# Joint Example-based Depth Map Super-Resolution

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Abstract—The fast development of time-of-flight (ToF) cameras in recent years enables capture of high frame-rate 3D depth maps of moving objects. However, the resolution of depth map captured by ToF is rather limited, and thus it cannot be directly used to build a high quality 3D model. In order to handle this problem, we propose a novel joint example-based depth map super-resolution method, which converts a low resolution depth map to a high resolution depth map, using a registered high resolution color image as a reference. Different from previous depth map SR methods without training stage, we learn a mapping function from a set of training samples and enhance the resolution of the depth map via sparse coding algorithm. We further use a reconstruction constraint to make object edges sharper. Experimental results show that our method outperforms state-of-the-art methods for depth map super-resolution.

*Keywords*-Depth map; Super-resolution (SR); Registered color image; Sparse representation;

# I. INTRODUCTION

A depth map, representing the relative distance of each object point to the video camera, is widely used in 3D object presentation. Current depth sensors for capturing depth maps can be grouped into two categories: passive sensors and active sensors. Passive sensors, like a stereo vision camera system, is time-consuming and not accurate at textureless or occluded regions. On the contrary, active sensors generate more accurate result, and two most popular active sensors are laser scanners and Time-of-Flight (ToF) sensors. Laser scanners, despite the high-quality depth map they generated, they have limited applications in static environments, as they can only measure a single point at a time. Compared with the these sensors, ToF sensors are much cheaper and can capture a depth map of fast moving objects, which are drawing more and more interest in recent years [1], [2]. However, a depth map captured by a ToF sensor has very low resolution. For example, the resolution of PMD CamCube 3.0 is  $200 \times 200$ resolution and the resolution of MESA SR 4000 is  $176 \times 144$ . In order to improve the quality, a postprocess step is needed to enhance the resolution of the depth map [3], [4], which is called depth map super-resolution (SR) in the literature.

Some previous SR approaches [3] recover a high resolution depth map from multiple depth maps of the same static



Figure 1. Depth map super-resolution. (a) Raw depth map captured by a ToF camera. (b) Corresponding registered color image. (c) Recovered high resolution depth map.

object (taken from slightly displaced viewpoints). For the situation with only one depth map captured from the a single viewpoint, most state-of-the-art methods focus on recovering a high resolution depth map from the low resolution depth map with the help of a registered high resolution color image, as shown in Figure 1. A common approach is to apply a joint bilateral filter with color information to raise the resolution [4]. This approach can obtain a depth map with sharper boundaries. However, since the joint bilateral filter do not have the training stage, it is sensitive to noise in the color image and a recovered depth map often contains some false edges. Other algorithms, such as detecting edges of the registered high resolution color image to direct SR [5] and utilizing color value to calculate weights for SR to achieve sharp boundaries [6], have similar principle and problems as the bilateral filter method.

Recently there is a rapid development of the examplebased 2D image SR. In this approach, the algorithm learns the fine details that correspond to different low resolution image patches from a set of low resolution and high resolution training pairs, then use learned correspondence to predict the details of a low resolution testing image [7]. Sun et al. propose a Bayesian approach to sharpen the boundary of interpolation result by inferring high resolution patches from input low resolution patches based on primal sketch priors [8]. Change et al. propose to learn a mapping function from low resolution patches to high resolution ones using locally linear embedding (LLE) [9]. Glasner et al. propose to directly collect training patch pairs from the single low resolution input image, instead of precollected data set [10]. Yang et al. proposed an example-based 2D image SR approach using sparse signal representation [11]. And Dong et al. extend this works using multiple sets of



bases learned from training data set to adapt to the content variation across different images [12]. However, all these example-based methods use input of a single image only and do not well fit to our application where the input includes a depth map and a registered 2D color image.

In this paper, we propose a novel joint example based SR method to enhance the resolution of a depth map captured by a ToF camera. Unlike traditional example based SR methods, which only utilize a single 2D image as input, our joint example based SR use both a low resolution depth map and a registered color image to recover a high resolution depth map. We design a mapping function from a low resolution depth patch and a color patch to a high resolution depth patch, according to their sparse presentations on three related dictionaries. Furthermore, we use a reconstruction and make object edges sharper. Experimental results demonstrate that our SR method obtains a high resolution depth map with clearer boundaries and fewer false edges than state-of-the-art methods.

