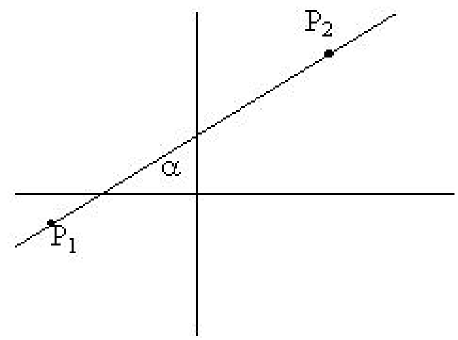
Analytic Geometry

Points and Lines:

For any points P_1 (x_1, y_1) and P_2 (x_2, y_2) in a rectangular coordinate plane,

Distance between P_1 and P_2 : $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Slope m of P_1 and P_2 : $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \tan a$

Angle q between two lines of slopes m_1 and m_2 : $\tan q = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

Distance from $Ax + By + C = 0$ to P_1 : $\left| \dfrac{Ax_1 + Bx_2 + C}{\sqrt{A^2 + B^2}} \right|$

Triangles:

For a triangle with vertices P_1 (x_1, y_1) , P_2 (x_2, y_2) and P_3 (x_3, y_3),

Area: $\dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

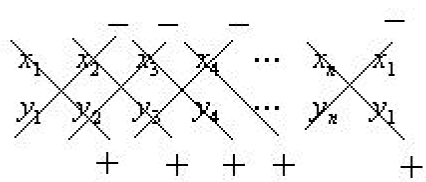
Coordinates of Centroid: $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3} \right)$

Polygons:

Area of Polygon $P_1\ P_2 \ldots P_n$:

$$\frac{1}{2}\left(x_1y_2 + x_2y_3 + \cdots x_{n-1}y_n + x_ny_1 - y_1x_2 - y_2x_3 - \ldots - y_{n-1}x_n - y_nx_1\right)$$

This is the sums of the products of the coordinates on lines slanting downward minus the products of the coordinates on lines slanting upwards (like a 3 by 3 determinant).



Pick’s Theorem:

For any polygon whose vertices are lattice points, the area is given by $K = \frac{1}{2}B + I - 1$, where B is the number of lattice points on the boundary of the polygon and I is the number of lattice points in the interior.