A Novel Full-Search Vector Quantization Algorithm Based on the Law of Cosines

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***Abstract*—Vector quantization (VQ) is an essential tool in signal processing. Although many algorithms for vector quantizer design have been developed, the classical generalized Lloyd algorithm (GLA) is still widely used, mainly for its simplicity and relatively good performance. Using law of cosines this letter presents a simple improved method for nearest-neighbor search in GLA. Experiments show that the proposed algorithm outperforms the traditional GLA.**

***Index Terms*—Fast nearest-neighbor search, law of cosines, vector quantization.**

I. Introduction

VECTOR quantization (VQ) is a popular asymmetric technique used for data compression [4], [5], [12]. While VQ coding (compression) can be computationally demanding, decoding (decompression) is a computationally inexpensive table lookup process. The performance of the VQ techniques depends largely on the quality of the codebook used in coding the data. There are several known methods for generating a codebook [4]. Although there are a number of ways of obtaining the vector quantizer codebook, most of them are based on one particular approach, known as the generalized Lloyd algorithm (GLA) [7]. The GLA algorithm is sometimes referred to as the Linde–Buzo–Gray (LBG) algorithm. Vector quantization encoding is the minimum-distortion quantization of an input vector $v = (v_1, v_2, \ldots, v_k)$, using C code vectors $u = (u_1, u_2, \ldots, u_k)$ in the codebook, under some distance measure $d(u, v)$. This means finding the nearest neighbor of v in the codebook, which requires C vector distance computations using the exhaustive search of the codebook. The computational complexity of the nearest-neighbor search is very high when C and k are large. Therefore, the research in developing fast nearest-neighbor searches has been very active. To reduce the computation time needed for an exhaustive search, many fast algorithms have been proposed [1], [8], [10], [11]. However, many of them decrease the coding time at the expense of coding quality. The proposed method has the advantage of being simple and producing the same results as the GLA algorithm. Using Winograd's Identity [3] or partial distance search [2], the search time could be improved even more. Quality of the codebook could be improved with iterative methods that have been proposed for GLA [6], [13].

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This letter is organized as follows. In the introduction we give a short review of the motivation for the full codebook search in VQ. Our improvement for the full codebook search in the GLA algorithm is presented in the second section. Results are given in the third section. The last section concludes our work and results.

II. Proposed Algorithm

By the law of cosines, the Euclidian distance between two vectors is

$$d(u, v) = \|u - v\|^2 = \|u\|^2 + \|v\|^2 - 2\|u\|\,\|v\| \cos\theta \quad (1)$$

where $0^\circ \leq \theta \leq 180^\circ$ is the angle between the two vectors u and v. If the estimate $d^* \leq d$ is larger than the smallest distance found so far ($d_{\min}$), then there is no need to calculate the exact distance between the two vectors. The proposed method for calculating an estimate of the distance of

$$d^* = \|u\|^2 + \|v\|^2 - 2\|u\|\,\|v\| \cos(\theta_1 - \theta_2) \quad (2)$$

is based on the observation that for a fixed vector x and the angles θ_1 between v and x and θ_2 between u and x the angle θ is bounded below $|\theta_1 - \theta_2|$. Therefore $\cos(\theta) \leq \cos(\theta_1 - \theta_2)$ and so $d^* \leq d$. Use the well-known connection between the inner product and the angle between two line segments θ_1

$$v \cdot x = \|v\|\,\|x\| \cos\theta_1 \quad (3)$$

to calculate the cosine θ_1 and from that calculate the sine of θ_1. Similarly calculate angles θ_2 from each code vector u to vector x. Distances near zero have to be handled as special cases, due to limited precision of the floating-point numbers and the possibility for a division by zero. Using cosine identity,

$$\cos(\theta_1 - \theta_2) = \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2) \quad (4)$$

the estimate for distance d^* can be calculated as follows:

$$d^* = d_1 + d_2 - 2\sqrt{d_1 d_2}\,[\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2)]. \quad (5)$$

If the estimate d^* is larger than the current minimum $d_{\min}$ then the exact distance d does not have to be calculated. Since the codeword-searching area is reduced with this inequality, the proposed algorithm requires less computational time than the GLA.

The distances from input vectors to origin v, their squares, sines, and cosines are saved to table in order to save computational time. Similar calculations are performed for code vectors u, whenever the code vectors change. So the additional memory

TABLE I
RESULTS FOR THE AVIRIS IMAGE

Codebook Size	PSNR	GLA [s]	Proposed Method [s]
2048	41.59	394	65
1024	41.41	186	41
512	41.21	96	26
256	41.01	50	19
128	40.80	28	14
64	40.57	18	11
32	40.24	13	9
16	38.72	7	6

requirements are $4(N+C)$, where N is the number of input vectors and C is the size of the codebook.

III. EXPERIMENTAL RESULTS

We performed the experiments on a 500 MHz Pentium III personal computer and the compiler used was GCC 2.95.2 with -O3 optimizations and loop unrolling. The developed C program was based on Khalid Sayood's LBG program [12]. The used multispectral image was an AVIRIS image from the Jasper Ridge [14]. The size of the image was 256 × 256 × 32, i.e., the image had 256 vertical and horizontal pixels and each pixel had 32 spectral components, and all the values were scaled to the range from 0 to 255. Vector quantization was performed in spectral dimension so that input vectors were 32-D. The information loss is measured by the peak-signal-to-noise ratio (PSNR), which we define for multispectral images as

$$\mathrm{PSNR} = 10 \lg \frac{MNs^2}{E^{cr}} \tag{6}$$

where s is the peak value of the image, 255 in our case. E^{cr} is the difference between the energy of the original image and the energy of the compressed image; N is the number of pixels in the image; and M is the number of the bands in the image [9]. Table I shows the computation times for the GLA and the proposed method as a function of the codebook size. Also in the table is the PSNR, which was used to check that both methods give the same results, which they did.

IV. CONCLUSIONS

A new fast search algorithm for vector quantization has been proposed. The algorithm uses 1) the knowledge about distances and angles between input and code vectors and 2) a third constant vector to reduce the codeword-searching area. The proposed algorithm performed better than the GLA in experiments. Moreover, the algorithm is very simple to implement and is effective for large-dimension vectors.

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