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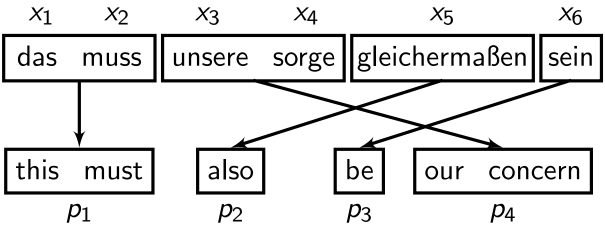

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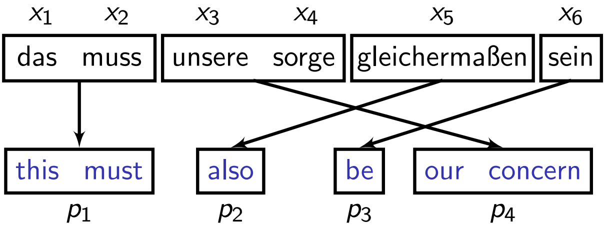
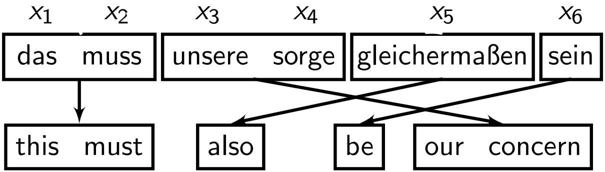


$y = \{(1, 2, \text{this must}), (5, 5, \text{also}), (6, 6, \text{be}), (3, 4, \text{our concern})\}$

Scoring Derivations

derivation:

$$y = \{(1, 2, \text{this must}), (5, 5, \text{also}), (6, 6, \text{be}), (3, 4, \text{our concern})\}$$



objective:

$$f(y) = h(e(y)) + \sum_{k=1}^{L} g(p_k) + \sum_{k=1}^{L-1} \eta |t(p_k) + 1 - s(p_{k+1})|$$

- language model score h
- phrase translation score g
- distortion penalty η

Scoring Derivations

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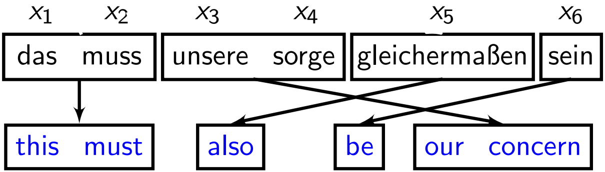
$$y = \{(1, 2, \text{this must}), (5, 5, \text{also}), (6, 6, \text{be}), (3, 4, \text{our concern})\}$$



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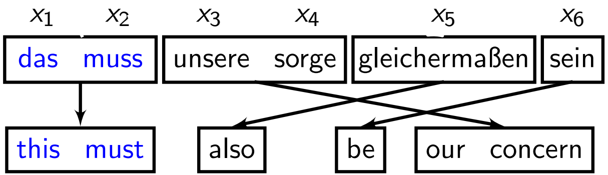

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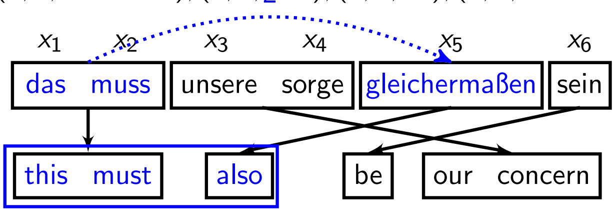

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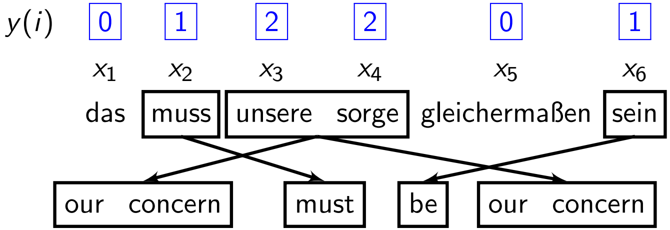
Relaxed Problem

$\mathcal{Y}'$: only requires the total number of words translated to be N

$$\mathcal{Y}' = \{y : \sum_{i=1}^{N} y(i) = N \text{ and the distortion limit } d \text{ is satisfied}\}$$

example:



(3, 4, our concern)(2, 2, must)(6, 6, be)(3, 4, our concern)

Relaxed Problem

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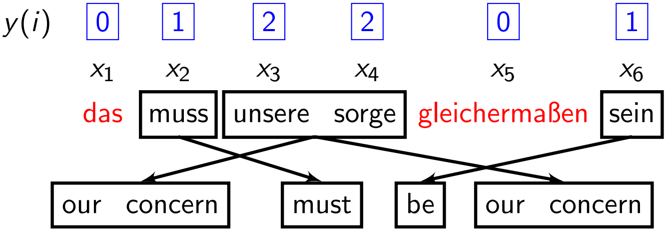

$(3, 4, \text{our concern})(2, 2, \text{must})(6, 6, \text{be})(3, 4, \text{our concern})$

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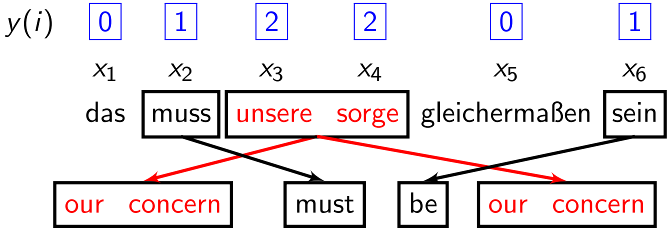

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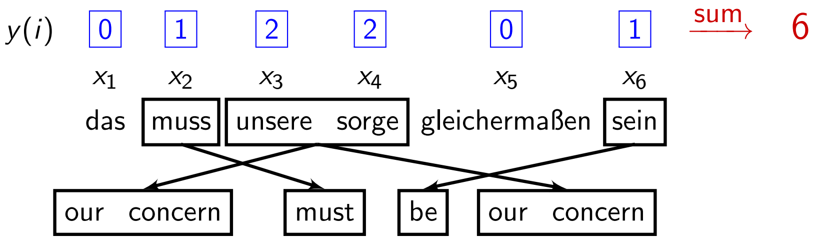

$(3, 4, \text{our concern})(2, 2, \text{must})(6, 6, \text{be})(3, 4, \text{our concern})$

Lagrangian Relaxation Method

original:

$$\underbrace{\arg\max_{y\in\mathcal{Y}} f(y)}$$

$$\mathcal{Y} = \{y : y(i) = 1 \ \forall i = 1 \ldots N\} \qquad \boxed{1}\,\boxed{1}\ldots\boxed{1}$$

rewrite:

$$\underbrace{\arg\max_{y\in\mathcal{Y}'} f(y)} \qquad \text{such that} \qquad \underbrace{y(i) = 1 \ \forall i = 1 \ldots N}$$

$$\mathcal{Y}' = \{y : \textstyle\sum_{i=1}^{N} y(i) = N\} \qquad \underbrace{\boxed{2}\,\boxed{0}\ldots\boxed{1}}_{\text{sum to } N}$$

