On Multiple Moving Objects

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Abstract: This paper explores the motion planning problem for multiple moving objects. The approach taken consists of assigning priorities to the objects, then planning motions one object at a time. For each moving object, the planner constructs a configuration space-time that represents the time-varying constraints imposed on the moving object by the other moving and stationary objects. The planner represents this space-time approximately, using two-dimensional slices. The space-time is then searched for a collision-free path. The paper demonstrates this approach in two domains. One domain consists of translating planar objects; the other domain consists of two-link planar articulated arms.

1. Introduction

A planner solving complex manipulation problems should be able to synthesize motion strategies for multiple moving objects. The need for this capability is evident both in large assembly operations during which it is impractical to move only one part at a time, and in tasks whose solutions involve the cooperation of several robots.

1.1. Examples

We have implemented planners for multiple moving objects in two domains. The first domain consists of translating planar objects, while the second domain consists of two-link planar articulated arms. A detailed discussion of these domains will be given later.

The example of Figure 1 displays the solution determined by our planner for a problem involving four translating polygons. The example of Figure 2 displays the solution determined by our planner for a problem involving three articulated arms.

In both examples, the planner generated a series of collision-free motions taking the objects from their start configurations to their desired goal configurations. The objects generally move simultaneously, although the planner will also consider stopping an object to wait for other objects to pass, if doing so is advantageous.

1.2. Problem Statement

This paper concentrates on the motion planning problem. There are, however, other important issues that a task planner should understand. In particular, the dynamics of object interactions, the effect of uncertainty on object motions, and the design of environments conducive to particular tasks, are problems that deserve attention. These issues are beyond the scope of this paper.

The assumptions of this paper are:

- The environment consists of a set of stationary objects and a set of moving objects, modelled as polyhedra.
- All objects perform rigid motions.
- Object interactions may be specified geometrically.
- Planned motions should be correct to some resolution. Some of our implemented planners have resolution bounds.

See Erdmann and Lozano-Pérez [1986] for a more detailed discussion of the material presented in this paper.

2. Previous Work

We are aware of four lines of previous work on multiple moving objects. The first of these seeks to find an optimal path of a manipulator between a sequence of edges in space [Campbell and Luh 1980]. The positions of the edges may be time-varying. Thus, given a collection of moving objects, this approach may be used to compute the trajectory of an additional moving object.

The second line of approach has focused on the special case of coordinating the motions of several circular bodies in two dimensional regions bounded by collections of polygonal walls. See the work by Schwartz and Sharir [1983], Yap [1984], and Ramanathan and Alagar [1985]. These authors demonstrate various algorithms for solving the coordinated disk motion problem. The time complexities of these algorithms are shown to be polynomial in the number of walls and exponential in the number of disks.

The third line of approach decomposes the multiple moving objects problem into two subproblems [Kant and Zucker 1984]. The first subproblem consists of planning a path for each of the moving objects that avoids collisions with the static objects in the environment. The second subproblem consists of varying the velocities of the moving objects along their specified trajectories so as to avoid collisions. The velocities may be so chosen as to ensure minimum traversal times.

Along the fourth line of approach, Hopcroft, Schwartz, and Sharir [1984] have examined the complexity classification of the coordinated motion problem. In particular, they have shown that the two-dimensional problem of coordinating the motions of an arbitrary number of rectangles in a rectangular region is PSPACE-hard.

3. General Problem Discussion

3.1. Autonomous and Centralized Planning

The problem of planning motions in the presence of multiple moving objects arises in at least two contexts. The first context consists of planning motions for a single autonomous object in the presence of other, possibly moving, objects. The second context consists of centrally planning motions for several moving objects.

3.2. Configuration Space

One approach to planning motions for a single moving object in an environment consisting solely of stationary objects is to transform the problem into that of planning point motions in the object's configuration space. The configuration space [Arnold 1978; Lozano-Pérez 1981, 1983; Schwartz and Sharir 1982; Donald 1984; Canny 1984] of an object is the parameter space representing the degrees of freedom of the object. Obstacles in real space constitute constraints on the object's degrees of freedom. These may be represented as hypersurfaces in the object's configuration space. The planning process consists of determining a path of a point in configuration space that does not violate any of these hypersurfaces.

