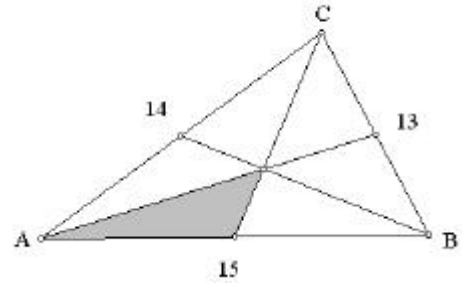


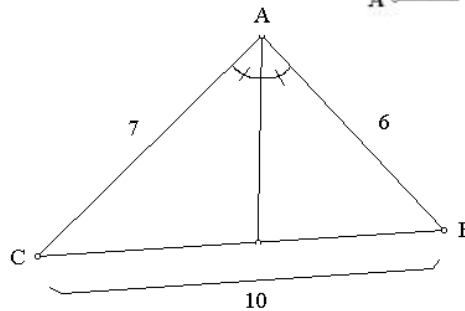
## Problems

Solutions are left as an exercise for the reader. All answers must be simplified and exact answers unless otherwise specified (irrational decimal answers require infinitely many decimal places.)

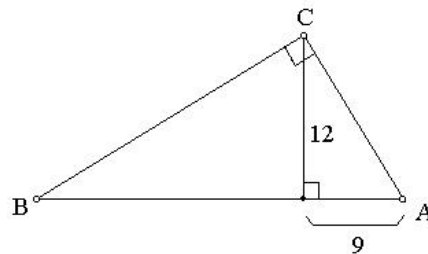
- What is the ratio of the side of a regular octahedron with equal volume and surface area to the side of a tetrahedron with equal volume and surface area?
- What is the sum of the product and the sum of the roots of the equation  $x^2 + 7x - 1$ ?  
What about  $x^3 - 3x^2 + 7x - 25$ ?
- The medians of a triangle with sides 13, 14, and 15 intersect as shown below. What is the area of the shaded region?



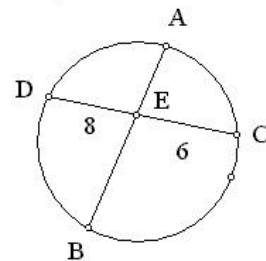
- What is the ratio of the area of  $\triangle ADC$  to the area of  $\triangle ADB$ ?



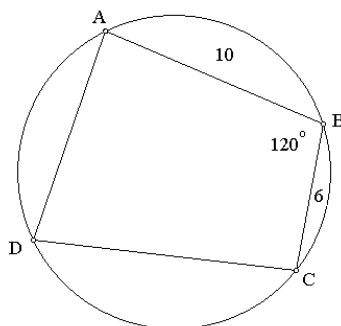
- What is the length of the median to side  $\overline{AB}$ ?



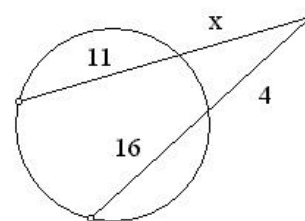
- $\overline{AB}$  and  $\overline{CD}$  are intersecting chords in the circle. The radius of the circle is 10, and the distance from the center to  $\overline{AB}$  is 6. What are the lengths of the segments  $\overline{AE}$  and  $\overline{BE}$ ?



- $\overline{AB} \cong \overline{CD}$ . What is the area of  $ABCD$ ?

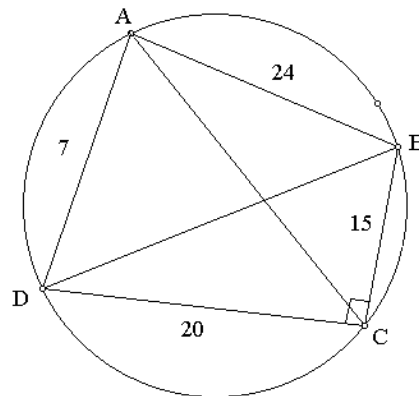


- What is  $x$ ?