Lagrangian Relaxation Method

original:

$$\underbrace{\arg\max_{y \in \mathcal{Y}} f(y)}_{\text{exact DP is NP-hard}}$$

$$\mathcal{Y} = \{y : y(i) = 1 \ \forall i = 1 \ldots N\}$$

$$\boxed{1}\ \boxed{1} \ldots \boxed{1}$$

rewrite:

$$\underbrace{\arg\max_{y \in \mathcal{Y}'} f(y)}_{} \qquad \text{such that} \qquad \underbrace{y(i) = 1 \ \forall i = 1 \ldots N}_{}$$

$$\mathcal{Y}' = \{y : \sum_{i=1}^{N} y(i) = N\}$$

$$\underbrace{\boxed{2}\ \boxed{0} \ldots \boxed{1}}_{\text{sum to } N}$$

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$$\underbrace{\arg\max_{y \in \mathcal{Y}} f(y)}_{\text{exact DP is NP-hard}}$$

$$\mathcal{Y} = \{y : y(i) = 1 \ \forall i = 1 \ldots N\} \qquad \boxed{1}\boxed{1}\ldots\boxed{1}$$

rewrite:

$$\underbrace{\arg\max_{y \in \mathcal{Y}'} f(y)}_{\text{can be solved efficiently by DP}} \quad \text{such that} \quad \underbrace{y(i) = 1 \ \forall i = 1 \ldots N}$$

$$\mathcal{Y}' = \{y : \textstyle\sum_{i=1}^{N} y(i) = N\} \qquad \underbrace{\boxed{2}\boxed{0}\ldots\boxed{1}}_{\text{sum to } N}$$

Lagrangian Relaxation Method

original:

$$\underbrace{\arg\max_{y \in \mathcal{Y}} f(y)}_{\text{exact DP is NP-hard}}$$

$$\mathcal{Y} = \{y : y(i) = 1 \; \forall i = 1 \ldots N\} \qquad \boxed{1}\,\boxed{1} \ldots \boxed{1}$$

rewrite:

$$\underbrace{\arg\max_{y \in \mathcal{Y}'} f(y)}_{\text{can be solved efficiently by DP}} \quad \text{such that} \quad \underbrace{y(i) = 1 \; \forall i = 1 \ldots N}_{\text{using Lagrangian relaxation}}$$

$$\mathcal{Y}' = \{y : \textstyle\sum_{i=1}^{N} y(i) = N\} \qquad \underbrace{\boxed{2}\,\boxed{0} \ldots \boxed{1}}_{\text{sum to } N}$$

Algorithm

Iteration 1:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 1$

$u(i)$	0	0	0	0	0	0
$y(i)$						
	x_1	x_2	x_3	x_4	x_5	x_6
	das	muss	unsere	sorge	gleichermaßen	sein

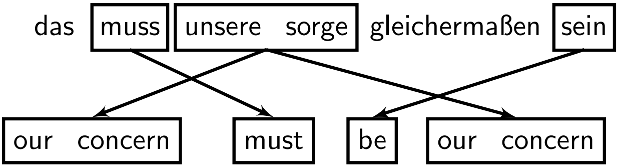
Algorithm

Iteration 1:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 1$

$u(i)$	0	0	0	0	0	0
$y(i)$	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	$\boxed{2}$	$\boxed{0}$	$\boxed{1}$
	x_1	x_2	x_3	x_4	x_5	x_6

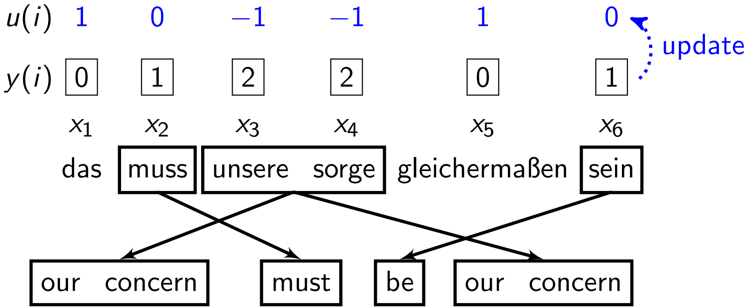


Algorithm

Iteration 1:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $$\alpha = 1$$



Algorithm

Iteration 2:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 0.5$

$u(i)$	1	0	-1	-1	1	0
$y(i)$						
	x_1	x_2	x_3	x_4	x_5	x_6
	das	muss	unsere	sorge	gleichermaßen	sein

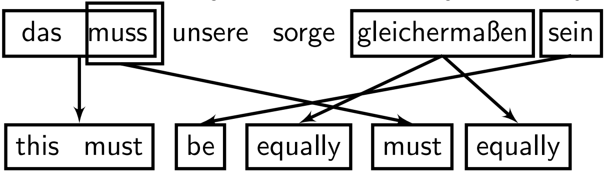
Algorithm

Iteration 2:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 0.5$

$u(i)$	1	0	-1	-1	1	0
$y(i)$	$\boxed{1}$	$\boxed{2}$	$\boxed{0}$	$\boxed{0}$	$\boxed{2}$	$\boxed{1}$
	x_1	x_2	x_3	x_4	x_5	x_6

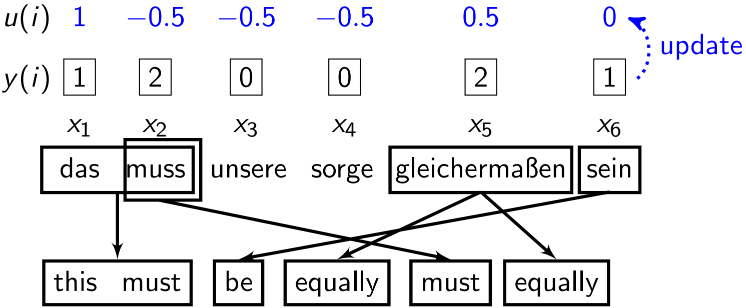


Algorithm

Iteration 2:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 0.5$



Algorithm

Iteration 3:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 0.5$

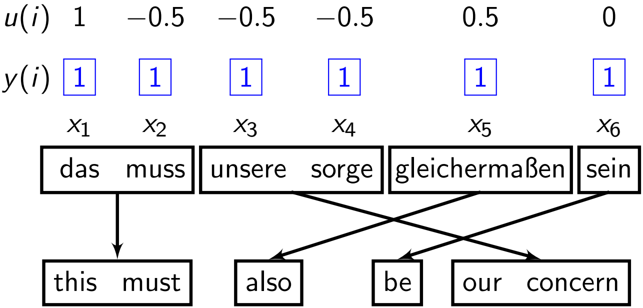
$u(i)$	1	-0.5	-0.5	-0.5	0.5	0
$y(i)$						
	x_1	x_2	x_3	x_4	x_5	x_6
	das	muss	unsere	sorge	gleichermaßen	sein

Algorithm

Iteration 3:

- update $u(i)$: $u(i) \leftarrow u(i) - \alpha(y(i) - 1)$

  $\alpha = 0.5$



Tightening the Relaxation

In some cases, we never reach $y(i) = 1$ for $i = 1 \ldots N$

If dual $L(u)$ is not decreasing fast enough

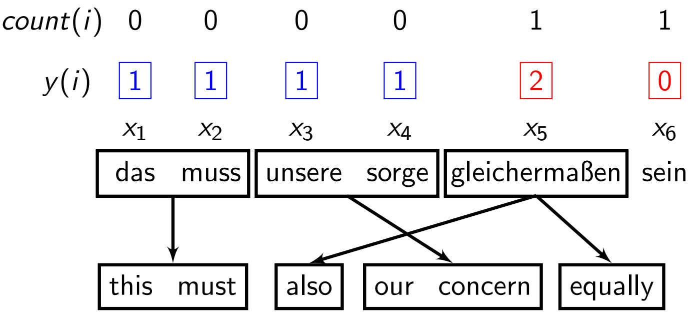
run for 10 more iterations

count number of times each constraint is violated

add 3 most often violated constraints

Tightening the Relaxation

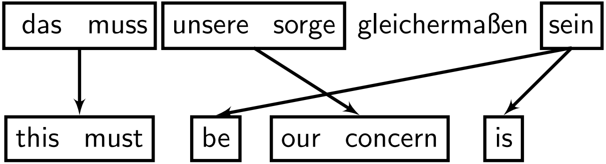
Iteration 41:



