

Trigonometric Identities

$$\begin{aligned}
 \sin^2 x + \cos^2 x &= 1 \\
 1 + \tan^2 x &= \sec^2 x \\
 1 + \cot^2 x &= \csc^2 x \\
 \sin x &= \cos(90 - x) = \sin(180 - x) \\
 \cos x &= \sin(90 - x) = -\cos(180 - x) \\
 \tan x &= \cot(90 - x) = -\tan(180 - x)
 \end{aligned}$$

Angle-sum and angle-difference formulas

$$\begin{aligned}
 \sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\
 \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\
 \tan(a \pm b) &= \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \\
 \cot(a \pm b) &= \frac{\cot a \cot b \mp 1}{\cot b \pm \cot a} \\
 \sin(a+b)\sin(a-b) &= \sin^2 a - \sin^2 b = \cos^2 b - \cos^2 a \\
 \cos(a+b)\cos(a-b) &= \cos^2 a - \sin^2 b = \cos^2 b - \sin^2 a
 \end{aligned}$$

Double-angle relations

$$\begin{aligned}
 \sin 2a &= 2 \sin a \cos a = \frac{2 \tan a}{1 + \tan^2 a} \\
 \cos 2a &= \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a = \frac{1 - \tan^2 a}{1 + \tan^2 a} \\
 \tan 2a &= \frac{2 \tan a}{1 - \tan^2 a} & \cot 2a &= \frac{\cot^2 a - 1}{2 \cot a}
 \end{aligned}$$

Multiple-angle relations

$$\begin{aligned}
 \sin 3a &= 3 \sin a - 4 \sin^3 a & \tan 3a &= \frac{3 \tan a - \tan^3 a}{1 - 3 \tan^2 a} \\
 \cos 3a &= 4 \cos^3 a - 3 \cos a & \tan 4a &= \frac{4 \tan a - 4 \tan^3 a}{1 - 6 \tan^2 a + \tan^4 a} \\
 \sin 4a &= 4 \sin a \cos a - 8 \sin^3 a \cos a \\
 \cos 4a &= 8 \cos^4 a - 8 \cos^2 a + 1 \\
 \sin 5a &= 5 \sin a - 20 \sin^3 a + 16 \sin^5 a \\
 \cos 5a &= 16 \cos^5 a - 20 \cos^3 a + 5 \cos a \\
 \sin 6a &= 32 \cos^5 a \sin a - 32 \cos^3 \sin a + 6 \cos a \sin a \\
 \cos 6a &= 32 \cos^6 a - 48 \cos^4 a + 18 \cos^2 a - 1
 \end{aligned}$$

$$\sin na = 2 \sin(n-1)a \cos a - \sin(n-2)a$$

$$\tan na = \frac{\tan(n-1)a + \tan a}{1 - \tan(n-1)a \tan a}$$

$$\cos na = 2 \cos(n-1) \cos a - \cos(n-2)a$$

Function-product relations

$$\sin a \sin b = \frac{1}{2} (\cos(a-b) - \cos(a+b))$$

$$\cos a \cos b = \frac{1}{2} (\cos(a-b) + \cos(a+b))$$

$$\sin a \cos b = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$\cos a \sin b = \frac{1}{2} (\sin(a+b) - \sin(a-b))$$

Function-sum and function-difference relations

$$\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\cos a + \cos b = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\cos a - \cos b = -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cos b}$$

$$\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cos b}$$

Half-angle relations

$$\sin \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{2}}$$

$$\cos \frac{a}{2} = \pm \sqrt{\frac{1+\cos a}{2}}$$

$$\tan \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{1+\cos a}} = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\cot \frac{a}{2} = \pm \sqrt{\frac{1+\cos a}{1-\cos a}} = \frac{1+\cos a}{\sin a} = \frac{\sin a}{1-\cos a}$$

Trig functions of special angles

Angle	\sin	\cos	\tan
0	0	1	0
15	$\frac{\sqrt{2}}{4}(\sqrt{3}-1)$	$\frac{\sqrt{2}}{4}(\sqrt{3}+1)$	$2-\sqrt{3}$
18	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{5}+\sqrt{5}}{2\sqrt{2}}$	$\frac{\sqrt{2}(\sqrt{5}-1)}{2\sqrt{5}+\sqrt{5}}$
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
36	$\frac{\sqrt{5}-\sqrt{5}}{2\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{(\sqrt{5}-1)\sqrt{5}-\sqrt{5}}{2\sqrt{2}}$
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
54	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{5}-\sqrt{5}}{2\sqrt{2}}$	$\frac{(\sqrt{5}+1)\sqrt{2}}{2\sqrt{5}-\sqrt{5}}$
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
72	$\frac{\sqrt{5}+\sqrt{5}}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{(\sqrt{5}+1)\sqrt{5}-\sqrt{5}}{2\sqrt{2}}$
75	$\frac{\sqrt{2}}{4}(\sqrt{3}+1)$	$\frac{\sqrt{2}}{4}(\sqrt{3}-1)$	$2+\sqrt{3}$
90	1	0	...