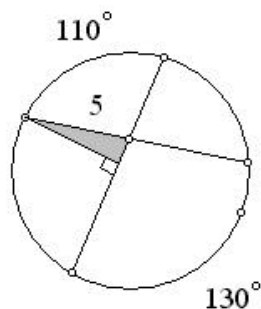


9.  $\bar{x}34\bar{y}73$  is a 6 digit number that is divisible by 7. What is  $y - x$ ?
10. Find an even number that has 7 factors. Is this the only such number?
11. Compute  $\sin\left(3\cos^{-1}\left(\frac{3}{5}\right)\right)$ .
12. A regular polygon has an exterior angle whose measure is equal to  $\frac{1}{8}$  of its interior angle. How many sides does the polygon have?
13. What is the sum of the factors of 1572?
14. There are 4 different types of monitors, 5 different CPU's, and 3 different types of printers that can be purchased. Two of the CPU's are not compatible with one of the monitors. How many different systems can be purchased?
15. A committee of 3 people must be chosen from a group of 10 individuals. One must be appointed the leader and another the secretary. How many different ways can a committee be chosen?
16. How many 5 digit numbers exist whose digits are all in descending order?
17. Out of a group of 100 people, 70 people are taking math, 60 are taking science, and 50 are taking history. 40 are taking both math and science, 25 are taking both math and history, and 35 are taking both science and history. How many are taking all three subjects?
18. What is the probability that if an integer between 1 and 1000 is chosen, that it is divisible by either 2 or 5?

19. What is the area of  $ABCD$ ? What are the lengths of the diagonals?

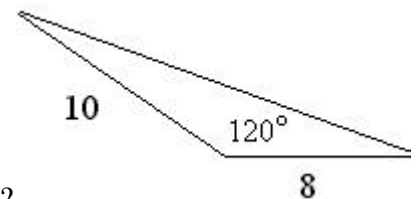


20. What is the area of the shaded region?



21. If  $r$ ,  $s$ , and  $t$  are the roots of the equation  $x^3 - 3x^2 + 8x - 5$ , what is the value of  $r^3 + s^3 + t^3$ ?

22. What is the radius of the circumscribed circle of the triangle?



23. How many negative roots does  $x^{11} - 5x^6 + 4x^3 + 2x^2 - x + 1 = 0$  have?
24. If  $q$  is the angle the line  $3x + 5y = 10$  makes with the x-axis, then what is  $\cos q$ ?
25. What is the distance from the point  $(7, 12)$  to the line  $x = -y$ ?

26. A triangle has vertices at  $(2, 5)$ ,  $(0, 8)$ , and  $(4, 12)$ . What is its area?
27. What is the inverse of the matrix  $\begin{bmatrix} 5 & -2 \\ -7 & 1 \end{bmatrix}$ ?
28. What is the remainder when  $13^8$  is divided by 7?
29. What is the lateral area of a cone with radius 4 and height 3?
30. Find all solutions to the equation  $8x + 28y = 34$ , where  $x$  and  $y$  are integers.
31. What is the sum of the first  $n$  triangular numbers?
32.  $34\overline{a}743164$  is divisible by 11. What is the value of  $a$ ?
33. A biased coin is flipped 10 times. If the probability of getting heads is  $\frac{2}{3}$ , then what is the probability of getting exactly 6 tails?
34. The 2<sup>nd</sup> term of an arithmetic sequence is 2, and the 10<sup>th</sup> term is 26. What is the sum of the first 15 terms of the sequence?
35. What is  $\sin(22.5^\circ)$ ?
36. A sphere is inscribed inside a regular tetrahedron with side length 6. What is its surface area?
37. In the expansion of  $(3a - b + c + d - 2e)^{10}$ , what is the coefficient of the  $a^2b^3cd^3e$  term?
38.  $\cos 2x + \cos 4x$  is equivalent to  $2\cos ax \cos bx$ . What is  $a + b$ ?
39. What is  $\cos^2 18^\circ$ ?
40.  $n$  is the sum of the first 25 digits in the decimal approximation of  $e^p$ . It is also a factor of 703987968917520. A sphere is inscribed inside a regular icosahedron with side length 108. Its radius has a length of  $z\sqrt{a+b\sqrt{c}}$ , where  $z$ ,  $a$ ,  $b$ , and  $c$  are all integers, and the expression is simplified as much as possible. Let  $q$  equal the sum of the first 5 decimal digits in  $\mathbf{p}$ . The 6<sup>th</sup> triangular number is equal to  $k + 1$ . If  $n = z + a + b + c + q + k$ , what is the value of  $n$ ?