## II. DEPTH MAP SR

In our work, depth map SR is to reconstruct a high resolution depth map  $I_h$  from a low resolution depth map  $I_l$  and a high resolution color image  $I_c$ .

First, the reconstructed high resolution depth map  $I_h$  should be consistent with the low resolution depth map  $I_l$  as:

$$I_l = DI_h \tag{1}$$

where D is a downsampling operator.

Furthermore, as the reconstructed high resolution depth map corresponds with the registered color map  $I_c$  in pixel level, there is a correlation between two images  $I_c$  and  $I_h$ .

Based on the analysis above, we model the correlation between  $I_l$ ,  $I_c$  and  $I_h$  as a mapping function  $r(\cdot, \cdot)$  from a low resolution depth patch and its corresponding color patch to a high resolution depth patch as:

$$H_i = r(L_i, C_i) \tag{2}$$

where  $L_i$  is a patch in  $I_l$ ,  $C_i$  is the corresponding color patch in  $I_c$ , and  $H_i$  is the recovered high resolution depth patch in  $I_h$ . In this paper, we define the mapping function  $r(\cdot, \cdot)$ using the sparse representation of patches over a pre-trained dictionary, which will be discuss in section II-B.

#### A. Feature Representation for Patches

To enhance the robustness, we do not use the raw depth map patches and color patches as input for (2). Instead, we extract features from them and use these features to represent the raw patches as shown in Figure 2.

For a low resolution depth patch  $L_i^{raw}$ , we use the first and second derivatives of its bicubic interpolation result to form a feature vector to represent this patch:

$$L_{i} = \left[\frac{\partial}{\partial x}H_{i}^{int}, \frac{\partial}{\partial y}H_{i}^{int}\frac{\partial^{2}}{\partial x^{2}}H_{i}^{int}, \frac{\partial^{2}}{\partial y^{2}}H_{i}^{int}\right]$$
(3)

where  $H_i^{int}$  is the bicubic interpolation result of  $L_i^{raw}$ . According to [11], the first and second derivatives of patches can better reflect the similarity between patches.

For a color patch  $C_i^{raw}$ , we use its edge map as features. Since some edges on the color map are caused by the texture of the object and do not correspond to the edges on the depth map, we need to remove them to enhance the correlation between high resolution depth patches and color patches. Therefore, we first upsample the low resolution depth patch using bicubic interpolation and extract both the edges of the color image and the upsampled depth image. Then the pixel-wise product between these two edge maps is used as the feature to represent a color patch, which can efficiently remove the texture edges of the color image with:

$$C_i = (\nabla C_i^{raw}) \times (\nabla H_i^{int}) \tag{4}$$

where  $\nabla$  is the edge extraction operator,  $H_i^{int}$  is the bicubic interpolation result of  $L_i^{raw}$  and  $C_i^{raw}$  is the color patch.

The feature to represent a high resolution depth patch  $H_i^{raw}$  is:

$$H_i = H_i^{raw} - mean(H_i^{raw}) \tag{5}$$

where  $mean(H_i^{raw})$  is the vector with all its elements equal to the mean depth value of  $H_i^{raw}$ . For the practical SR procedure,  $mean(H_i^{raw})$  is unknown before reconstruction, so we could use  $mean(H_i^{int})$  to represent  $mean(H_i^{raw})$ . As  $H_i^{int}$  and  $H_i^{raw}$  share similar mean depth value, this replacement is reasonable.

#### B. Mapping Function via Sparse Representation

The mapping function  $r(\cdot, \cdot)$  in (2) is defined using the sparse representation over low resolution depth patch and color patch. We first co-train three dictionaries:  $D_h$  consists of high resolution patches;  $D_l$  consists of low resolution patches;  $D_c$  consists of color patches. Notice that patches in

 $D_l$ ,  $D_c$  and  $D_h$  are correspondent, i.e., the *i*th low resolution patch in  $D_l$  corresponds with the *i*th high resolution patch in  $D_h$  and the *i*th color patch in  $D_c$ . The details of training will be introduced in section II-C.

