

Why Graham-Scan Needs to Sort Vertices Before Scanning

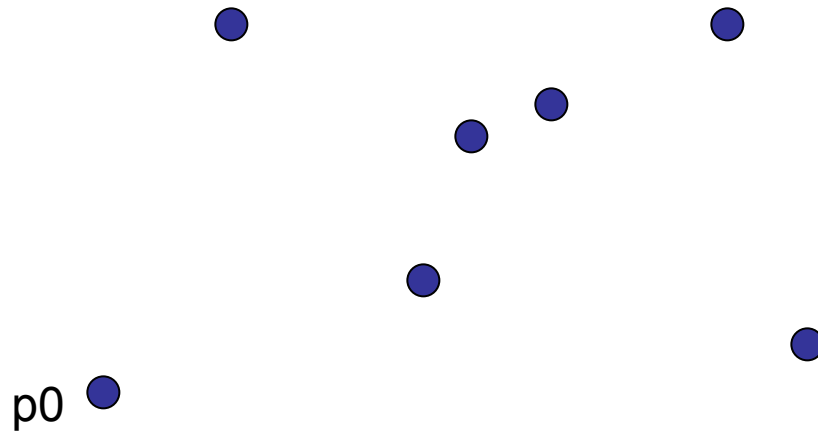
Bill Thies

November 12, 2004

6.046 Recitation Supplement

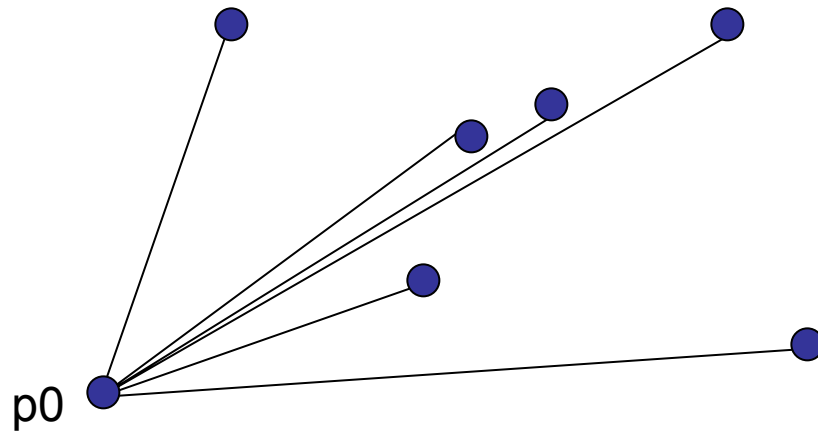
Graham Scan: Sorting Step

- The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, p_0 :



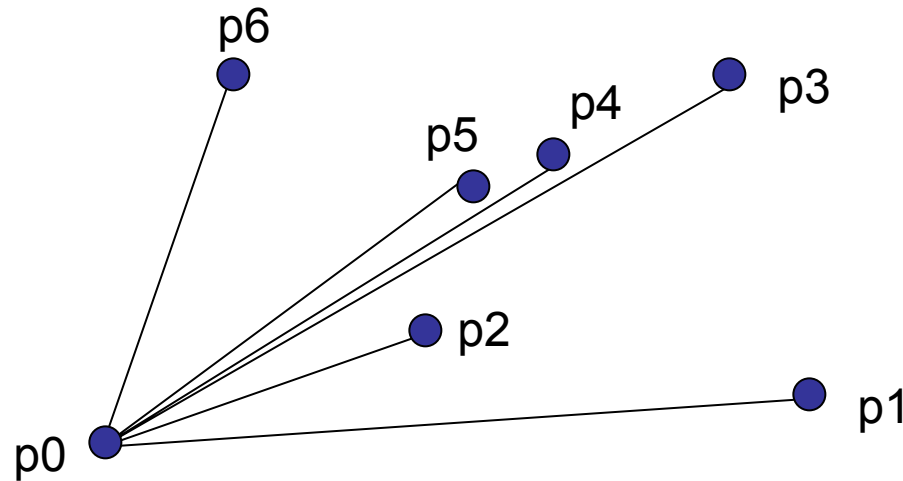
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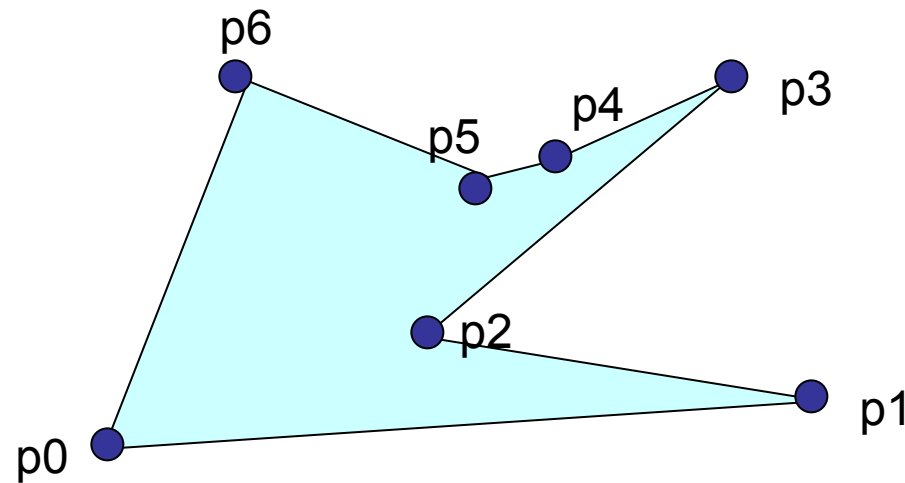
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Graham Scan: Sorting Step