Tightening the Relaxation

Iteration 42:

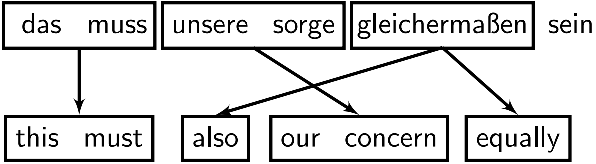
$count(i)$	0	0	0	0	2	2
$y(i)$	$\boxed{1}$	$\boxed{1}$	$\boxed{1}$	$\boxed{1}$	$\boxed{0}$	$\boxed{2}$
	x_1	x_2	x_3	x_4	x_5	x_6



Tightening the Relaxation

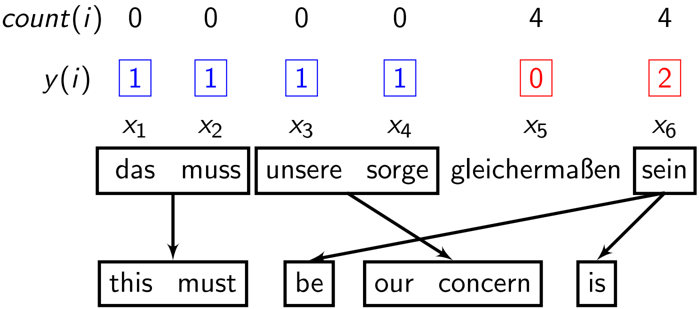
Iteration 43:

$count(i)$	0	0	0	0	3	3
$y(i)$	$\boxed{1}$	$\boxed{1}$	$\boxed{1}$	$\boxed{1}$	$\boxed{2}$	$\boxed{0}$
	x_1	x_2	x_3	x_4	x_5	x_6



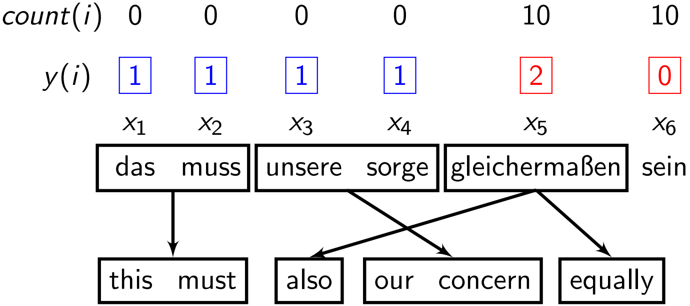
Tightening the Relaxation

Iteration 44:



Tightening the Relaxation

Iteration 50:



Tightening the Relaxation

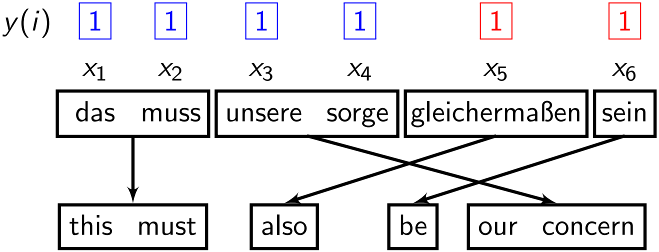
Iteration 51:

$y(i)$					$\boxed{1}$	$\boxed{1}$
	x_1	x_2	x_3	x_4	x_5	x_6
	das	muss	unsere	sorge	gleichermaßen	sein

Add 2 hard constraints (x_5, x_6) to the dynamic program

Tightening the Relaxation

Iteration 51:

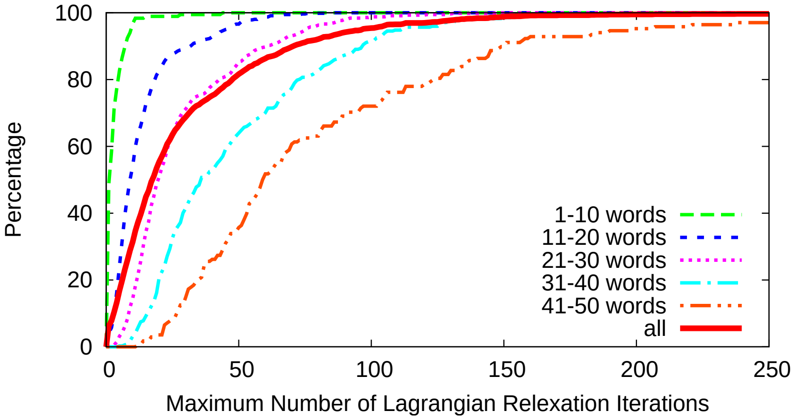


Add 2 hard constraints (x_5, x_6) to the dynamic program

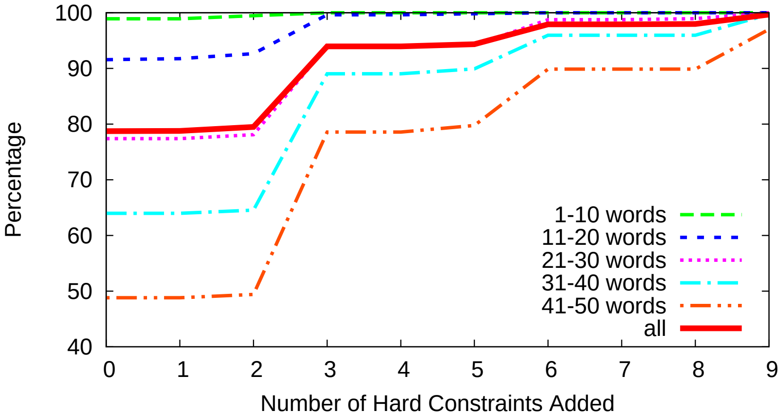
Experiments: German to English

- Europarl data: German to English
- Test on 1,824 sentences with length 1-50 words
- Converged: 1,818 sentences (99.67%)

Experiments: Number of Iterations



Experiments: Number of Hard Constraints Required



Experiments: Mean Time in Seconds

# words	1-10	11-20	21-30	31-40	41-50	All
mean	0.8	10.9	57.2	203.4	679.9	120.9
median	0.7	8.9	48.3	169.7	484.0	35.2

Comparison to ILP Decoding

	(sec.)	(sec.)
1-10	275.2	132.9
11-15	2,707.8	1,138.5
16-20	20,583.1	3,692.6

Summary

presented Lagrangian relaxation as a method for decoding in NLP

formal guarantees

- gives certificate or approximate solution
- can improve approximate solutions by tightening relaxation

efficient algorithms

- uses fast combinatorial algorithms
- can improve speed with lazy decoding

widely applicable

- demonstrated algorithms for a wide range of NLP tasks (parsing, tagging, alignment, mt decoding)

Higher-order non-projective dependency parsing

setup: given a model for higher-order non-projective dependency parsing (sibling features)

problem: find non-projective dependency parse that maximizes the score of this model

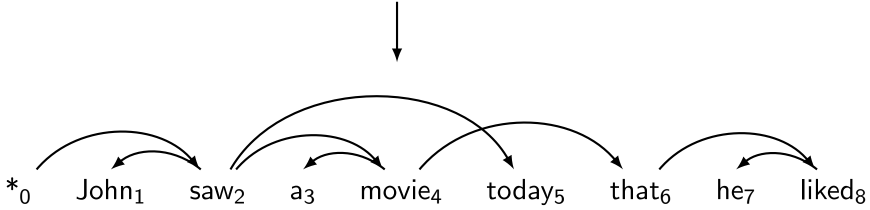
difficulty:

- model is NP-hard to decode
- complexity of the model comes from enforcing combinatorial constraints

strategy: design a decomposition that separates combinatorial constraints from direct implementation of the scoring function

Non-Projective Dependency Parsing

$*_0$ $\quad$ John$_1$ $\quad$ saw$_2$ $\quad$ a$_3$ $\quad$ movie$_4$ $\quad$ today$_5$ $\quad$ that$_6$ $\quad$ he$_7$ $\quad$ liked$_8$



Important problem in many languages.