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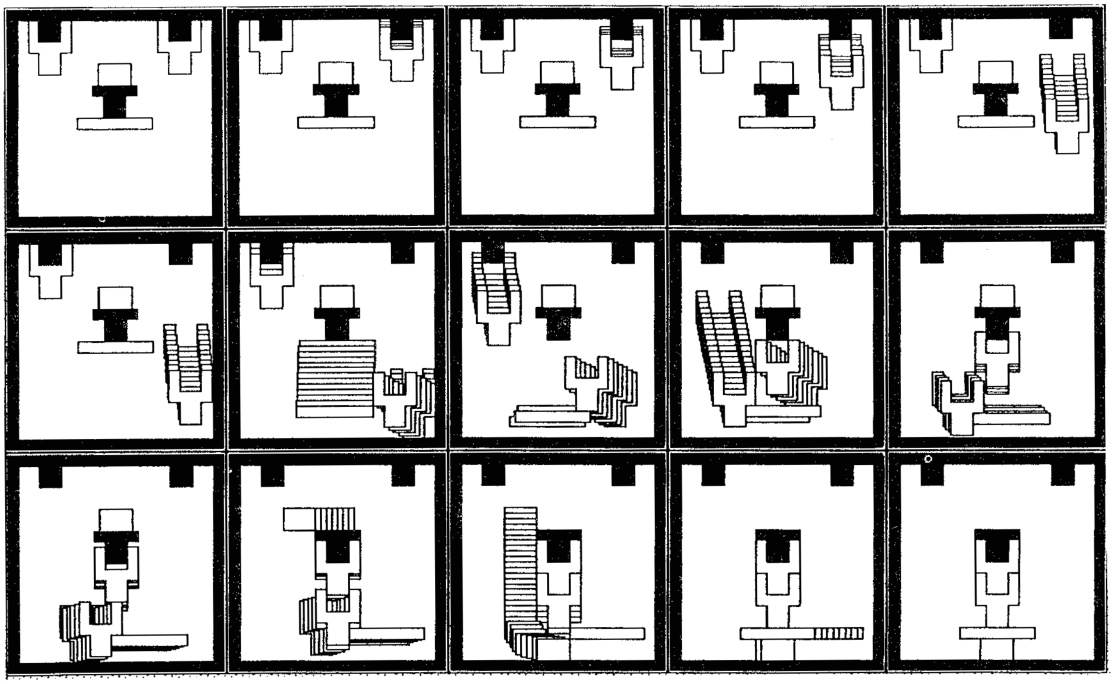

Figure 1. This figure traces a solution determined by our planner for moving four translating objects. The start configurations are shown in the top left frame; the goals, in the bottom right.