Then for each new input low resolution patch  $L_i$  and its corresponding color patch  $C_i$ , we can find the output high resolution patch  $H_i$  as follows. We first find a sparse representation of these two patches on the dictionaries  $D_l$ and  $D_c$  respectively. Here we enforce  $L_i$  and  $C_i$  have the same sparse coefficients on dictionaries  $D_l$  and  $D_c$ . Then the high resolution patch is recovered using the same coefficients. The mapping function is defined as:

$$H_i = r(L_i, C_i) = D_h \alpha_i^*$$

where: 
$$\alpha_i^* = \underset{||\alpha_i||_0 \le \epsilon}{\operatorname{argmin}} \{\lambda_l ||D_l \alpha_i - L_i||_2^2 + \lambda_c ||D_c \alpha_i - C_i||_2^2 + f(H_i, L_i) \}$$

where  $\alpha_i$  is the coefficient vector consisting of all the coefficients, each of  $D_h$ ,  $D_l$  and  $D_c$  is a matrix with each prototype patch being a column vector,  $\lambda_l$  and  $\lambda_c$  are two balance parameters, and  $|| \cdot ||_0$  denotes the  $l^0$ -norm. f is a constraint function to ensure reconstruction constraint defined in (1). The detailed definition of  $f(H_i, L_i)$  is given in section II-D.

Here we enforce the sparsity constraint  $||\alpha||_0 \le \epsilon$  for two reasons: first, with the sparsity constraint, it is reasonable to reconstruct  $H_i$  using coefficients  $\alpha_i$ , which are the linear coefficients for representing  $L_i$  (or  $C_i$ ) using patches in  $D_l$ (or  $D_c$ ). Second, as discussed in [13], if a high resolution patch  $H_i$  can be represented as a sufficient sparse linear combination of patches in  $D_h$ , it can be perfectly recovered from a low resolution patch.

Same as previous works on sparse representation, we replace  $l^0$ -norm by  $l^1$ -norm in (6) for computational efficiency. As discussed in [13], the  $l^1$ -norm constraint still ensures the sparsity of the coefficients  $\alpha_i$ . Then while ignoring f, (6) becomes

$$\alpha_{i}^{*} = \underset{||\alpha_{i}||_{1} \leq \epsilon}{\operatorname{argmin}} \{\lambda_{l} ||D_{l}\alpha_{i} - L_{i}||_{2}^{2} + \lambda_{c} ||D_{c}\alpha_{i} - C_{i}||_{2}^{2} \}$$
(7)

## C. Dictionary Training

The dictionaries  $D_l$ ,  $D_c$  and  $D_h$  are trained from a set of corresponded low resolution depth patches, high resolution depth patches (ground-truth) and color patches. The training is to minimize the following estimation error:

$$E = \sum_{i} ||\vec{r}(L_i, C_i) - H_i||_2^2$$
(8)

Combining (7) and (8), we have:

$$\min_{D_l, D_c, D_h, \alpha_i} \sum_i \lambda_l ||D_l \alpha_i - L_i||_2^2 + \lambda_c ||D_c \alpha_i - C_i||_2^2 + ||D_h \alpha_i - H_i||_2^2$$
  
subject to:  $||\alpha_i||_1 \le \epsilon \forall i$  (9)

In our work, we use about  $10^5$  patches for training and after training, each dictionary contains only 1024 patches. We use dictionary size which is much smaller than the number of training samples for robustness and efficiency. The above dictionary training formulation (9) is common in sparse representation, and can be solved using an iterative optimization method [14], [11].

## D. Mapping Function with a Reconstruction Constraint

The reconstruction constraint defined in (1) is important for SR. Without this constraint, the downsampling result of the recovered high resolution depth map is not guaranteed to be close to the input low resolution depth map, and a serious artifact will appear when the mapping function fails to get the correct high resolution patch.