Problem is NP-Hard for all but the simplest models.

Dual Decomposition

A classical technique for constructing decoding algorithms.

Solve complicated models

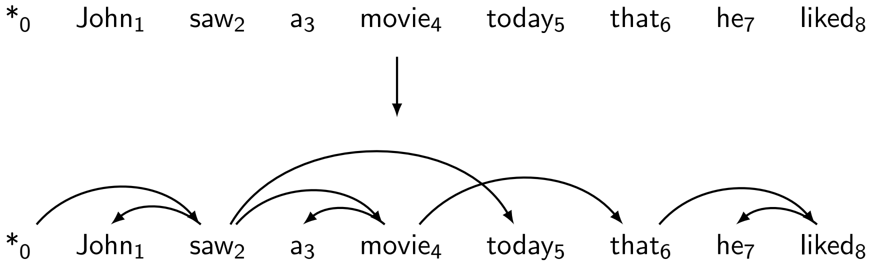
$$y^* = \arg\max_y f(y)$$

by decomposing into smaller problems.

Upshot: Can utilize a toolbox of combinatorial algorithms.

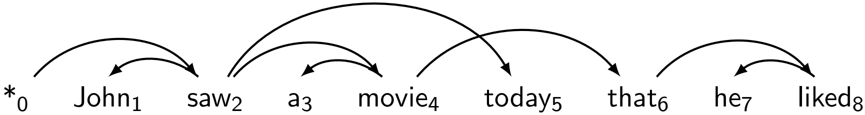
- Dynamic programming
- Minimum spanning tree
- Shortest path
- Min-Cut
- ...

Non-Projective Dependency Parsing



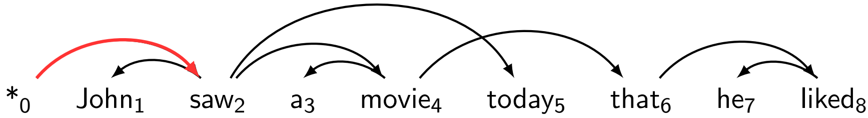
- Starts at the root symbol *
- Each word has a exactly one parent word
- Produces a tree structure (no cycles)
- Dependencies can cross

Arc-Factored



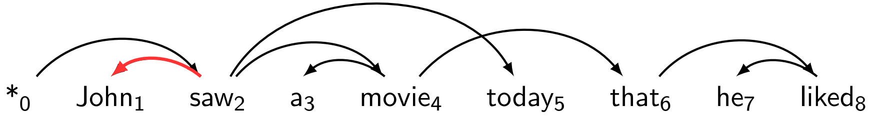
$f(y) =$

Arc-Factored



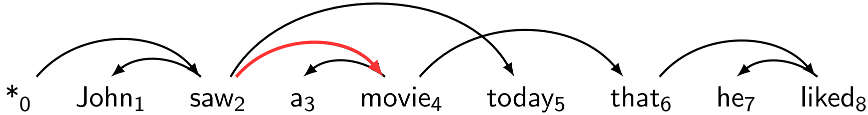
$f(y) = \textit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2)$

Arc-Factored



$$f(y) = \mathit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \mathit{score}(\text{saw}_2, \text{John}_1)$$

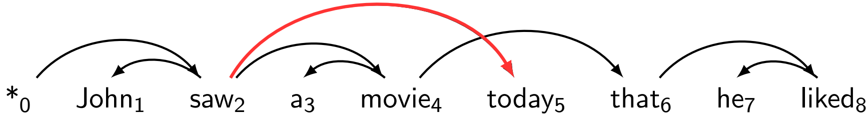
Arc-Factored



$$f(y) = \mathit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \mathit{score}(\text{saw}_2, \text{John}_1)$$

$$+\mathit{score}(\text{saw}_2, \text{movie}_4)$$

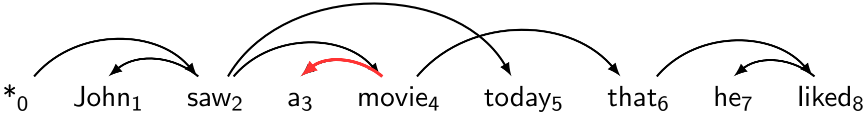
Arc-Factored



$$f(y) = \mathit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \mathit{score}(\text{saw}_2, \text{John}_1)$$

$$+\mathit{score}(\text{saw}_2, \text{movie}_4) \; {\color{red} +\mathit{score}(\text{saw}_2, \text{today}_5)}$$

Arc-Factored

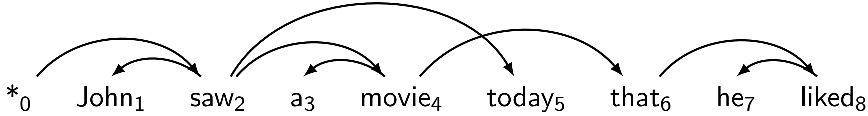


$$f(y) = \mathit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \mathit{score}(\text{saw}_2, \text{John}_1)$$

$$+\mathit{score}(\text{saw}_2, \text{movie}_4) + \mathit{score}(\text{saw}_2, \text{today}_5)$$

$$+\mathit{score}(\text{movie}_4, \text{a}_3) + \ldots$$

Arc-Factored



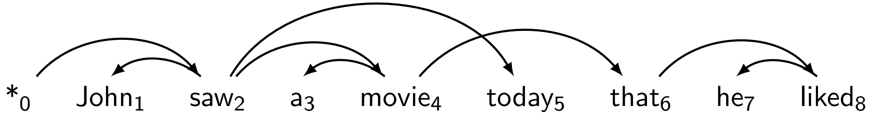
$$f(y) = \textit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \textit{score}(\text{saw}_2, \text{John}_1)$$

$$+ \textit{score}(\text{saw}_2, \text{movie}_4) + \textit{score}(\text{saw}_2, \text{today}_5)$$

$$+ \textit{score}(\text{movie}_4, \text{a}_3) + \ldots$$

e.g. $\textit{score}(*_0, \text{saw}_2) = \log p(\text{saw}_2 | *_0)$ (generative model)

Arc-Factored



$$f(y) = \textit{score}(\text{head} = *_0, \text{mod} = \text{saw}_2) + \textit{score}(\text{saw}_2, \text{John}_1)$$

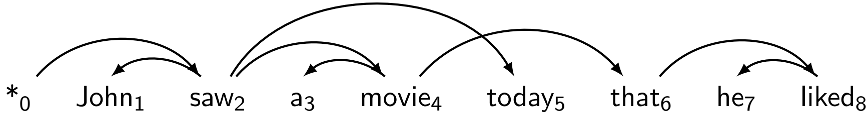
$$+\textit{score}(\text{saw}_2, \text{movie}_4) + \textit{score}(\text{saw}_2, \text{today}_5)$$

$$+\textit{score}(\text{movie}_4, \text{a}_3) + \ldots$$

e.g. $\textit{score}(*_0, \text{saw}_2) = \log p(\text{saw}_2 | *_0)$ (generative model)

or $\textit{score}(*_0, \text{saw}_2) = w \cdot \phi(\text{saw}_2, *_0)$ (CRF/perceptron model)