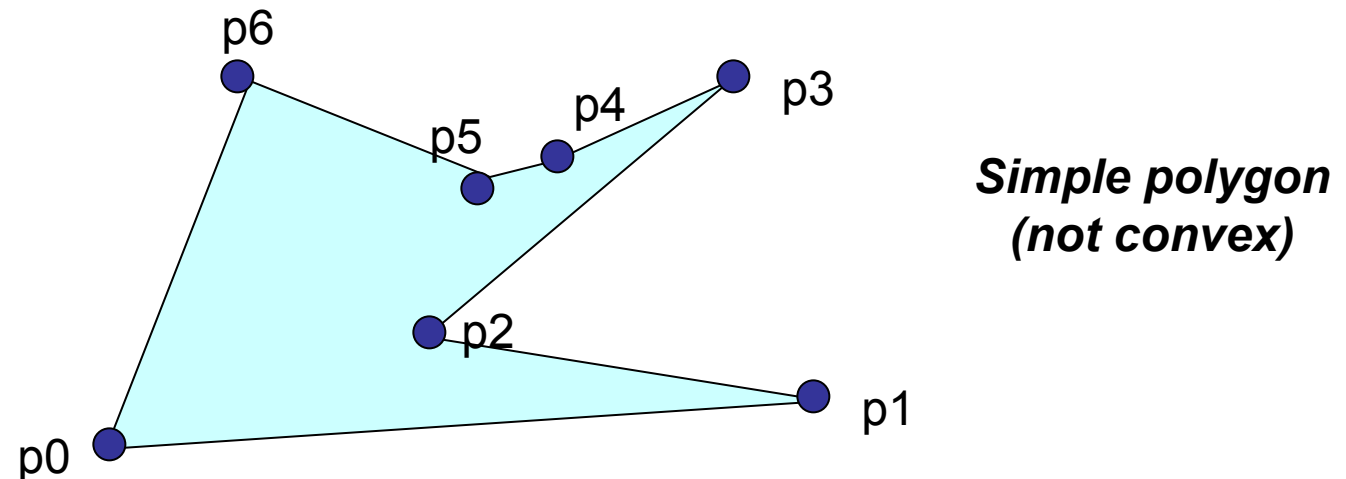
- The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, p_0 :



*Simple polygon
(not convex)*

Graham Scan on Simple Polygons

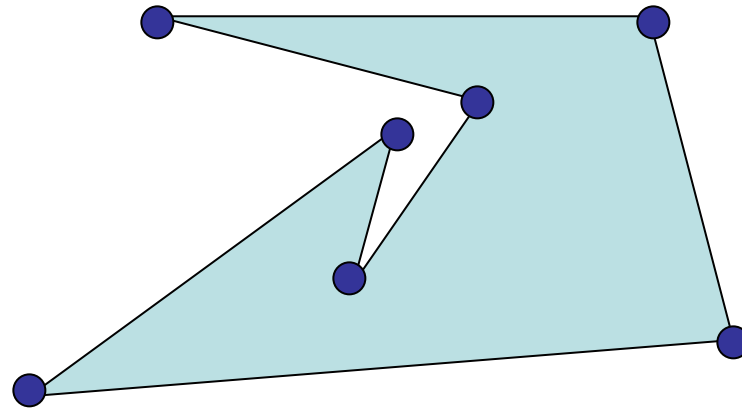
- The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, p_0 :



- In recitation, we asked: would the scanning phase of Graham-Scan work on any **simple polygon**? That is, can you omit this sorting phase if you start from a simple polygon?
 - The answer is **NO**.

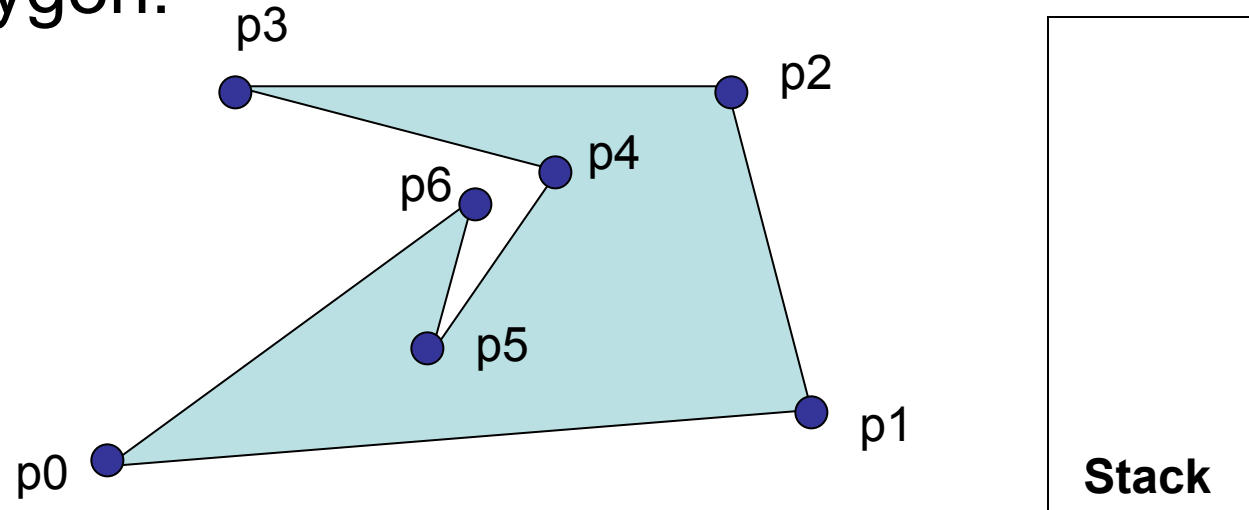
Counter-Example

- Consider the same set of points with the following simple polygon:



Counter-Example

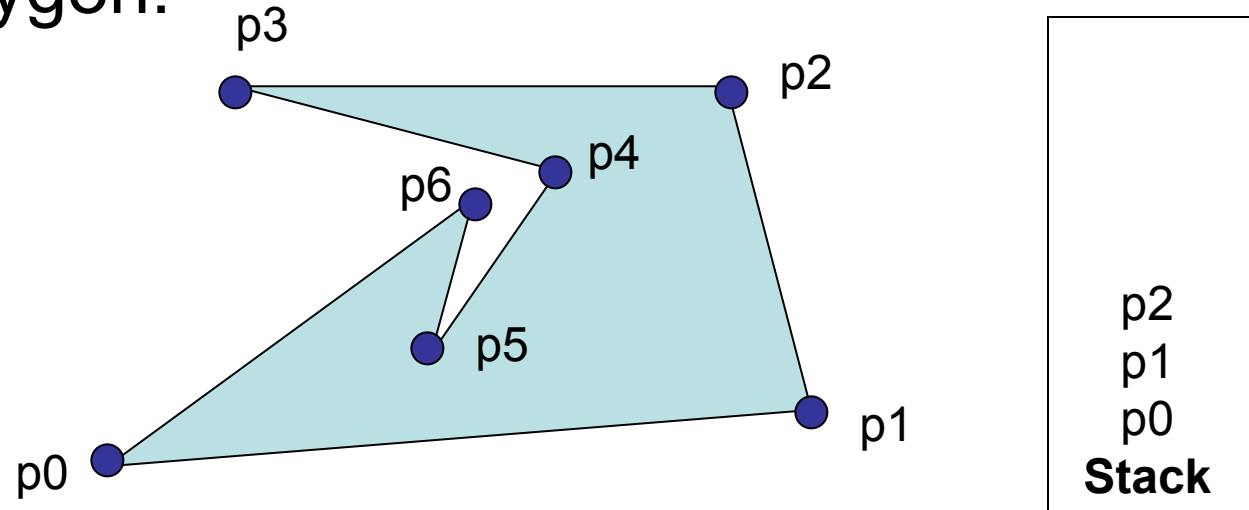
- Consider the same set of points with the following simple polygon:



- Run Graham's Scan starting from p0

Counter-Example

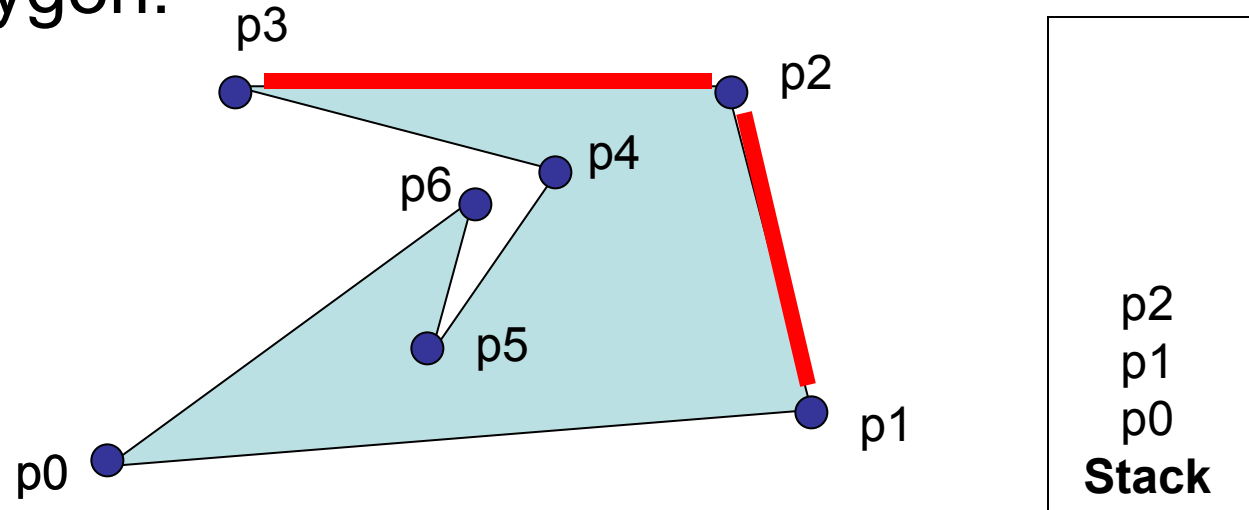
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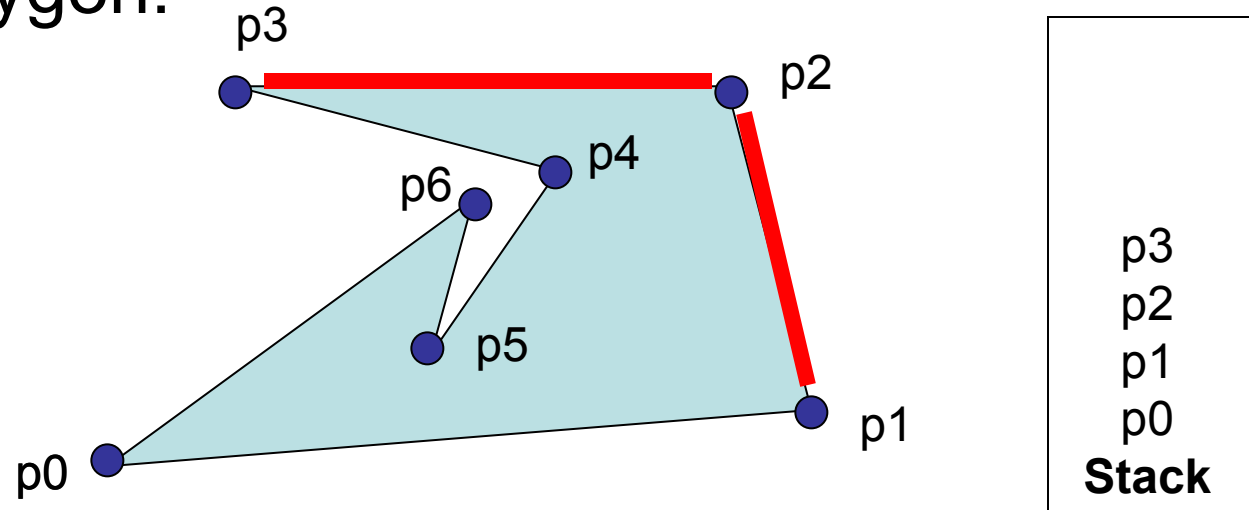
- Consider the same set of points with the following simple polygon:



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Counter-Example

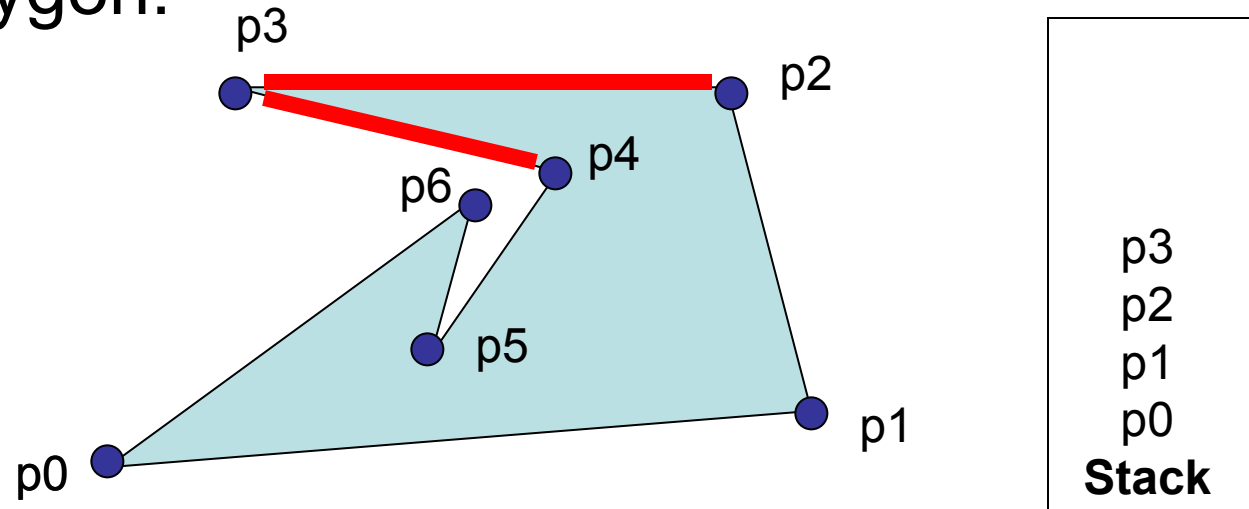
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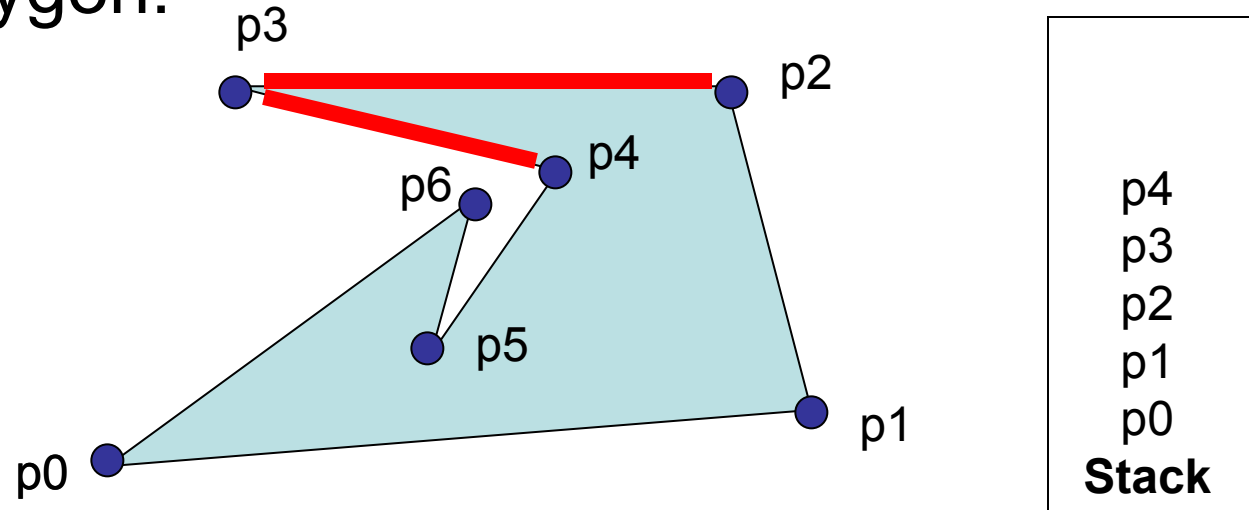
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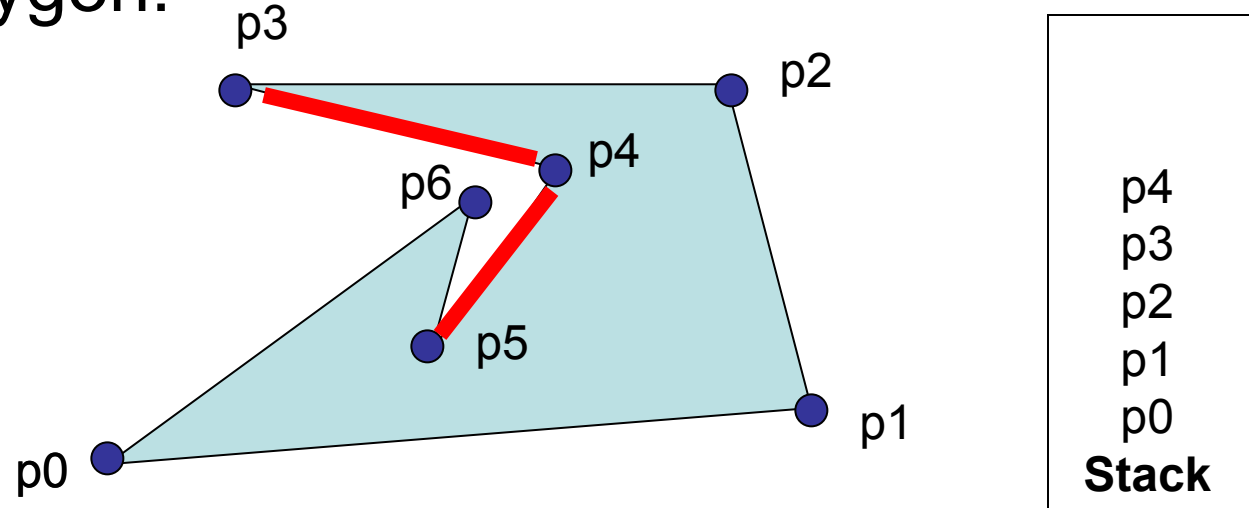
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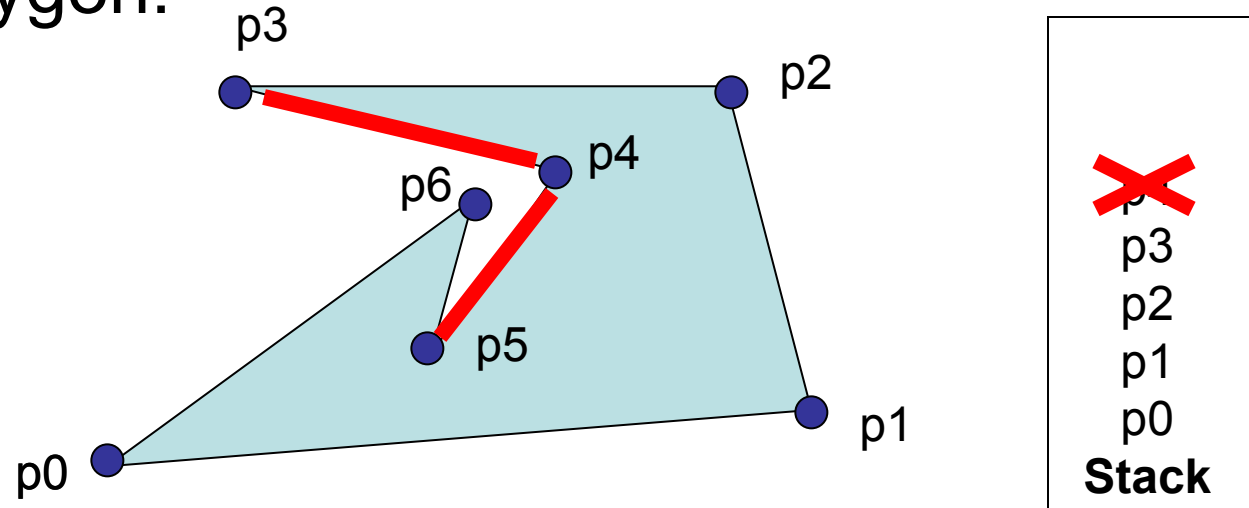
