A Log-Euclidean Statistical Analysis of DTI Brain Deformations

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Summary

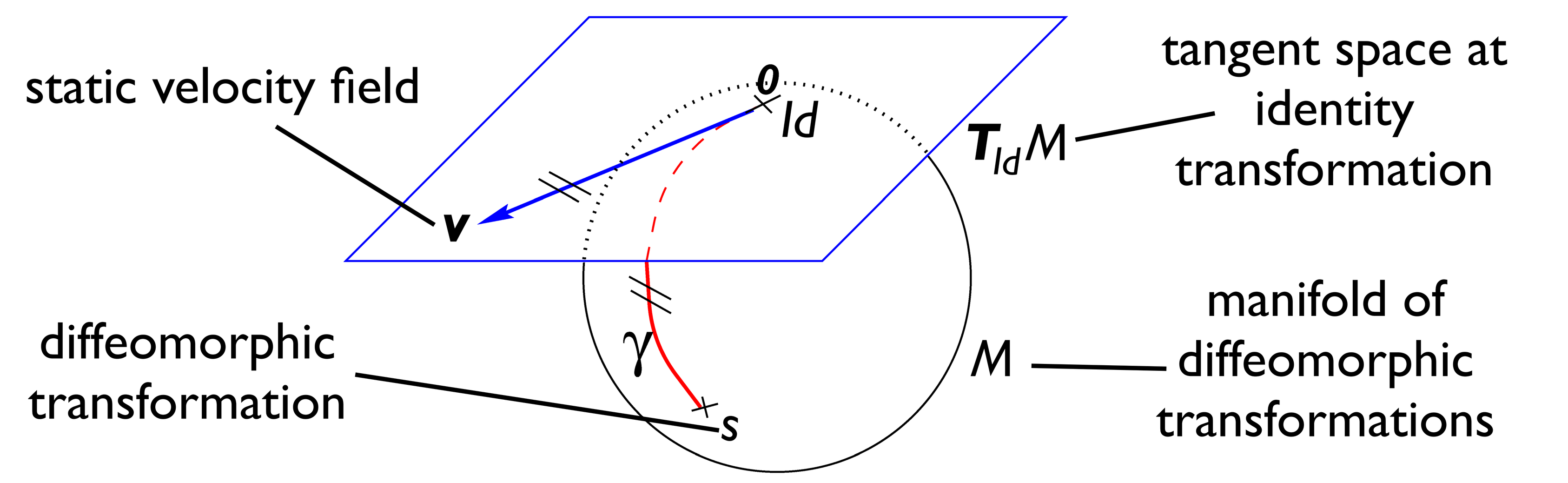
Problems
- Lack of well defined spatial statistics for DTI deformations
- Registration assumes global spatial regularity, w/o covariance

Approach
- Vector space deformation parameterization for proper statistics
- Modify existing method to use statistical regularization criterion

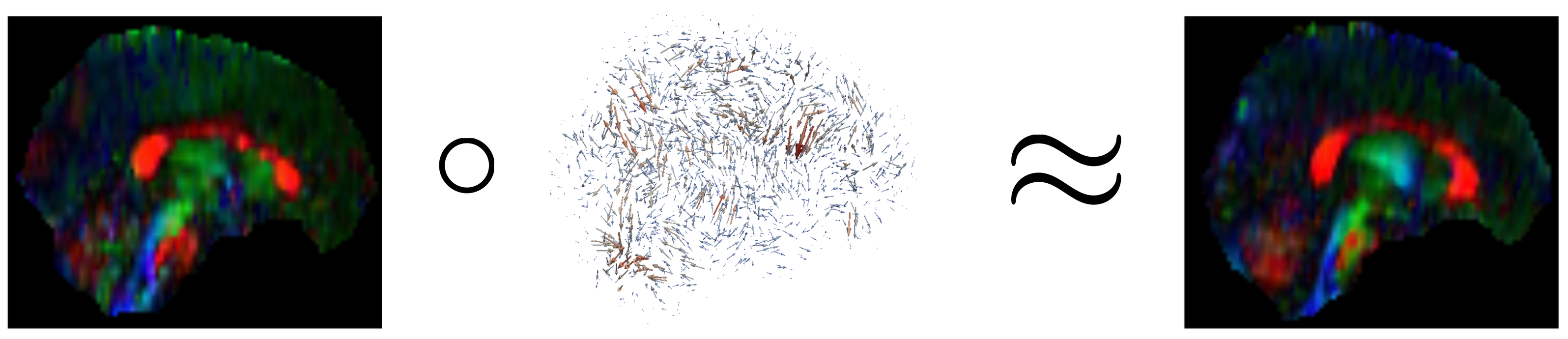
Contributions
- Anatomically meaningful statistics validate existing methods
- Statistical regularization exhibits success in some brain areas