Arc-Factored



$$f(y) = \textit{score}(\mathrm{head} = *_0, \mathrm{mod} = \mathrm{saw}_2) + \textit{score}(\mathrm{saw}_2, \mathrm{John}_1)$$

$$+\textit{score}(\mathrm{saw}_2, \mathrm{movie}_4) + \textit{score}(\mathrm{saw}_2, \mathrm{today}_5)$$

$$+\textit{score}(\mathrm{movie}_4, \mathrm{a}_3) + \ldots$$

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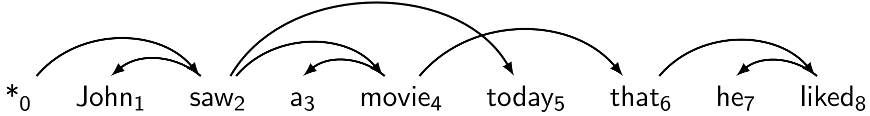
or $\textit{score}(*_0, \mathrm{saw}_2) = w \cdot \phi(\mathrm{saw}_2, *_0)$ (CRF/perceptron model)

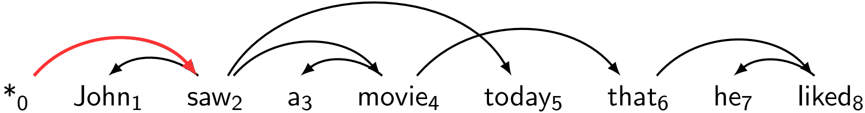
$$y^* = \arg\max_y f(y) \Leftarrow$$ Minimum Spanning Tree Algorithm

Sibling Models



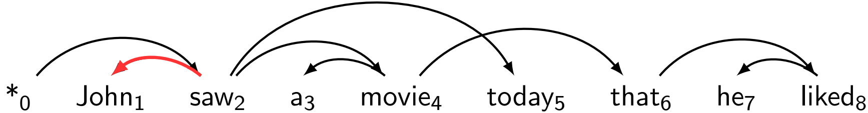
$f(y) =$

Sibling Models



$$f(y) = \textit{score}(\textit{head} = *_0, \textit{prev} = \mathrm{NULL}, \textit{mod} = \mathrm{saw}_2)$$

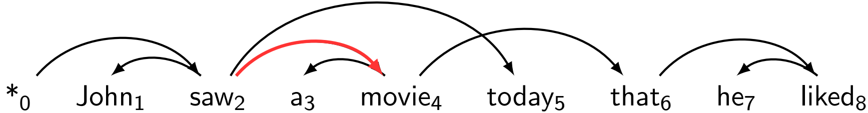
Sibling Models



$f(y) = \textit{score}(\textit{head} = *_0, \textit{prev} = \mathrm{NULL}, \textit{mod} = \mathrm{saw}_2)$

$+\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1)$

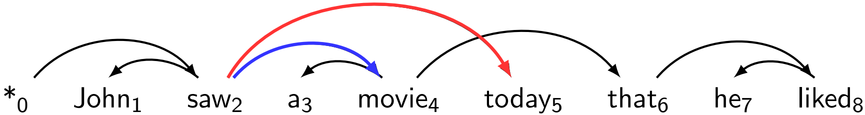
Sibling Models



$$f(y) = \textit{score}(\textit{head} = *_0, \textit{prev} = \mathrm{NULL}, \textit{mod} = \mathrm{saw}_2)$$

$$+\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) \; +\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{movie}_4)$$

Sibling Models

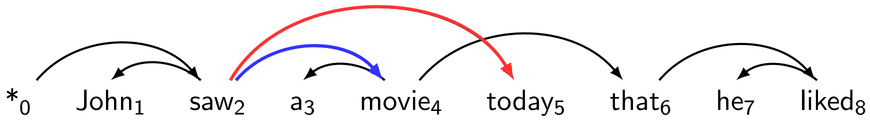


$$f(y) = \mathit{score}(\mathit{head} = *_0, \mathit{prev} = \text{NULL}, \mathit{mod} = \text{saw}_2)$$

$$+\mathit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \mathit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4)$$

$$+\mathit{score}(\text{saw}_2, {\color{blue}\text{movie}_4}, {\color{red}\text{today}_5}) + \ldots$$

Sibling Models



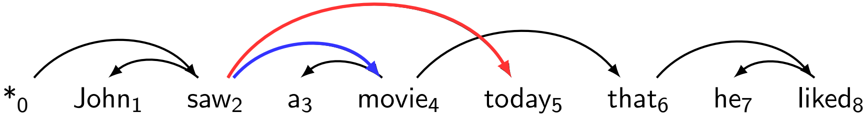
$$f(y) = \textit{score}(\textit{head} = *_0, \textit{prev} = \text{NULL}, \textit{mod} = \text{saw}_2)$$

$$+\textit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \textit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4)$$

$$+\textit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5) + \ldots$$

e.g. $\textit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5) = \log p(\text{today}_5 | \text{saw}_2, \text{movie}_4)$

Sibling Models



$$f(y) = \mathit{score}(\mathit{head} = *_0, \mathit{prev} = \mathrm{NULL}, \mathit{mod} = \mathrm{saw}_2)$$

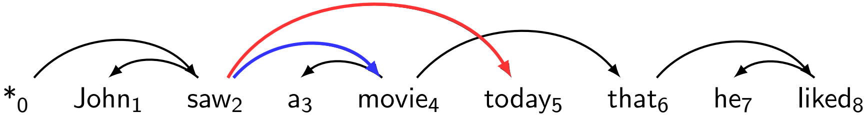
$$+\mathit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) + \mathit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{movie}_4)$$

$$+\mathit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5) + \ldots$$

e.g. $\mathit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5) = \log p(\mathrm{today}_5 | \mathrm{saw}_2, \mathrm{movie}_4)$

or $\mathit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5) = w \cdot \phi(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5)$

Sibling Models



$$f(y) = \mathit{score}(\mathit{head} = *_0, \mathit{prev} = \text{NULL}, \mathit{mod} = \text{saw}_2)$$

$$+\mathit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \mathit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4)$$

$$+\mathit{score}(\text{saw}_2, {\color{blue}\text{movie}_4}, {\color{red}\text{today}_5}) + \ldots$$

e.g. $\mathit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5) = \log p(\text{today}_5 | \text{saw}_2, \text{movie}_4)$

or $\mathit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5) = w \cdot \phi(\text{saw}_2, \text{movie}_4, \text{today}_5)$

$$y^* = \arg\max_y f(y) \Leftarrow \text{NP-Hard}$$

Thought Experiment: Individual Decoding

$*_0$ $\quad$ John_1 $\quad$ saw_2 $\quad$ a_3 $\quad$ movie_4 $\quad$ today_5 $\quad$ that_6 $\quad$ he_7 $\quad$ liked_8

Thought Experiment: Individual Decoding

$*_0$ John_1 saw_2 a_3 movie_4 today_5 that_6 he_7 liked_8

$$\textit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \textit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4)$$
$$+\textit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5)$$

Thought Experiment: Individual Decoding

$*_0$ John_1 saw_2 a_3 movie_4 today_5 that_6 he_7 liked_8

$$\textit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \textit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4)$$
$$+\textit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5)$$

$$\textit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \textit{score}(\text{saw}_2, \text{NULL}, \text{that}_6)$$

Thought Experiment: Individual Decoding

$*_0$ $\quad$ John$_1$ $\quad$ saw$_2$ $\quad$ a$_3$ $\quad$ movie$_4$ $\quad$ today$_5$ $\quad$ that$_6$ $\quad$ he$_7$ $\quad$ liked$_8$

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) + \textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{movie}_4)$$
$$+\textit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5)$$

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) + \textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{that}_6)$$

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{a}_3) + \textit{score}(\mathrm{saw}_2, \mathrm{a}_3, \mathrm{he}_7)$$