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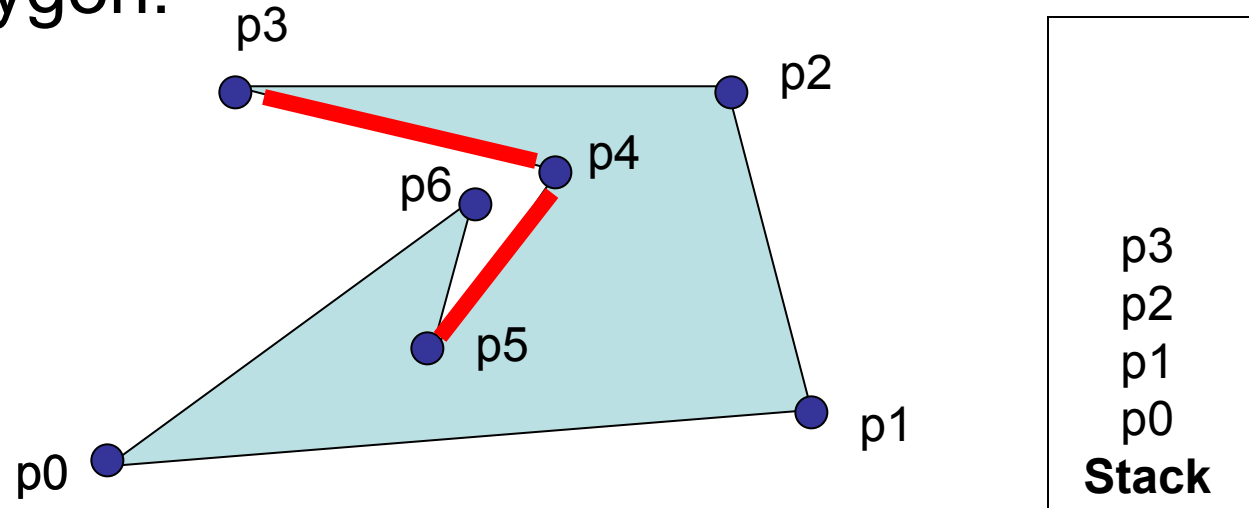
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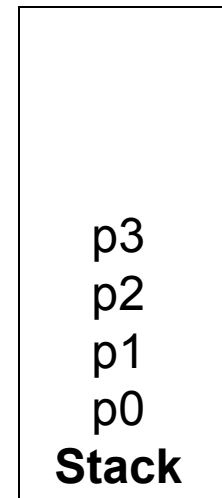
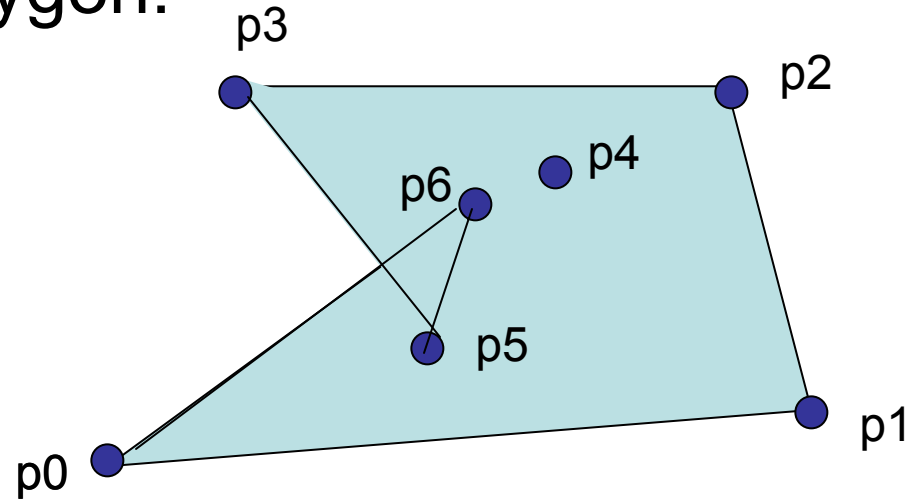
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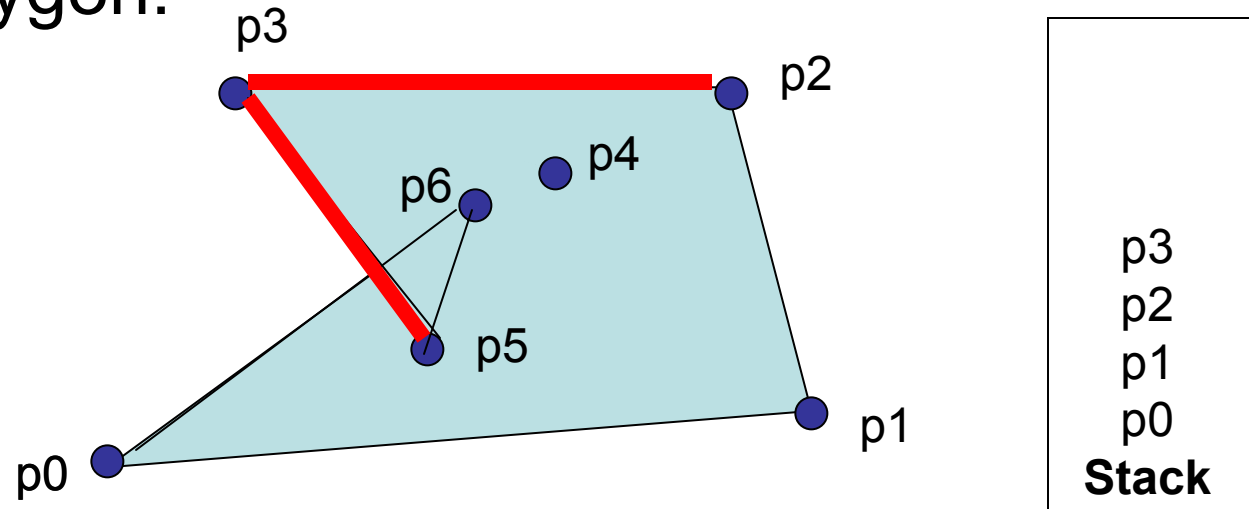
- Consider the same set of points with the following simple polygon:



- Run Graham's Scan starting from p_0

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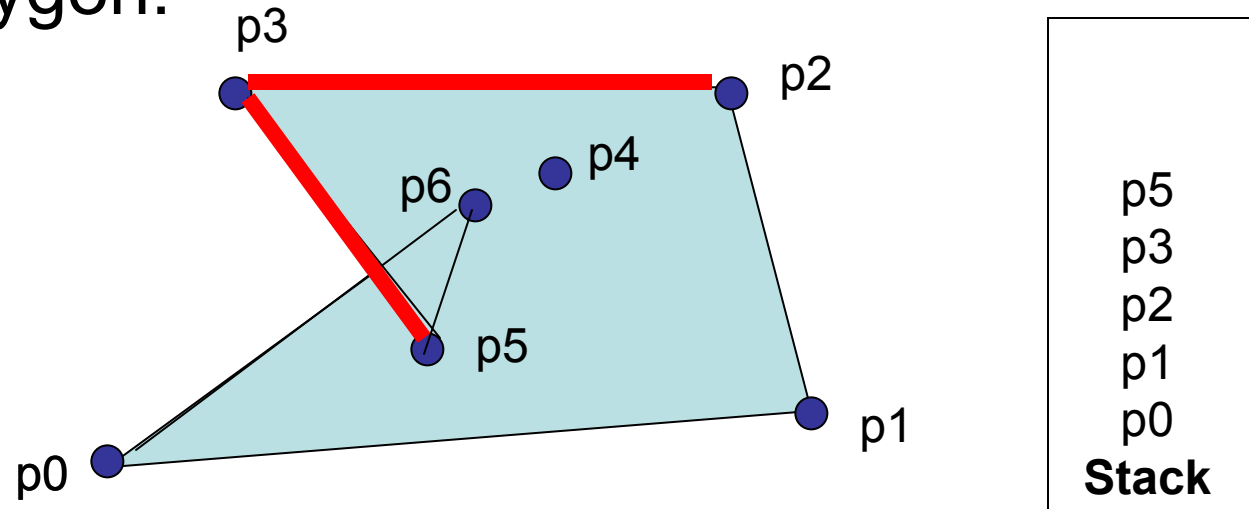
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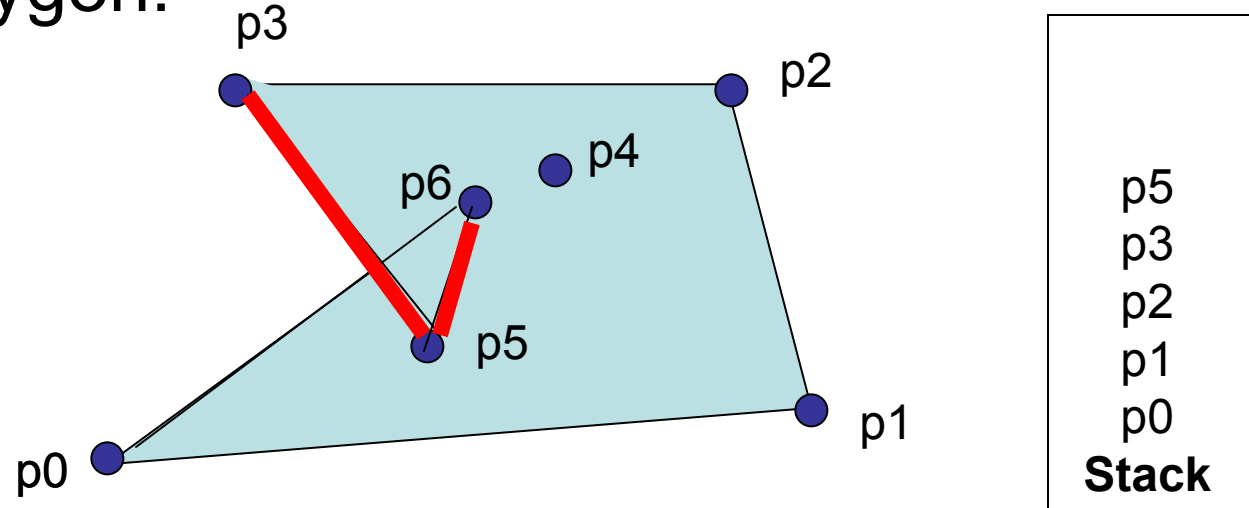
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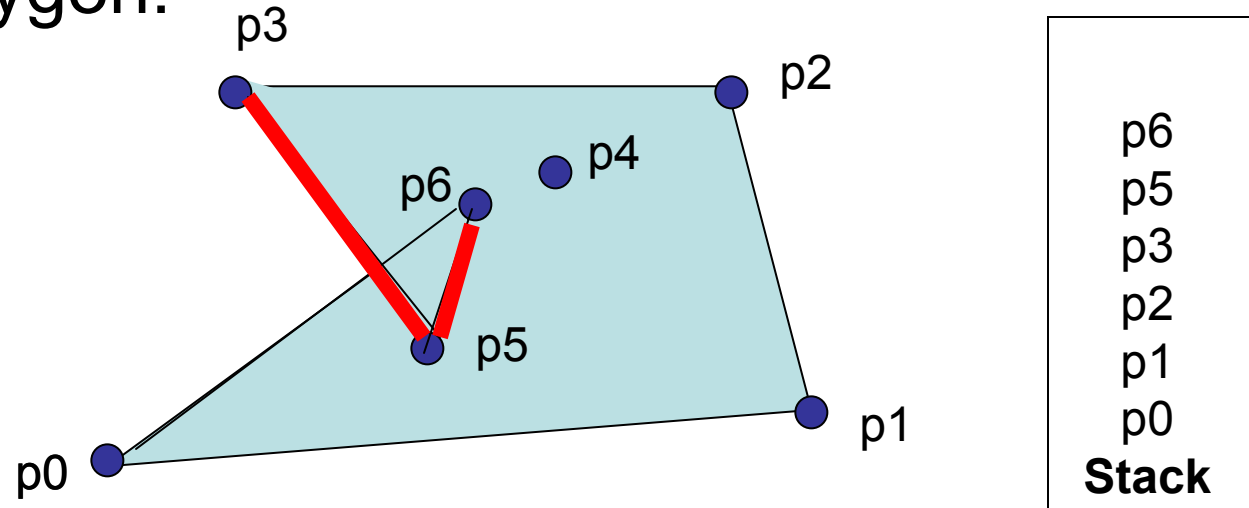
