Why Graham-Scan Needs to Sort Vertices Before Scanning

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6.046 Recitation Supplement
Graham Scan: Sorting Step

- The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, \( p_0 \):
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\[ p_0 \quad p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6 \]
Graham Scan: Sorting Step

- The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, \( p_0 \):

![Diagram of a simple polygon](image-url)

- Simple polygon (not convex)
Graham Scan on Simple Polygons

• The first stage of Graham-Scan sorts the points by their polar angle from the bottom-left vertex, \( p_0 \):

![Diagram of a simple polygon](image)

• In recitation, we asked: would the scanning phase of Graham-Scan work on any *simple polygon*? That is, can you omit this sorting phase if you start from a simple polygon?
  – The answer is **NO**.
Counter-Example

• Consider the same set of points with the following simple polygon:
Counter-Example

- Consider the same set of points with the following simple polygon:

- Run Graham’s Scan starting from p0
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Diagram showing a simple polygon with labeled points and a stack on the right side.
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At end, get convex hull = \(<p0, p1, p2, p3, p5, p6>\)

But actually, convex hull = \(<p0, p1, p2, p3>\)
Counter-Example

- Consider the same set of points with the following simple polygon:

What went wrong? Vertex p3 was not “visible” from p0, so the line p3→p5 crossed p6→p0 when building the convex hull. That is, the polygon became non-simple during course of the algorithm.
Graham-Scan Builds Star-Shaped Polygons

• When vertices are sorted by polar angle from p0, all other vertices are visible from p0 in resulting polygon:

• A polygon with a point visible from each vertex is called star-shaped (CLRS p. 957, Ex. 33.3-4).

• Graham-Scan works for all star-shaped polygons, but not for all simple ones.
For More Information

• There do exist linear-time algorithms for building the convex hull of a simple polygon. Many of the first algorithms proposed were actually incorrect!

See here for an interesting history:

• Is the question we considered a million-dollar question? Probably not, but it can be worth up to $200! Click here for a good time:
http://answers.google.com/answers/threadview?id=137349