Thought Experiment: Individual Decoding

$*_0$ $\quad$ John_1 $\quad$ saw_2 $\quad$ a_3 $\quad$ movie_4 $\quad$ today_5 $\quad$ that_6 $\quad$ he_7 $\quad$ liked_8

2^{n-1}
possibilities

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) + \textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{movie}_4)$$
$$+\textit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5)$$

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{John}_1) + \textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{that}_6)$$

$$\textit{score}(\mathrm{saw}_2, \mathrm{NULL}, \mathrm{a}_3) + \textit{score}(\mathrm{saw}_2, \mathrm{a}_3, \mathrm{he}_7)$$

Thought Experiment: Individual Decoding

$*_0$ $\quad$ John_1 $\quad$ saw_2 $\quad$ a_3 $\quad$ movie_4 $\quad$ today_5 $\quad$ that_6 $\quad$ he_7 $\quad$ liked_8

$$\begin{array}{c}2^{n-1}\\ \textbf{possibilities}\end{array} \left\{ \begin{array}{l} \mathit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \mathit{score}(\text{saw}_2, \text{NULL}, \text{movie}_4) \\ +\mathit{score}(\text{saw}_2, \text{movie}_4, \text{today}_5) \\ \hline \mathit{score}(\text{saw}_2, \text{NULL}, \text{John}_1) + \mathit{score}(\text{saw}_2, \text{NULL}, \text{that}_6) \\ \hline \mathit{score}(\text{saw}_2, \text{NULL}, \text{a}_3) + \mathit{score}(\text{saw}_2, \text{a}_3, \text{he}_7) \end{array} \right.$$

Under Sibling Model, can solve for each word with Viterbi decoding.

Thought Experiment Continued

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

Idea: Do individual decoding for each head word using dynamic programming.

If we're lucky, we'll end up with a valid final tree.

Thought Experiment Continued

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

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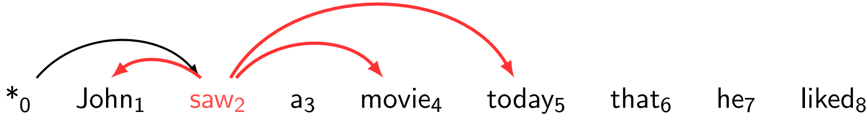
Thought Experiment Continued



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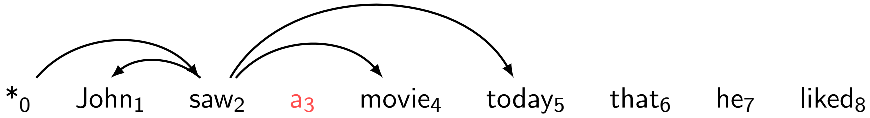

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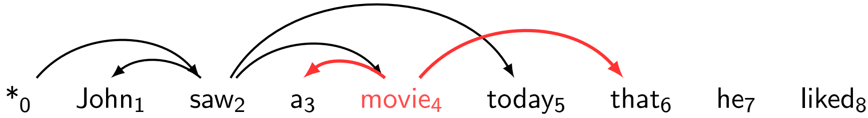
Thought Experiment Continued



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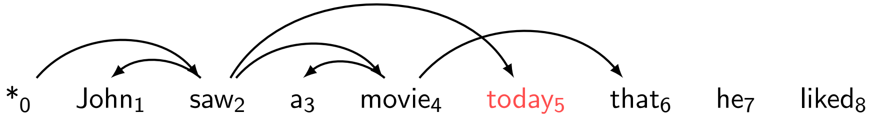

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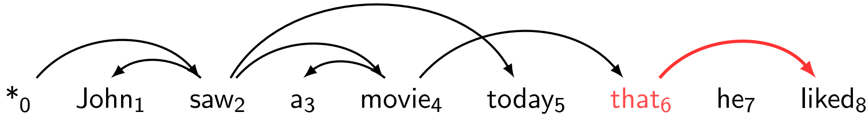

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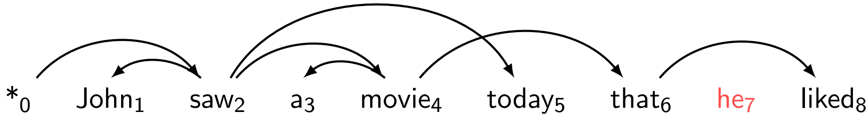

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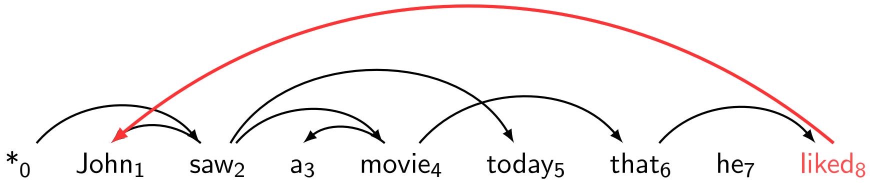
Thought Experiment Continued



Idea: Do individual decoding for each head word using dynamic programming.

If we're lucky, we'll end up with a valid final tree.

But we might violate some constraints.

Dual Decomposition Structure

$$\text{Goal } y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

Dual Decomposition Structure

Goal $y^* = \arg\max_{y \in \mathcal{Y}} f(y)$

Rewrite as $\underset{z \in \mathcal{Z},\ y \in \mathcal{Y}}{\operatorname{argmax}} \quad f(z) + g(y)$

such that $z = y$

Dual Decomposition Structure

$$\text{Goal } y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

$$\text{Rewrite as} \quad \underset{z \in \mathcal{Z},\ y \in \mathcal{Y}}{\text{argmax}} \quad f(z) + g(y)$$

All Possible

$$\text{such that } z = y$$

Dual Decomposition Structure

Goal $y^* = \arg\max_{y \in \mathcal{Y}} f(y)$

Rewrite as $\underset{\underset{\text{All Possible}}{z \in \mathcal{Z}},\ \underset{\text{Valid Trees}}{y \in \mathcal{Y}}}{\text{argmax}} \quad f(z) + g(y)$

such that $z = y$

Dual Decomposition Structure

$$\text{Goal } y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

$$\text{Rewrite as} \quad \operatorname*{argmax}_{\underbrace{z \in \mathcal{Z}}_{\textbf{All Possible}},\ \underbrace{y \in \mathcal{Y}}_{\textbf{Valid Trees}}} \quad \overbrace{f(z)}^{\textbf{Sibling}} + g(y)$$

$$\text{such that } z = y$$

Dual Decomposition Structure

$$\text{Goal } y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

$$\text{Rewrite as} \quad \underbrace{\operatorname{argmax}}_{z \in \underbrace{\mathcal{Z}}_{\textbf{All Possible}},\ y \in \underbrace{\mathcal{Y}}_{\textbf{Valid Trees}}} \quad \overbrace{f(z)}^{\textbf{Sibling}} + \overbrace{g(y)}^{\textbf{Arc-Factored}}$$

$$\text{such that } z = y$$

Dual Decomposition Structure

$$\text{Goal } y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

$$\text{Rewrite as} \quad \operatorname*{argmax}_{\underset{\textbf{All Possible}}{z \in \mathcal{Z}},\ \underset{\textbf{Valid Trees}}{y \in \mathcal{Y}}} \quad \overset{\textbf{Sibling}}{f(z)} + \overset{\textbf{Arc-Factored}}{g(y)}$$

$$\text{such that } z = \underset{\textbf{Constraint}}{y}$$

Algorithm Sketch

Set penalty weights equal to 0 for all edges.

For $k = 1$ **to** K

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Algorithm Sketch

Set penalty weights equal to 0 for all edges.

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$z^{(k)} \leftarrow$ Decode $(f(z) + \mathrm{penalty})$ by Individual Decoding

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If $y^{(k)}(i,j) = z^{(k)}(i,j)$ for all i,j **Return** $(y^{(k)}, z^{(k)})$

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Set penalty weights equal to 0 for all edges.