Directly combining the reconstruction constraint (1) with the mapping function defined in previous section is not easy. So we first apply an upsampling operator U on both sides of (1), resulting in:

$$UI_l = UDI_h \approx I_h \tag{10}$$

The simplest way for upsampling is the bicubic interpolation. However, there is an obvious blurring effect on the boundaries of the object in the depth map  $H^{int}$  obtained from the interpolation. To remove this effect, we apply the joint-bilateral filter proposed in [4], which can generate clearer object boundaries than  $H^{int}$ :

$$H^{b}(x) = \frac{1}{Z} \sum_{x' \in N(x)} e^{-\frac{||x-x'||_{2}^{2}}{\theta_{s}^{2}} - \frac{||C(x) - C(x')||_{2}^{2}}{\theta_{c}^{2}}} H^{int}(x')$$
(11)

where Z is the normalization factor, N(x) is a neighborhood of x, C(x) is the RGB color vector of the pixel at position x in the registered color image  $I_c$ , and  $H^{int}(x)$  is the depth value of the pixel at position x in  $H^{int}$ . After filtering, pixels with similar colors tend to have similar depth values. Therefore, the filtering result  $H^b$  normally has a sharper boundary compared with the interpolation result  $H^{int}$ .

Then we use  $H^b = UI_l \approx I_h$  as the reconstruction constraint. Let  $H_i^b$  be a patch on  $H^b$ . Then the recovered high resolution depth patch  $H_i$  in  $I_h$  should be as near to  $H_i^b$ as possible. We use the  $l^2$ -norm to model this reconstruction constraint f and add it to the mapping function (7):

 $H_i = D_h \alpha_i^*$ 

where: 
$$\alpha_i^* = \underset{\substack{||\alpha||_1 \le \epsilon}}{\operatorname{argmin}} \{\lambda_l ||D_l \alpha - L_i||_2^2 + \lambda_c ||D_c \alpha - C_i||_2^2 \quad (12)$$
$$+ \lambda_r ||D_h \alpha - H_i^b||_2^2 \}$$

where  $\lambda_r$  is the weight for reconstruction constraint. (12) is a LASSO linear regression problem, and can be efficiently solved by [15]. In the experiment, we simply set all the weighting parameters  $\lambda_l$ ,  $\lambda_c$  and  $\lambda_r$  to 1.

#### E. Proposed Joint Example-based Depth Map SR

Utilizing the final mapping function as (12), the proposed example-based depth map SR algorithm can be summarized in Algorithm 1. To remove the blocking effect, we divide the low resolution depth map into overlapping patches, and obtain the high resolution patch using the mapping function (12). Then we combine these patches to a whole high resolution depth map by averaging the depth values over the overlapping regions.

## Algorithm 1 Proposed Example-based depth map SR

**Input:** A low-resolution depth map  $I_l$ , the corresponding color image  $I_c$  and dictionaries  $D_l$ ,  $D_c$  and  $D_h$ 

- 1) Upsample  $I_l$  to the same resolution as  $I_c$ , and apply bilateral filter (11) to it to get filtered image  $H^b$
- 2) for each patch of  $L_i$  in  $I_l$
- 3) Get the corresponding color patch  $C_i$  and depth patch  $H_i^b$  from  $I_c$  and  $H^b$  respectively
- 4) Get the high resolution depth patch  $H_i$  from  $L_i$ ,  $C_i$ and  $H_i^b$  by solving the optimization problem (12)
- 5) endfor
- 6) Recover the whole high resolution depth map by combining all patches  $H_i$  obtained in step 4

# **III. EXPERIMENTS AND RESULTS**

#### A. Comparison with other approaches

We collect 34 pairs of color images and depth maps from 4 videos of Philips 3DTV. The low resolution depth maps for training are obtained from high resolution depth maps by a Gaussian blurring and downsampling. In the training stage, we only train dictionaries to increase the resolution of a depth map by factor 2. Examples of training patches are shown in Figure 3. For larger magnifying factors, such as 8, we get the high resolution depth map by applying the SR three times. For each training patch triple (low and high resolution depth patches and color patches), we randomly select a  $4 \times 4$  patche from the low resolution depth maps and its corresponding  $8 \times 8$  patches from the high resolution depth maps and color images. We extract 100,000 patch triples from the 34 groups of images to train the dictionaries  $D_l$ ,  $D_h$  and  $D_c$ , each of which contains 1024 patches after training.