Methods

Log-Euclidean Diffeomorphic Parameterization
- Parameterize diffeomorphism *s* by static velocity field **v** in tangent space of identity transformation (Arsigny, MICCAI 2006)
- Exponential map gives *s*=exp(**v**) and inverse s^{-1}=exp(-**v**)



Symmetric Diffeomorphic Demons DTI Registration
- Find velocity field **v** that parameterizes transformation exp(**v**) from moving DTI *M* to fixed DTI *F* and exp(-**v**) from *F* to *M* (Vercauteren, MICCAI 2008; Yeo, TMI 2009; Sweet, WBIR 2010)
- Represent exp(**v**) as a displacement field



Moving DTI *M* ∘ exp(**v**) ≈ Fixed DTI *F*

- Find **v** by iteratively minimizing unique energy over 2 variables

1. Minimize correspondence energy wrt update **u**, where **w=v+u**

$$E_{\mathrm{corr}} = \sigma_i^{-2}\|F - R^{\mathrm{T}}(M \circ \exp(\mathbf{w})R\|^2 + \sigma_i^{-2}\|M - R(\exp(-\mathbf{w}) \circ F)R^{\mathrm{T}}\|^2 + 2\sigma_d^{-2}\|\mathbf{u}\|^2$$

image errors (forwards and inverse), where R reorients tensor (Alexander, TMI 2001)

controls update size

2. Smooth **w** to minimize regularization energy of new **v**

$$E_{\mathrm{reg}} = \sigma_d^{-2}\|\mathbf{u}\|^2 + \sigma_r^{-2}\boxed{\|\nabla\mathbf{v}\|_K^2}$$

approx harmonic energy

Statistical Regularization Criterion
- Inverse consistency implies $\mu_{\mathbf{v}} = \mathbf{0}$
- Full covariance estimation is ill-posed so use PCA to define rank *k* covariance

$$\Sigma_{\mathbf{v}}^{(k)} = \sum_{i=1}^{k} \lambda_i \mathbf{y}_i \mathbf{y}_i^{\mathrm{T}}$$

- Replace regularization criterion with statistical prior on **v**

$$\mathbf{v}^{(k)} \sim \mathcal{N}[\mathbf{0}, \Sigma_{\mathbf{v}}^{(k)}]$$

- New demons step 2: project **w** onto eigenvectors to give new **v**

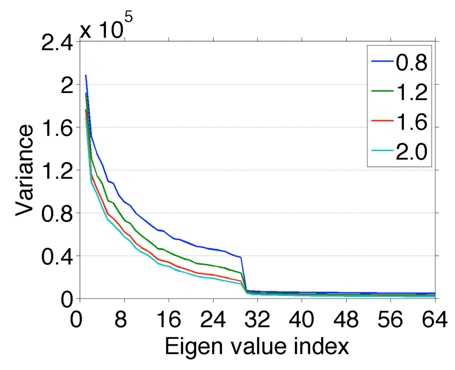
$$\mathbf{v}^{(k)} = \sum_{i=1}^{k} (\mathbf{w}^{\mathrm{T}}\mathbf{y}_i)\mathbf{y}_i$$

Data
- Neuradapt study (M Vassallo, C Lebrun, S Chanalet: CHU Nice)
- 37 HIV/AIDS patients' DWIs: 1×b=0, 23×b=700 s/mm^2
- Log-Gaussian DTI reconstruction with pre-processing
- 30 patients for learning (group A), 7 for validation (group B)