For $k = 1$ **to** K

$z^{(k)} \leftarrow$ Decode $(f(z) + \mathrm{penalty})$ by Individual Decoding

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If $y^{(k)}(i,j) = z^{(k)}(i,j)$ for all i,j **Return** $(y^{(k)}, z^{(k)})$

Else Update penalty weights based on $y^{(k)}(i,j) - z^{(k)}(i,j)$

Individual Decoding

Penalties

$u(i,j) = 0$ for all i,j

$*_0$ John_1 saw_2 a_3 movie_4 today_5 that_6 he_7 liked_8

$$z^* = \arg\max_{z \in \mathcal{Z}} (f(z) + \sum_{i,j} u(i,j) z(i,j))$$

Minimum Spanning Tree

$*_0$ John_1 saw_2 a_3 movie_4 today_5 that_6 he_7 liked_8

$$y^* = \arg\max_{y \in \mathcal{Y}} (g(y) - \sum_{i,j} u(i,j) y(i,j))$$

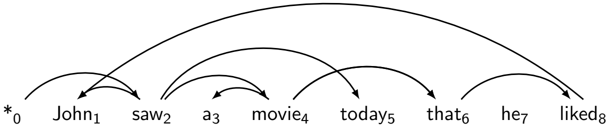
Key

$f(z)$	$\Leftarrow$	Sibling Model	$g(y)$	$\Leftarrow$	Arc-Factored Model
$\mathcal{Z}$	$\Leftarrow$	No Constraints	$\mathcal{Y}$	$\Leftarrow$	Tree Constraints
$y(i,j) = 1$	if	y contains dependency i,j			

Individual Decoding

Penalties

$u(i, j) = 0$ for all i,j



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Minimum Spanning Tree

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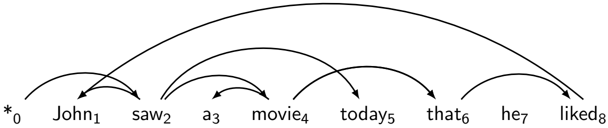

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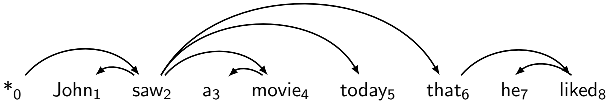
Minimum Spanning Tree



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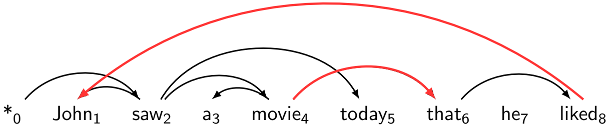

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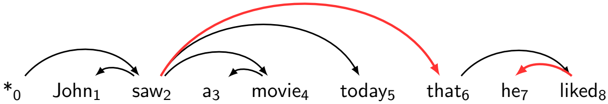
Minimum Spanning Tree



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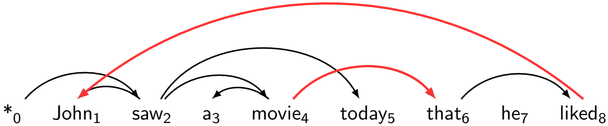

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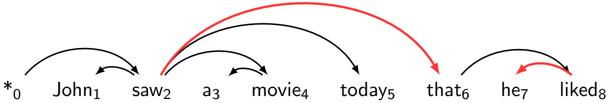
Minimum Spanning Tree



$$y^* = \arg\max_{y \in \mathcal{Y}} (g(y) - \sum_{i,j} u(i,j) y(i,j))$$

Penalties

$u(i,j) = 0$ for all i,j

Iteration 1	
$u(8,1)$	-1
$u(4,6)$	-1
$u(2,6)$	1
$u(8,7)$	1

Key

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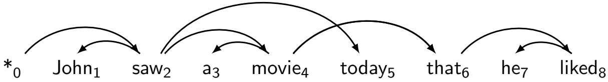

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Minimum Spanning Tree

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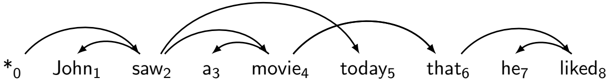

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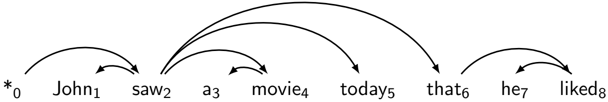

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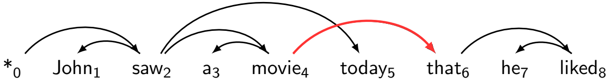

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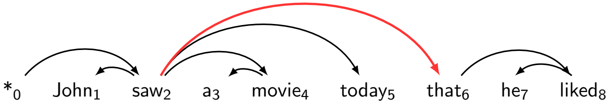

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$u(8,7)$	1

Iteration 2	
$u(8,1)$	-1
$u(4,6)$	-2
$u(2,6)$	2
$u(8,7)$	1

Key

$f(z)$	$\Leftarrow$	Sibling Model	$g(y)$	$\Leftarrow$	Arc-Factored Model
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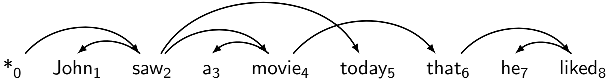
Individual Decoding



$$z^* = \arg\max_{z \in \mathcal{Z}} (f(z) + \sum_{i,j} u(i,j) z(i,j))$$

Minimum Spanning Tree

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Key

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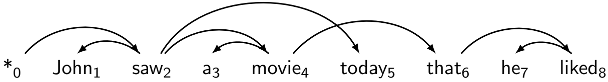
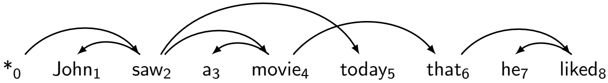
Individual Decoding



$$z^* = \arg\max_{z \in \mathcal{Z}} (f(z) + \sum_{i,j} u(i,j) z(i,j))$$

Minimum Spanning Tree



$$y^* = \arg\max_{y \in \mathcal{Y}} (g(y) - \sum_{i,j} u(i,j) y(i,j))$$

Penalties

$u(i,j) = 0$ for all i,j

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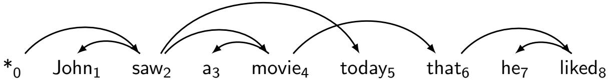

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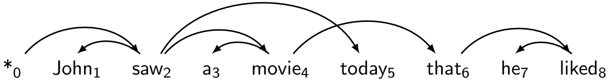
Minimum Spanning Tree



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Penalties

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Iteration 2	
$u(8,1)$	-1
$u(4,6)$	-2
$u(2,6)$	2
$u(8,7)$	1

Converged

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y) + g(y)$$

Key

$f(z)$	$\Leftarrow$	Sibling Model	$g(y)$	$\Leftarrow$	Arc-Factored Model
$\mathcal{Z}$	$\Leftarrow$	No Constraints	$\mathcal{Y}$	$\Leftarrow$	Tree Constraints
$y(i,j) = 1$	if	y contains dependency i,j			

Guarantees

Theorem

If at any iteration $y^{(k)} = z^{(k)}$, then $(y^{(k)}, z^{(k)})$ is the global optimum.

In experiments, we find the global optimum on 98% of examples.

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If we do not converge to a match, we can still return an approximate solution (more in the paper).

Extensions

- Grandparent Models

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

$$f(y) = \ldots + score(gp = *_0, head = \text{saw}_2, prev = \text{movie}_4, mod = \text{today}_5)$$

- Head Automata (Eisner, 2000)

  Generalization of Sibling models

  Allow arbitrary automata as local scoring function.