To evaluate the performance of our algorithm, we compare it with five other SR methods. Among them, there are three methods use no registered color image information: bicubic interpolation (Bc), bilateral filtering using only depth information (D-Bi)) and sparse representation method proposed by Yang et al. in [11] (Sc-Y). The other two methods are the state-of-the-art methods designed specially for depth map SR, which take into account the registered color image information: the joint-bilateral filtering as defined in [4] (J-Bi) and the algorithm using registered color image to direct calculation of weights for interpolation as in [6] (C-W). From another angle, among the five methods, Sc-Y is an example-based algorithm, just like our method. The same training set, and the dictionary size, patch size, and overlapping size are used in both our algorithm and Sc-Y. The other four methods do not have a training stage. The testing set consists of 13 images randomly selected from 6 videos. Four of these videos are also from Philips 3DTV website as the training set (Girl. Football, Dandelion and Frisbee) and two are from Microsoft Research Asia website (Ballet and Breakdancers).

We first evaluate the performance of these six algorithms



Figure 3. Example of training patches. The first row shows color patches, the second row shows corresponding low resolution depth patches, and third row shows corresponding high resolution depth patches.

 Table I

 ROOT MEAN SQUARE ERROR (RMSE) FOR SIX TESTING VIDEOS

TestVideos	Average RMSE					
	Ours	Sc-Y	J-Bi	C-W	D-Bi	Bc
Girl	6.5	7.03	7.3	9.24	7.35	7.36
Football	4.94	5.27	6.25	7.12	6.37	6.39
Dandelion	5.09	5.49	6.18	5.91	6.23	6.23
Frisbee	8.24	8.84	9.37	10.89	9.46	9.47
Ballet	8.83	9.63	9.98	9.42	10.16	10.25
Breakdancer	5.44	5.68	5.72	6.79	5.74	5.76

with a magnifying factor 8. A quantitative result is given in Table I. It shows that our algorithm achieves the best results on all the testing sequences. RMSE also shows that the example-based algorithms (our algorithm and Sc-Y) outperform those non-training methods. And the methods taking into account the registered color image information (J-Bi and C-W) perform better than the ones using only low resolution depth map (D-Bi and Bc).

To further evaluate these six methods, we compare the visual performance between them. Figure 4 shows examples of one MSRA video (Ballet) and one Philips video (Frisbee). We can see that the depth maps recovered by our algorithm obviously outperforms Bc, D-Bi, J-Bi and Sc-Y as having sharper boundaries and smoother surfaces, which can be easily found in the enlarged parts. Although the depth map obtained by C-W has sharper boundaries than ours, it contains some obvious artifacts (marked by circles) and its RMSE is much higher than ours (see Table1). This is because C-W is not robust and a small noise in the color image may greatly affect upsampled depth map. For instance, the woman's hand in *Ballet* is obviously uncorrect using C-W because the color of the woman's hand is much like the wooden bar behind her, and C-W mainly uses the background depth value to fill the unknown pixels. Another example is that the dandelion contains many errors because the lines are tiny and are easily affected by the surrounding backgrounds.

In conclusion, applying the machine learning method in depth map SR can improve its performance significantly. And by taking into consideration the registered color image information can further improve its accuracy. Also, our algorithm is robust to the errors in the color image and is not obviously affected by them, which is a big problem existing in some state-of-the-art depth map SR methods.



Figure 4. Visual comparison between different algorithms for *Ballet* (left) and *Frisbee* (right). Because depth map has little texture and the quality of the depth map is mainly evaluated by the quality of boundaries of objects, we mainly enlarge some part with many elaborate boundaries to show it clearer.

## B. Different magnifying factors

From the analysis and experiment above, we use Sc-Y to stand for the previous example-based algorithm, and J-Bi to stand for the algorithms using registered color image. Then we further test the performance of four algorithms (our algorithm, Sc-Y, J-Bi and Bc) with different magnifying factors. Figure 5 are the "magnifying factor-RMSE" curves for the sequences *Football* and *Frisbee*. It shows that our algorithm outperforms J-Bi and Bc under all the factors. Our algorithm has similar performance with Sc-Y under magnifying factor 2. However, when the magnifying factor increases, our algorithm performs better than Sc-Y.