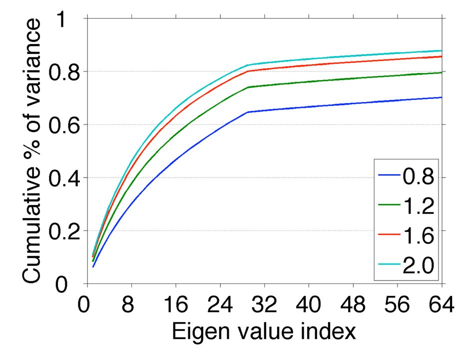
Results

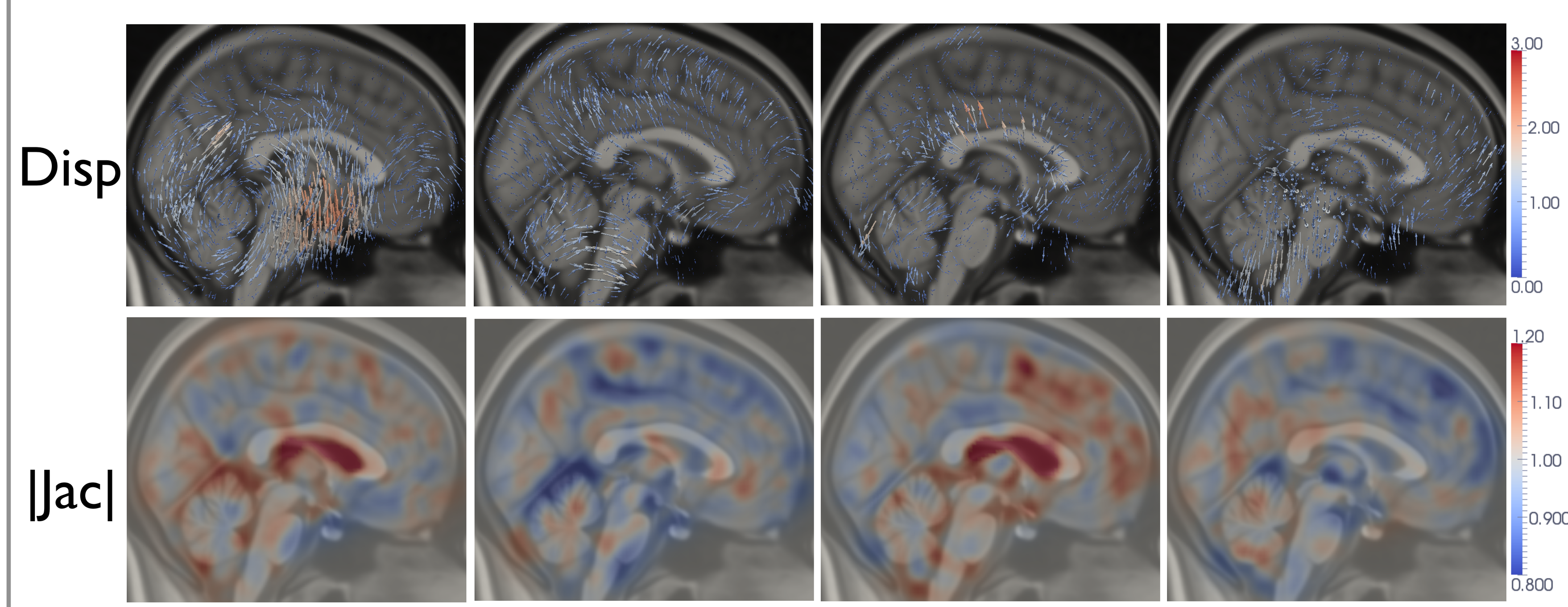
Principal Modes of Deformation
- All registrations in group A with σ_d = 1, σ_r = {0.8,1.2,1.6,2}
- Convergence with σ_r = 1.6 (first 4 modes shown underneath)