Experiments

Properties:

- Exactness
- Parsing Speed
- Parsing Accuracy
- Comparison to Individual Decoding
- Comparison to LP/ILP

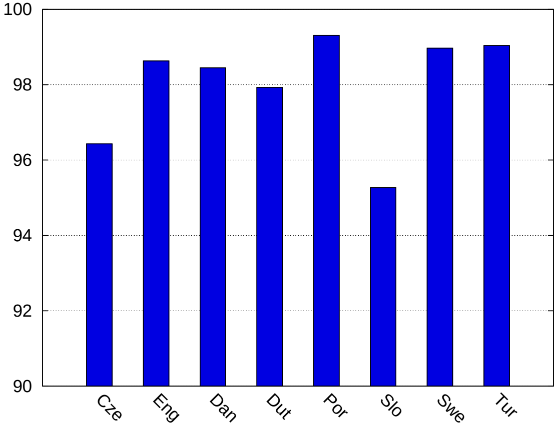
Training:

- Averaged Perceptron (more details in paper)

Experiments on:

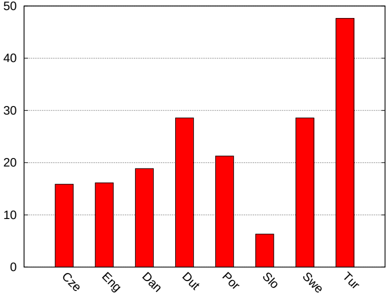
- CoNLL Datasets
- English Penn Treebank
- Czech Dependency Treebank

How often do we exactly solve the problem?

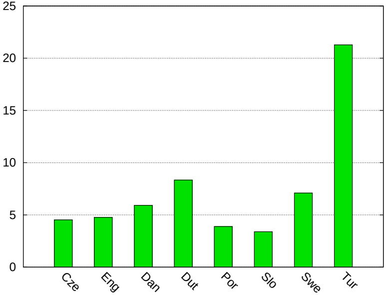


- Percentage of examples where the dual decomposition finds an exact solution.

Parsing Speed



Sibling model



Grandparent model

- Number of sentences parsed per second
- Comparable to dynamic programming for projective parsing

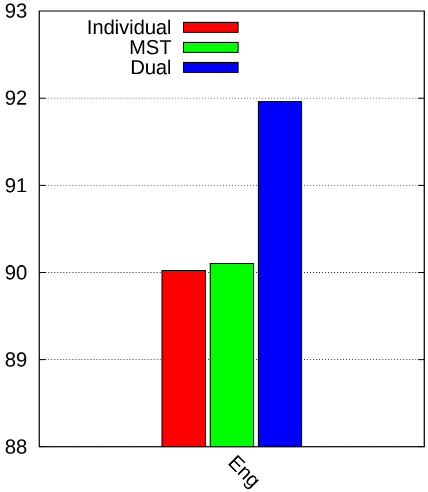
Accuracy

	Arc-Factored	Prev Best	Grandparent
Dan	89.7	91.5	**91.8**
Dut	82.3	85.6	**85.8**
Por	90.7	92.1	**93.0**
Slo	82.4	85.6	**86.2**
Swe	88.9	90.6	**91.4**
Tur	75.7	76.4	**77.6**
Eng	90.1	—	**92.5**
Cze	84.4	—	**87.3**

Prev Best - Best reported results for CoNLL-X data set, includes

- Approximate search (McDonald and Pereira, 2006)
- Loop belief propagation (Smith and Eisner, 2008)
- (Integer) Linear Programming (Martins et al., 2009)

Comparison to Subproblems



F_1 for dependency accuracy

Comparison to LP/ILP

Martins et al.(2009): Proposes two representations of non-projective dependency parsing as a linear programming relaxation as well as an exact ILP.

- LP (1)
- LP (2)
- ILP

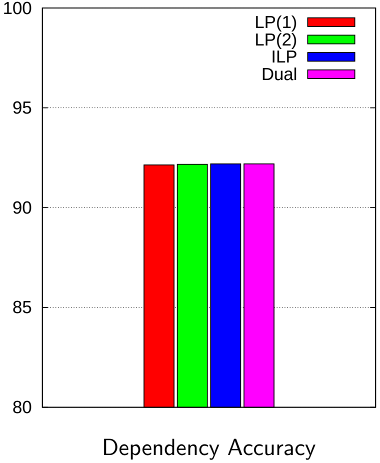
Use an LP/ILP Solver for decoding

We compare:

- Accuracy
- Exactness
- Speed

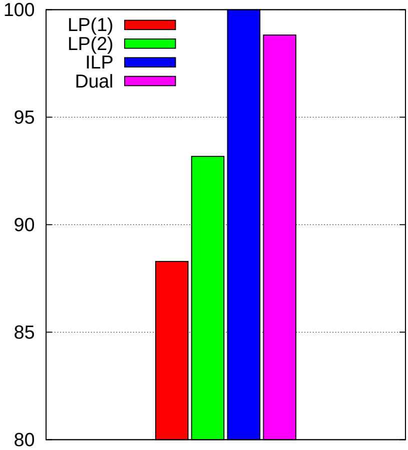
Both LP and dual decomposition methods use the same model, features, and weights w.

Comparison to LP/ILP: Accuracy

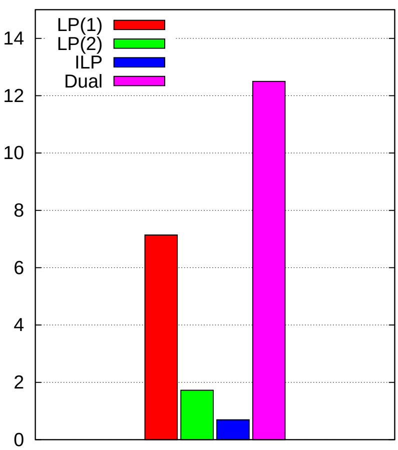


- All decoding methods have comparable accuracy

Comparison to LP/ILP: Exactness and Speed



Percentage with exact solution



Sentences per second

References I

Y. Chang and M. Collins. Exact Decoding of Phrase-based Translation Models through Lagrangian Relaxation. In *To appear proc. of EMNLP*, 2011.

J. DeNero and K. Macherey. Model-Based Aligner Combination Using Dual Decomposition. In *Proc. ACL*, 2011.

J. Duchi, D. Tarlow, G. Elidan, and D. Koller. Using Combinatorial Optimization within Max-Product Belief Propagation. In *NIPS*, pages 369–376, 2007.

D. Klein and C.D. Manning. Factored A* Search for Models over Sequences and Trees. In *Proc IJCAI*, volume 18, pages 1246–1251. Citeseer, 2003.

N. Komodakis, N. Paragios, and G. Tziritas. Mrf energy minimization and beyond via dual decomposition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2010. ISSN 0162-8828.

References II

Terry Koo, Alexander M. Rush, Michael Collins, Tommi Jaakkola, and David Sontag. Dual decomposition for parsing with non-projective head automata. In *EMNLP*, 2010. URL http://www.aclweb.org/anthology/D10-1125.

B.H. Korte and J. Vygen. *Combinatorial Optimization: Theory and Algorithms*. Springer Verlag, 2008.

A.M. Rush and M. Collins. Exact Decoding of Syntactic Translation Models through Lagrangian Relaxation. In *Proc. ACL*, 2011.

A.M. Rush, D. Sontag, M. Collins, and T. Jaakkola. On Dual Decomposition and Linear Programming Relaxations for Natural Language Processing. In *Proc. EMNLP*, 2010.

D.A. Smith and J. Eisner. Dependency Parsing by Belief Propagation. In *Proc. EMNLP*, pages 145–156, 2008. URL http://www.aclweb.org/anthology/D08-1016.