Figure 6 also shows the visual comparison between the algorithms under different magnifying factors, 8 and 16. The visual quality of depth maps obtained by Sc-Y and J-Bi are seriously affected as the magnifying factor increases, while the depth maps obtained by our method are still clear. It demonstrates that our algorithm has a good performance

even for a large magnifying factor.

This advantage is significant for practical applications, since the depth map captured by ToF has a very low resolution and has to be reconstructed with a high resolution for 3D modeling or 3DTV representation.

Another point worth mentioning is that the training patches are all taken from four synthesized videos, similar to the *Frisbee* image Figure 4, but the testing set contains both synthesized and real videos, such as the *Ballet* image in Figure 4.

# IV. CONCLUSION

In this paper, we have proposed a joint example-based depth map super-resolution method, using a registered high resolution color image as a reference. Previous example based SR methods only use a single low resolution image as an input and do not well fit to our application where the input includes a depth map and a registered 2D color image. We propose to learn a mapping function from both



Figure 5. Magnifying factor-RMSE curves for Football and Frisbee.



Figure 6. Comparison on *Girl* when the magnifying factor is equal to 8 and 16.

a patch in the low resolution depth map and a patch in the color image to a patch in the high resolution depth map. We also utilize a high resolution depth map reconstructed by the joint-bilateral filter as a reconstruction constraint, which can generate sharp object edges. Our experiments have shown that our algorithm outperforms state-of-the-art algorithms.

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#### REFERENCES

- D. Chan, "Noise vs. feature: Probabilistic denoising of timeof-flight range data," *Technical report, Stanford University*, 2008.
- [2] A. Kolb, E. Barth, R. Koch, and R. Larsen, "Time-of-flight sensors in computer graphics," *Eurographics State of the Art Reports*, pp. 119–134, 2009.
- [3] S. Schuon, C. Theobalt, J. Davis, and S. Thrun, "Lidarboost: Depth superresolution for tof 3d shape scanning," *CVPR*, 2009.
- [4] Q. Yang, R. Yang, J. Davis, and D. Nister, "Spatial-depth super resolution for range images," *CVPR*, 2007.
- [5] E. Ekmekcioglu, M. Mrak, S. Worrall, and A. Kondoz, "Utilisation of edge adaptive upsampling in compression of depth map videos for enhanced free-viewpoint rendering," *ICIP*, pp. 733–736, 2009.
- [6] Y. Li and L. Sun, "A novel upsampling scheme for depth map compression in 3dtv system," *Picture Coding Symposium*, pp. 186–189, 2010.
- [7] W. Freeman, T. Jones, and E. Pasztor, "Example-based superresolution," *IEEE Computer Graphics and Applications*, pp. 56–65, 2002.
- [8] J. Sun, N. Zheng, H. Tao, and H. Shum, "Image hallucination with primal sketch priors," *CVPR*, 2003.
- [9] H. Chang, D. Yeung, and Y. Xiong, "Super-resolution through neighbor embedding," *CVPR*, 2004.
- [10] D. Glasner, S. Bagon, and M. Irani, "Super-resolution from a single image," *ICCV*, pp. 349–356, 2009.
- [11] J. Yang, J. Wright, T. Huang, and Y. Ma, "Image superresolution via sparse representation," *IEEE Trans. Image Processing*, vol. 19, no. 11, pp. 2861–2873, 2010.
- [12] W. Dong, L. Zhang, G. Shi, and X. Wu, "Image deblurring and super-resolution by adaptive sparse domain selection and adaptive regularization," *IEEE Trans. Image Processing*, 2011.
- [13] D. Donoho, "For most large underdetermined systems of linear equations the minimal 11-norm solution is also the sparsest solution," *Communications on Pure and Applied Mathematics*, vol. 59, no. 6, pp. 797–829, 2006.
- [14] H. Lee, A. Battle, R. Raina, and A. Ng, "Efficient sparse coding algorithms," *NIPS*, 2007.
- [15] R. Tibshirani, "Regression shrinkage and selection via the lasso," *Journal of the Royal Statistical Society. Series B*, vol. 58, no. 1, pp. 267–288, 1996.