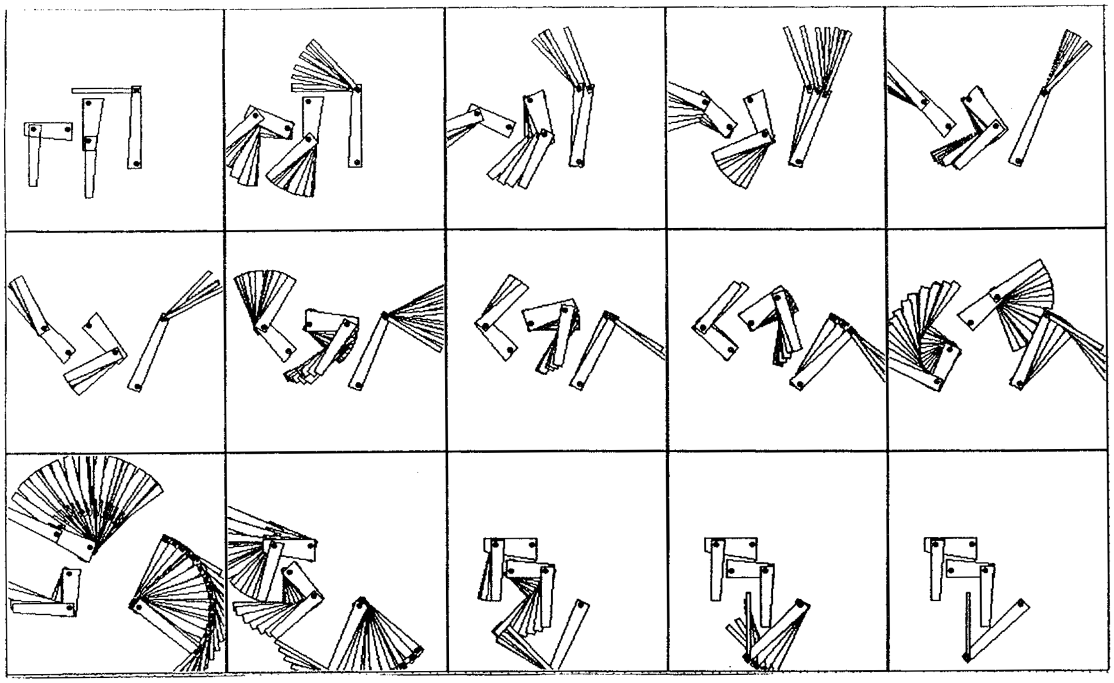

Figure 2. This figure traces a solution determined by our planner for moving three articulated arms. The start configurations are shown in the top left frame; the goals, in the bottom right.

This same approach may taken to plan motions for several moving objects in the presence of stationary obstacles. Specifically, one constructs a configuration space that represents all the degrees of freedom of all the moving objects. Constraint hypersurfaces in this space correspond to configurations at which some moving object is touching another moving or stationary object. The advantage of this approach is that it permits the construction of complete and correct planning solutions. The disadvantage is that the dimension of the configuration space for tasks involving large numbers of objects may be very high.

3.3. Prioritized Planning

An alternative approach is to plan motions for several moving objects by planning motions one object at a time. Thus the centralized planning problem is transformed into a series of autonomous planning problems. The appeal of this decomposition approach is that it reduces the problem from a single planning problem in a very high dimensional space to a sequence of planning problems in low dimensional spaces. The disadvantage of this approach is the loss of completeness. By not considering all moving objects at once, the planner runs the risk of choosing a trajectory for an object early on that prevents finding a solution for an object later in the planning sequence.

Any task in which a prioritization of motions may be assigned, may be approached using the decomposition scheme. Examples include tasks in which robots are cooperating in master/slave relationships and tasks in which the order of part assembly is highly constrained. Notice that a prioritization does not imply that an object of lower priority must follow or assist an object of higher priority. In general, an object of low priority may be performing independent operations. A prioritization simply states that the burden of avoiding collisions between two objects falls on the object of lower priority.

4. Outline of the Approach

We now outline a method for planning motions of several moving objects. It is assumed that the objects have been assigned priorities. Motions are planned one object at a time, according to the assigned priorities. Each object's motion is planned so as to avoid collisions with all stationary objects and all moving objects whose motions have already been determined.

4.1. Incorporating Time

The constraints on a single moving object in an otherwise static environment are readily captured by the configuration space of the object. Now suppose that the environment is no longer static. Notice that it is still possible to construct a configuration space at any fixed point in time. The configuration space at a particular point in time geometrically captures the constraints on the object's degrees of freedom at that time. Considering all points in time, this construction produces a space-time configuration space that reflects the time-varying constraints on the object's possible motions. Planning an object motion entails planning the motion of a point in the space-time that moves forward in time and does not violate any space-time constraints.

4.2. Issues

Configuration space-time correctly describes the problem of planning motions for a single moving object in a time-varying environment. Some issues that arise while solving this problem include:

- How to build the space-time configuration space.
- How much of the space-time configuration space to build.
- How to search the space for a collision-free trajectory.

We have explored these issues, and implemented algorithms, in two different domains. The remainder of this paper is a description of our observations and results.

5. Translating Planar Objects

The first domain that we will explore consists of two-dimensional polygons. The environment is composed of both stationary objects and moving objects. The moving objects are allowed to translate but not to rotate. The objective is to plan a collision-free motion for a moving object in the presence of the stationary objects and those other moving objects whose motions have already been planned.

5.1. Constructing the Configuration Space-Time

Configuration space obstacles are shape-invariant under translations. Given some moving object and some stationary object, suppose we construct the resulting configuration space obstacle. Now suppose that we change the position of the stationary object. Then the shape of the resulting configuration space obstacle remains unchanged. Furthermore, the position of the configuration space obstacle translates exactly as does the real space obstacle. See, for instance, Figure 3.

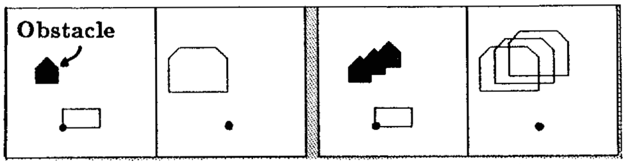

Figure 3. The left two frames show a rectangle and a stationary obstacle, along with the constraints in the rectangle's configuration space that are determined by the obstacle. The right two frames show how the translation of the real space obstacle is reflected in configuration space by a translation of the configuration space obstacle.

