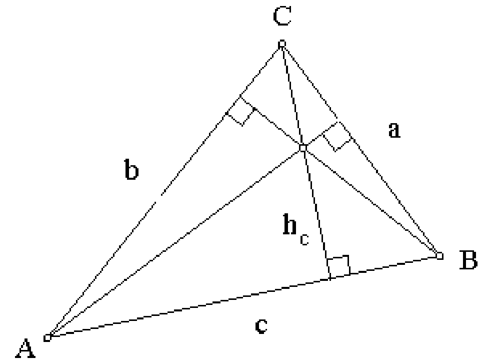
Cevians

A cevian is any segment drawn from the vertex of a triangle to the opposite side. Cevians with special properties include altitudes, angle bisectors, and medians. Let h_c, t_c, and m_c represent the altitude, angle bisector, and median to side c, respectively.

Altitudes:

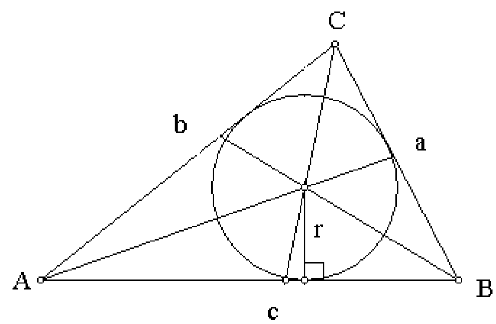
The altitudes of a triangle intersect at the *orthocenter*.

$$h_c = a \sin B \qquad h_c = \frac{2K}{c}$$



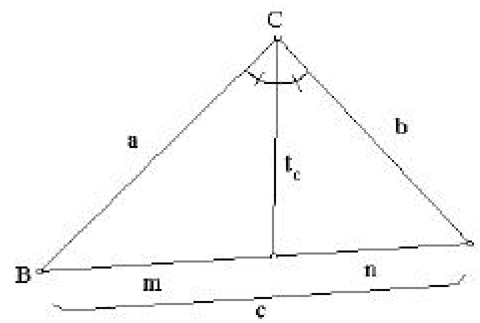
Angle Bisectors:

The angle bisectors of a triangle intersect at the *incenter*, the center of the triangle's inscribed circle.



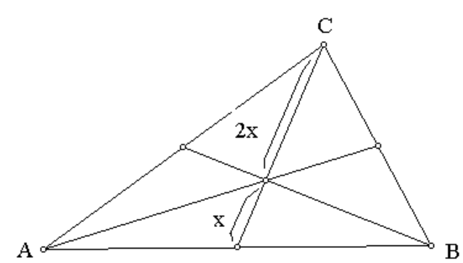
Angle Bisector Theorem: $\frac{a}{m} = \frac{b}{n}$

Length of an Angle Bisector: $t_c = \sqrt{ab\left(1 - \frac{c^2}{a^2 + b^2}\right)}$

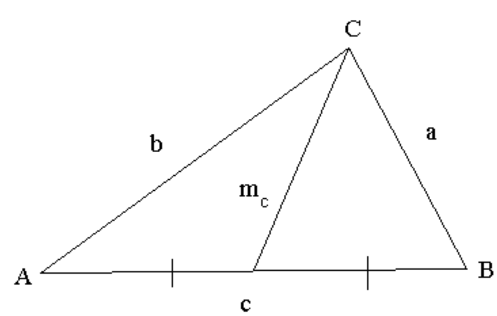


Medians:

The medians of a triangle intersect at the *centroid*. Along the median, the distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.



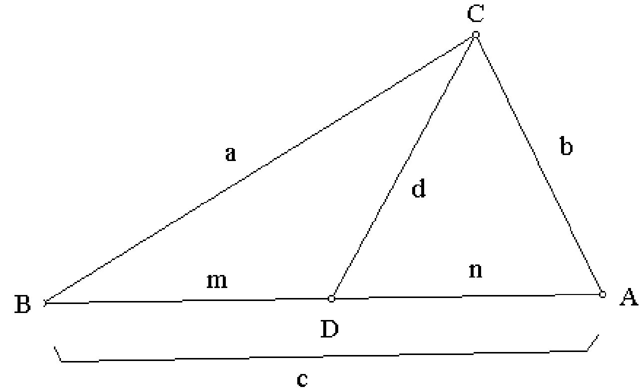
Length of a Median: $m_c = \sqrt{\frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{4}}$



Stewart's Theorem

If a cevian of length d is drawn and divides side c into segments m and n, then

$$a^2n + b^2m = c(d^2 + mn)$$

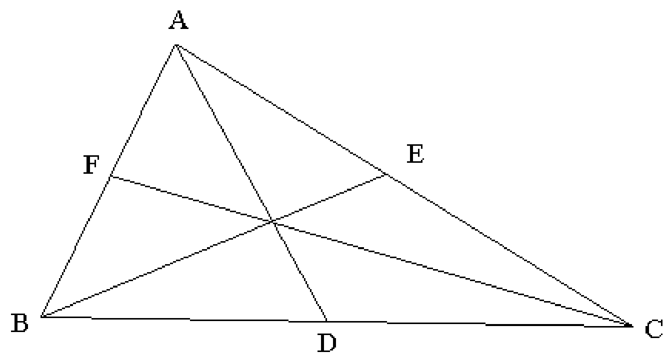


Ceva's Theorem

A necessary and sufficient condition for AD, BE, CF, where D, E, and F are points on the respective side lines BC, CA, AB of a triangle ABC, to be concurrent is that

$$BD \cdot CE \cdot AF = +DC \cdot EA \cdot FB$$

where all segments in the formula are directed segments.



Ex. Suppose AB, AC, and BC have lengths 13, 14, and 15. If AF:FB = 2:5 and CE:EA = 5:8. If BD = x and DC = y, then $10x = 40y$, and $x + y = 15$. Solving, we have $x = 12$ and $y = 3$.

Menelaus' Theorem

A necessary and sufficient condition for points D, E, F on the respective side lines BC, CA, AB of a triangle ABC to be collinear is that

$$BD \cdot CE \cdot AF = -DC \cdot EA \cdot FB$$

where all segments in the formula are directed segments.