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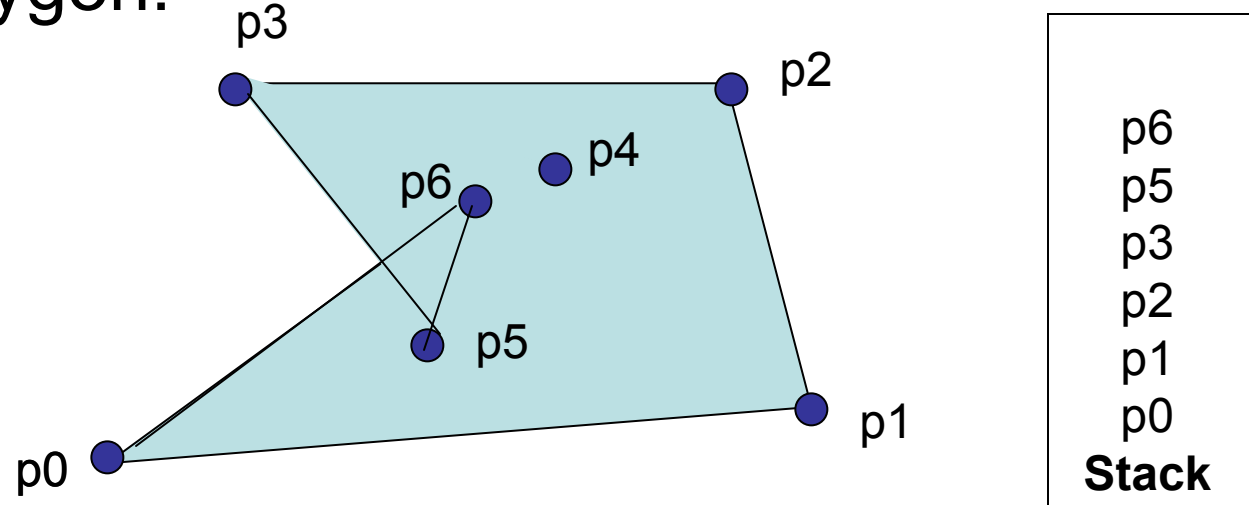
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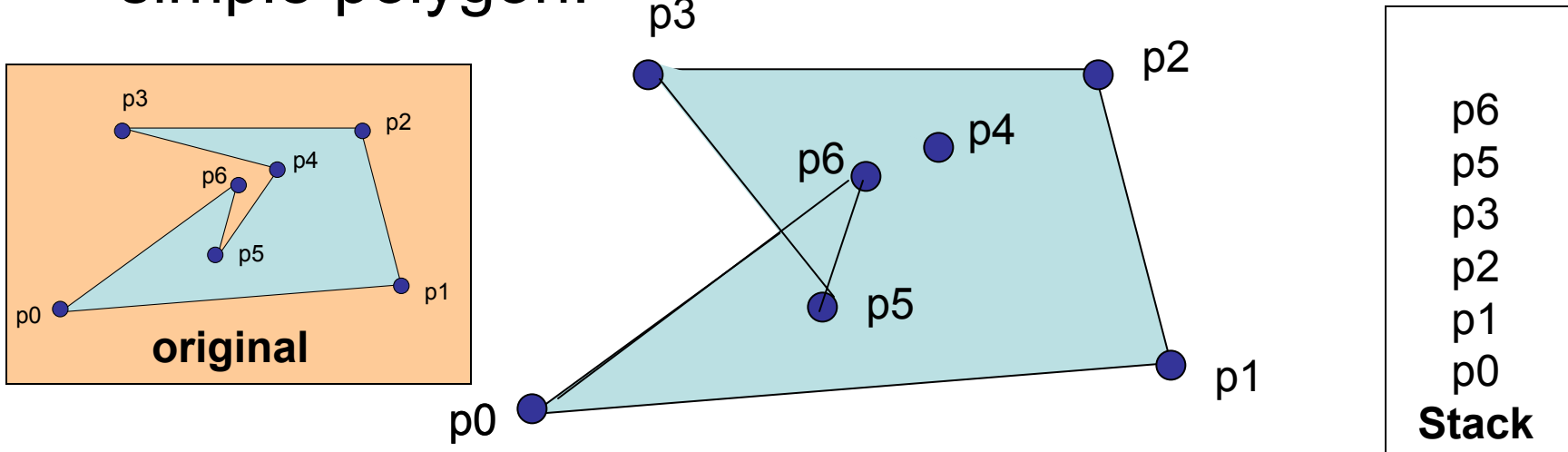
- Consider the same set of points with the following simple polygon:



- Run Graham's Scan starting from p0
- ➔ At end, get convex hull = $\langle p0, p1, p2, p3, p5, p6 \rangle$
But actually, convex hull = $\langle p0, p1, p2, p3 \rangle$

Counter-Example

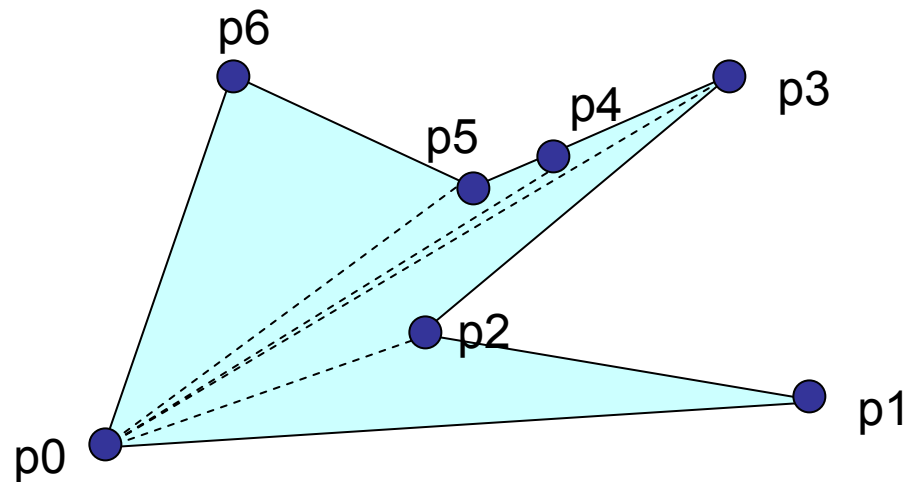
- Consider the same set of points with the following simple polygon:



What went wrong? Vertex p_3 was not “visible” from p_0 , so the line $p_3 \rightarrow p_5$ crossed $p_6 \rightarrow p_0$ when building the convex hull. That is, the polygon became non-simple during course of the algorithm.

Graham-Scan Builds Star-Shaped Polygons

- When vertices are sorted by polar angle from p_0 , all other vertices are visible from p_0 in resulting polygon:



- A polygon with a point visible from each vertex is called ***star-shaped*** (CLRS p. 957, Ex. 33.3-4).
- Graham-Scan works for all star-shaped polygons, but not for all simple ones

For More Information

- There do exist linear-time algorithms for building the convex hull of a simple polygon. Many of the first algorithms proposed were actually incorrect!

See here for an interesting history:

<http://cgm.cs.mcgill.ca/~athens/cs601/intro.html>

- Is the question we considered a million-dollar question? Probably not, but it can be worth up to \$200! Click here for a good time:

<http://answers.google.com/answers/threadview?id=137349>