Eigenvalue distribution



Cumulative relative eigenvalue distribution

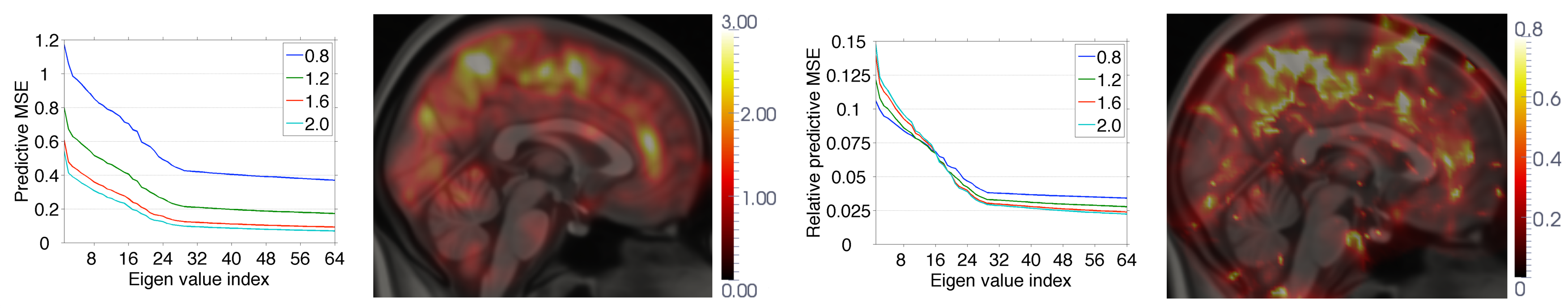




$\exp(\sqrt{\lambda_1}\mathbf{y}_1)$ $\exp(\sqrt{\lambda_2}\mathbf{y}_2)$ $\exp(\sqrt{\lambda_3}\mathbf{y}_3)$ $\exp(\sqrt{\lambda_4}\mathbf{y}_4)$

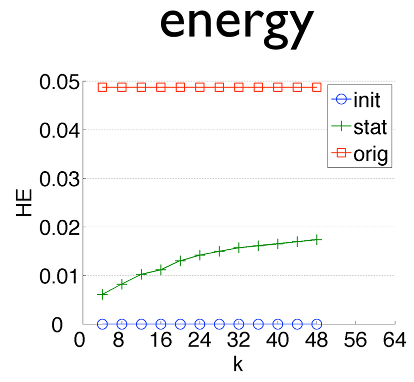
Generalization of Modes
- Predictive error of group A modes on group B velocity fields
- Errors mostly in cortical areas (shown for k = 29, σ_r = 1.6)

Mean absolute vel field error — Mean relative vel field error

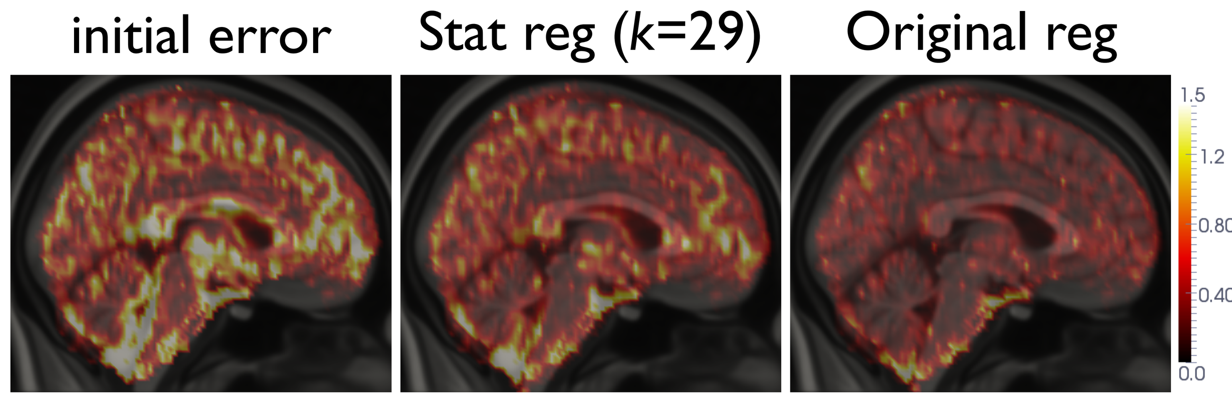


Statistical Regularization
- Group B statistical regularization (with A modes, σ_r=1.6) final errors mostly in cortices
- Original method reduces these errors, but at a cost to harmonic energy

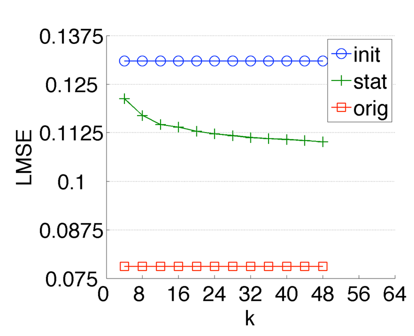
Mean harmonic energy



Mean initial error — Mean final errors: Stat reg (k=29), Original reg



Mean image error



Discussion

Conclusions
- PCA on static velocity fields captures majority of deformation in major deep white matter structures

Further Work
- Use DTI deformation statistics in T1 registration
- Multi-scale PCA could better capture space of deformations
- Log-Euclidean framework allows well defined atlas construction

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