This invariance greatly simplifies the computation of the configuration space-time. Specifically, the planner computes a standard configuration space obstacle for each of the stationary and moving objects. The actual constraint imposed by a moving object at a particular time may then be determined simply by performing a polygonal translation of the associated configuration space obstacle.

5.2. Representing the Configuration Space-Time

Let us assume that all translations of moving objects are piecewise linear. This assumption is reasonable in polyhedral environments. Then it is sufficient to represent the configuration space-time as a list of configuration space slices at particular points in time. The times are those at which some moving object changes its velocity. This is because all object motions between such points in time are straight-line motions in space.

Using this representation of configuration space-time, it is easy to decide whether a proposed path collides with any of the other stationary or moving objects. Specifically, the decision amounts to determining whether a moving point collides with a moving polygon. In turn, that computation may be reduced to deciding whether a stationary line segment intersects a stationary polygon (see Figure 4).

5.3. Searching for a Collision-Free Path

Once the configuration space-time has been constructed, a collision-free path may be determined by finding motions that do not intersect any of the constraints represented in the configuration space-time.

The particular algorithm that we implemented considers all path segments between adjacent slices that terminate at vertices of obstacles. Any path segment that pierces a stationary configuration space obstacle or that intersects an implicitly represented

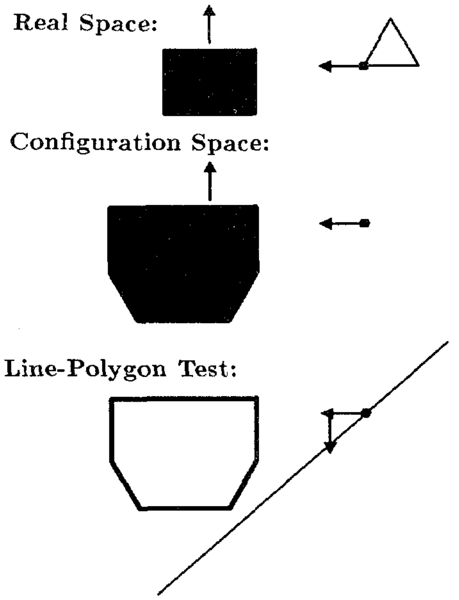

Figure 4. The problem of deciding whether two moving objects collide is first transformed into the problem of deciding whether a moving point collides with a configuration space obstacle. This problem is then transformed into a line-polygon intersection test.

moving obstacle is ignored. This algorithm is a variation of the Vgraph algorithm used in [Lozano-Pérez 1983].

The difficulty with this approach is that the algorithm may not find a path because it generates too few path segments. The fundamental cause of this difficulty lies in the discrete representation of time. There is no natural mechanism for performing motions over time intervals that are shorter than the interval between two adjacent slices. In order to alleviate this problem slightly, the planner does not use solely the slices arising from changes in object motions. Instead, the planner introduces a fixed number of extra configuration space slices between those that already are represented. This is equivalent to ignoring solutions that involve motions below some fixed time resolution.

An alternative approach consists of explicitly searching the free space-time between slices. This approach makes the planner complete.

5.4. Summary for Translating Planar Objects

- The position of a configuration space obstacle at a particular time may be determined from its position at time zero by translation.
- Configuration space-time is represented as a series of configuration space slices at fixed points in time.
- Configuration space-time is searched using a Vgraph algorithm.
- Collisions between proposed trajectories and moving objects are detected using line-polygon intersection tests.
- The planner is complete only to the time resolution between slices, unless the free-space between slices is also searched.

6. Linked Planar Arms with Rotary Joints

The second domain that we will explore consists of two-link articulated planar arms. The links of the arms are modelled as polygons. In addition to the arms, the environment contains stationary obstacles that are also modelled as polygons. The objective is to plan a collision-free path for an arm between specified start and goal configurations in the presence of the stationary obstacles and those other arms whose motions have already been planned.

6.1. Constructing the Configuration Space-Time

For rotating linked arms the basic motions performed are rotations of various polygons about various rotation centers. For convenience, let us assume that only one joint of any arm is allowed to move at a time. It is thus sufficient to concentrate on analyzing the interaction of two polygons, each rotating about its particular rotation center. The constraints resulting from the interaction of two arms may be built up from the constraints of several such pairs of polygons.

