Polynomials

For all polynomials of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$, where $a_i \in R$:

Fundamental Theorem of Algebra: *P(x)* has *n* roots

Sum of roots: $-\dfrac{a_{n-1}}{a_n}$

Product of roots: $\dfrac{a_0}{a_n}(-1)^n$

For any a_k, $\dfrac{a_k}{a_n}(-1)^{n+k}$ represents the sum of the product of the roots, taken $(n-k)$ at a time.

Ex. when *n=3*, $\dfrac{a_1}{a_3}(-1)^{3+1}$ is the sum of product of the roots, taken *3-1* or *2* at a time. $\dfrac{a_1}{a_3} = (r_1 r_2 + r_2 r_3 + r_3 r_1)$, where r_1, r_2, and r_3 are the roots of the polynomial.

Remainder Theorem:

The remainder when *P(x)* is divided by *(x – w)* is *P(w)*.

Descartes' Rule of Signs:

The number of positive real roots of *P(x)* is *z* decreased by some multiple of two, *(z, z-2, z-4, etc…)*. *z* is the number of sign changes in the coefficients of *P(x)*, counting from a_n to a_0. The number of negative real roots is found similarly by finding *z* for *P(-x)*.

Ex. For the polynomial $x^5 - 4x^4 + 3x^2 - 6x + 1$, there are possibly *4, 2,* or *0* positive roots and *1* negative root.

Rational Root Theorem:

If all a_i are integers, then the only possible rational roots of *P(x)* are of the form $\pm\dfrac{k}{a_n}$, where *k* is a factor of a_0.