6.2. Constraints Arising from Rotating Polygons

This section considers the constraints imposed on one rotating polygon, the *planning object*, by the motion of another rotating polygon, the *obstacle polygon*.

For a particular orientation of the obstacle polygon there are a finite number of orientations of the planning object at which the two polygons touch but do not overlap. As the obstacle polygon rotates about its rotation center, the orientations of the planning object at which these contacts occur change continuously. The basic strategy in constructing the configuration space-time entails tracing these touching orientations as the obstacle polygon rotates. The resulting constraint contours describe the boundaries of the forbidden regions in space-time.

Consider a specific constraint contour, arising from some vertex-edge or edge-vertex contact. As the obstacle polygon rotates, the point of contact between the vertex and the edge moves along the edge. A number of events can occur:

- The direction of travel of the contact along the edge may reverse sign.
- The direction of rotation of the planning object required to maintain contact may reverse sign.
- The contact may disappear, as when the obstacle rotates out of the reach of the planning object.
- The contact may run off one end of the edge, that is, vertex-vertex contact may occur.
- The edge defining the constraint may become aligned with one of the edges incident at the vertex defining the constraint, that is, edge-edge alignment may occur.

The planner analyzes the conditions under which these events occur. Of particular interest are orientations of the two polygons at which contacts appear or disappear, and orientations at which vertex-vertex contacts or edge-edge alignments occur. At these orientations the constraint contours change character, either merging with other contours or splitting into several contours.

6.3. Example

Figure 5 displays two rotating triangles along with their rotation centers. Figure 6 shows the construction of the forbidden regions representing the constraints imposed on the smaller triangle by a rotation of the larger triangle. For simplicity we represented the constraint contours using bounding rectangles. This was not, however, a fundamental restriction, as the constraint contours could be described analytically.

6.4. Representing Multiple Joints

The time-varying constraints imposed on a two-link arm define a three-dimensional configuration space-time. Our planner represents this space-time as a collection of space-time slices. Each space-time slice represents the time-varying constraints imposed on the second link for a fixed orientation of the first link.

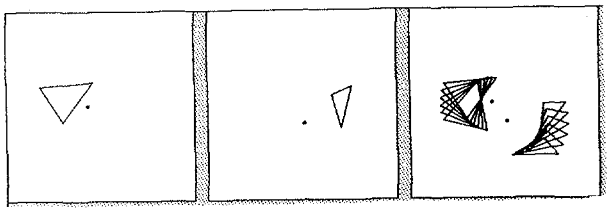

Figure 5. Two rotating triangles, along with their rotation centers.

The spacing between orientations of the first link at which space-time slices are computed limits the resolution to which solutions may be determined by the planner.

As an example, Figure 7 displays the construction of the space-time slice representing the constraints imposed on the second link of the right arm at a fixed orientation of the first link. The constraints were defined by the motion of the left arm and by the stationary obstacles.

6.5. Searching the Configuration Space-Time

The planner represents the free regions of configuration space-time as a collection of rectangles in each of the slices. The planner considers motions within slices as well as across slices, corresponding, respectively, to motions of the second joint alone and motions of the first joint alone. The planner determines a sequence of free-space rectangles that leads from the start to the goal configuration. Once this sequence has been found, the planner selects a particular path passing through the selected free-space regions.

Observe that the search must always move forward in time. Furthermore, any two rectangles that are adjacent in the sequence found by the planner must be spatially and temporally connected in the configuration space-time.

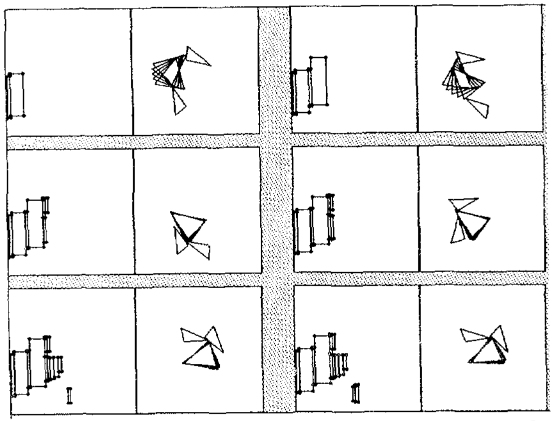

Figure 6. Construction of the constraints imposed on the smaller triangle by the motion of the larger triangle. The constraints are approximated by rectangles. The horizontal axis corresponds to the larger triangle's orientation, while the vertical axis corresponds to the smaller triangle's orientation. In an alternating fashion, the figures display the constraints constructed thus far, and the motion of the larger triangle over the most recently constructed constraint rectangle. The smaller triangle is displayed at the two extreme orientations of this constraint rectangle.

In the case of maximum velocity bounds on the joint velocities, the connectivity of adjacent regions depends on the particular partial sequence of regions being explored during the search. For example, consider Figure 8. Assume that the search is ex-

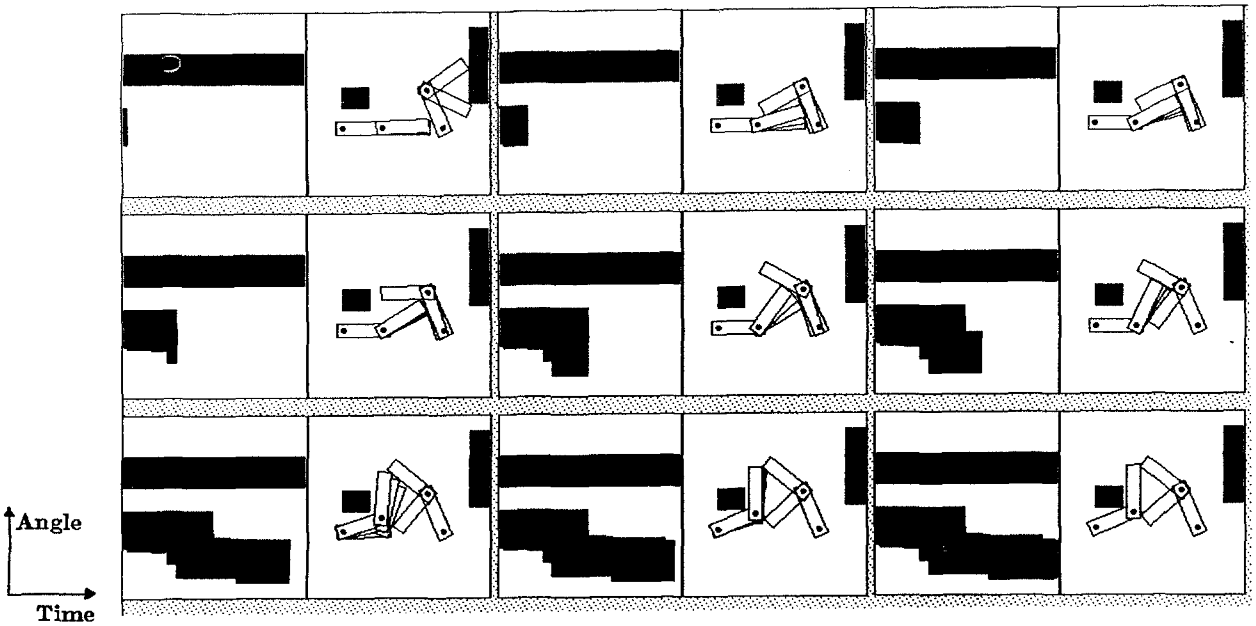

Figure 7. This figure displays the construction of the constraints on Link 2 of the right arm for a fixed orientation of Link 1. The constraints arise from the stationary obstacles and the motions of the left arm.

ploring a sequence leading from region R_0 to region R_2. Let C_{01} and C_{12} be the intersection regions, as shown. Suppose that a particular trajectory can pass from R_0 to R_1 anywhere within C_{01}. Then R_2 is reachable from R_1 if and only if the intersection region C_{12} is reachable from C_{01} along trajectories whose velocities remain within the prescribed bounds.

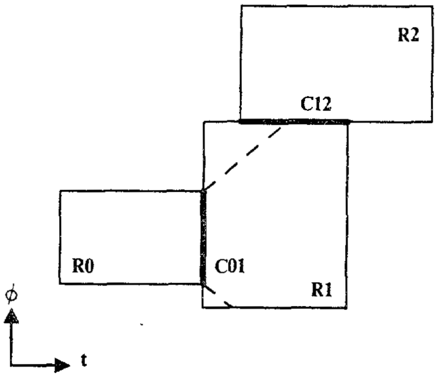

Figure 8. The intersection region C_{12} may be only partially reachable by motions from the previous intersection region C_{01} if joint velocities are bounded from above.

6.6. Summary for Two-Link Articulated Planar Arms

- The constraints imposed on one rotating polygon by another rotating polygon are determined by tracing the orientations required to maintain contact between the two polygons.
- The constraint contours change character at critical orientations. These include vertex-vertex contacts and edge-edge alignments.
- Configuration space-time is represented as a series of space-time slices. Each slice represents the time-varying constraints imposed on Link 2 of the arm at a particular orientation of Link 1.
- Configuration space-time is searched via connecting free-space regions.
- Solutions are exact and complete in Joint 2 motions. The planner is complete in Joint 1 motions only to the angular resolution between slices.

7. Summary

This paper has explored the motion planning problem for multiple moving objects. Two domains of application were considered. The first domain consisted of translating planar objects. The second domain consisted of two-link planar articulated arms. The approach taken consisted of assigning priorities to each of the moving objects. Motions were planned for the objects in sequence as determined by the prioritization.

The problem of planning for a single moving object in the presence of other moving and stationary objects was solved by constructing a configuration space-time. The configuration space-time captured the constraints imposed on the moving object by its time-varying environment. A motion for the object was then found by searching this space-time for a collision-free path from the start to the goal configuration.

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References

Arnold, V. I. 1978. *Mathematical Methods of Classical Mechanics.* New York: Springer-Verlag.

Campbell, C. E., and Luh, J. Y. S. 1980. A Preliminary Study on Path Planning of Collision Avoidance for Mechanical Manipulators. TR-EE 80-48. Lafayette, Indiana: Purdue University School of Electrical Engineering.

Canny, J. F. 1984 (Oct.). Collision Detection for Moving Polyhedra. A.I. Memo 806. Cambridge, Mass.: Massachusetts Institute of Technology Artificial Intelligence Laboratory.

Donald, B. R. 1984. Motion Planning with Six Degrees of Freedom. AI-TR-791. Cambridge, Mass.: Massachusetts Institute of Technology Artificial Intelligence Laboratory.

Erdmann, M., and Lozano-Pérez, T. 1986. On Multiple Moving Objects. Submitted to *Algorithmica.* Also A.I. Memo 883. Cambridge, Mass.: Massachusetts Institute of Technology Artificial Intelligence Laboratory.

Hopcroft, J. E., Schwartz, J. T., and Sharir, M. 1984 On the Complexity of Motion Planning for Multiple Independent Objects; *PSPACE*-Hardness of the "Warehouseman's Problem." *International Journal of Robotics Research.* 3(4):76–88.

Kant, K., and Zucker, S. W. 1984. Trajectory Planning in Time-varying Environments, 1: TPP = PPP + VPP. TR-84-7R. Montréal, Québec, Canada: McGill University, Computer Vision and Robotics Laboratory.

Lozano-Pérez, T. 1981. Automatic Planning of Manipulator Transfer Movements. *IEEE Transactions on Systems, Man, and Cybernetics.* SMC–11(10):681–689. Reprinted in Brady, M., et al., eds. 1982. *Robot Motion.* Cambridge, Mass.: MIT Press.

Lozano-Pérez, T. 1983. Spatial Planning: A Configuration Space Approach. *IEEE Transactions on Computers.* C-32(2):108–120.

Ramanathan, G., and Alagar, V. S. 1985 (March 25–28, St. Louis, Missouri). Algorithmic Motion Planning in Robotics: Coordinated Motion of Several Disks Amidst Polygonal Obstacles. *Proceedings of the 1985 IEEE International Conference on Robotics and Automation,* pp. 514–522.

Schwartz, J. T., and Sharir, M. 1982. On the Piano Movers' Problem: II. General Techniques for Computing Topological Properties of Real Algebraic Manifolds. Technical Report No. 41. New York University Computer Science Department, Courant Institute of Mathematical Sciences.

Schwartz, J. T., and Sharir, M. 1983. On the Piano Movers' Problem: III. Coordinating the Motion of Several Independent Bodies: The Special Case of Circular Bodies Amidst Polygonal Barriers. *International Journal of Robotics Research.* 2(3):46–75.

Yap, C. K. 1984. Coordinating the Motion of Several Discs. Technical Report No. 105. New York University Computer Science Department, Courant Institute of Mathematical Sciences.