### REGRASPING

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Abstract. Regrasping must be performed whenever a robot's grasp of an object is not compatible with the task it must perform. This paper presents an approach to the problem of regrasping for a robot arm equipped with parallel-jaw end-effector. The method employs a table surface to place the object in intermediate positions.

### 1. Introduction

Regrasping must be performed whenever a robot's grasp of an object is not compatible with the task it must perform. Imagine a robotic cell with an arm alternatively picking up parts from a conveyor or a pallet and inserting them. The parts are presented in arbitrary orientations. It can happen that the task cannot be achieved within a single grasp, due to a conjunction of constraints of the two following types, at pick-up and insertion:

- geometric interactions between the object, the manipulator's hand, and the environment,
- the kinematics of the manipulator, in particular, the mechanical limits on the joints.

In order to be complete, we should add the constraint that we must exhibit a collision-free path for the arm from the pickup to the insertion place. Yet we will drop this restriction by assuming that the robotic cell is so arranged that such a path exists for any grasp satisfying the above constraints.

Regrasping operations can be split into two main classes. One includes motions during which the object remains in contact with the hand. These manipulations can only be performed by a dextrous-hand [Fearing 84] as they rely on precise control of forces exerted on the object. For example, a robot would want to reorient its grasp on a pen of an angle of  $180^{\circ}$ . Then, by orienting the hand so that the grasp point is just below the center of mass of the pen, tilting it a little, relaxing the grasp so that the pen rotates slowly, the new grasp can be achieved. The other class of regrasping operations excludes all dynamic motions. It consists of manipulations for which the geometric relationship between the object and the hand is fixed during moves, the only changes being performed by putting the object on a table and regrasping it. We will restrict ourselves to the second class of regrasping problems.

We now define the *Regrasping Problem* informally: Given an initial and a final grasp of an object, construct a sequence of ungrasping and grasping operations connecting them. We assume that between grasp phases the object is placed on a given table, in a stable position. Figure 1 illustrates a one-step regrasping operation of an L-shaped part using a six degree of freedom manipulator. This regrasping operation was produced by the implementation of the algorithm described here within the Handey system [Lozano-Pérez et al 87].



Figure 1. A one-step regrasping operation: (a) the initial grasp, which is constrained by the presence of other objects in the environment, (b) the intermediadte placement on the table surface, (c) the new grasp, and (d) the final assembly operation.

If there were no constraints due to the presence of nearby objects or due to joint angle limits, all regrasping could be done in a single step. In practice, these constraints may require that more than one regrasp be done.

Our approach to regrasping is as follows: We characterize the possible grasps of an object and its stable placements on the table. Both grasps and placements are described by the discrete choice of a surface of contact, combined with the choice of a continuous parameter. The regrasping problem is then solved by computing transitions in a space where we represent all compatible conjunctions of grasps and placements.

A grasp and a placement are compatible if the placement can be reached with this grasp of the object. This has to be checked using the kinematics of the arm. In the sequel, we will assume the manipulator to be a 6 degrees of freedom revolute robot, with the last three rotation axes intersecting at a common point. We will thus take advantage of the decoupling of the last three degrees of freedom for the computation of joint angles.

A great deal of work has been done on the problem of choosing a single stable grasp on an object Paul 72, Lozano-Pérez 76, 81, Taylor 76, Brou 80, Wingham 77, Laugier 81, Hanafusa and Asada 82, Laugier and Pertin-Troccaz 83, Cutkosky 84, Fearing 84, Abel, Holzmann and McCarthy 85, Baker, Fortune and Grosse 85, Holzmann and McCarthy 85, Barber et al. 86, Nguyen 86a, 86b, Jameson and Leifer 86] As far as we are aware, however, the only previous systematic exploration of regrasping, as defined above, was by Paul [72] in the context of the Stanford Hand-Eye system. Paul's method assumed that the regrasping could be done in a single step and did not deal completely with the kinematic constraints. Many of the ideas presented here, however, can be traced to his pioneering work.

### 2. Grasps of an Object

### 2.1. Selecting Stable Grasps

The object to be manipulated is described as a three dimensional polyhedron, not necessarily convex. The hand can grasp the object by closing its two fingers, composed of two parallel planar surfaces. Ignoring friction, a stable grasp of a convex object can be realized with the fingers in contact with:

(i) two parallel faces of the object whose projections normal to the surfaces overlap (meaning their interiors intersect),

(ii) a face and a parallel edge whose projections overlap,

(iii) a face and a vertex whose projection is interior to the face.

In the case of non-convex objects, a first class of stable grasps is again obtained with the above pairs of features. In addition, two faces in a concave corner can give rise to another class of stable grasps. But as concavities constrain the positions of the hand to belong to a discrete set of orientations, these grasps do not constitute relevant tools for the problem of regrasping. Thus the grasps we will deal with are achieved with contacts of type (i), (ii) or (iii).

In all cases, such pairs of features constitute a discrete collection  $G_i, i \in \{1, ..., n\}$ , representing planes in which the fingers will lie during the grasp. We will write  $\mathbf{n}_i$  the unit vector normal to such a plane. It is the normal to a face of the object.

### 2.2. The Parameters of a Grasp

For each class of grasp  $G_i$  we define a standard frame bound to the object, whose x-y axes lie in the plane of the grasp. Let  $(\mathbf{C}, \mathbf{u}, \mathbf{v}, \mathbf{w})$  be the frame in which the object is described, with  $\mathbf{C}$  its center of mass. The frame attached to the grasp class  $G_i$ is written  $(\mathbf{C}_i, \mathbf{u}_i, \mathbf{v}_i, \mathbf{w}_i)$ , with  $\mathbf{w}_i = \mathbf{n}_i$ .

Let  $\mathbf{M}_1$  and  $\mathbf{M}_2$  be arbitrary points of contact between the object and the fingers, lying on opposite surfaces of the grasp. We call

$$e_i = rac{1}{2} (\overline{\mathbf{CM}}_1 + \overline{\mathbf{CM}}_2) \cdot \mathbf{n}_i$$

the eccentricity of grasp  $G_i$ . We impose the condition  $\overline{\mathbf{CC}}_i \cdot \mathbf{n}_i = e_i$  on the translation of the origins of the frames. Thus point  $\mathbf{C}_i$  is equidistant to the faces of contact of the grasp. In the sequel  $(\mathbf{C}_i, \mathbf{u}_i, \mathbf{v}_i)$  will be called the grasp plane.

We assume a standard model of a parallel-jaw hand with two rectangular fingers arranged symmetrically around  $\mathbf{W}$ , the *wrist point* (where the last three joint axes intersect). The coordinate frame attached to the hand is  $(\mathbf{W}, \mathbf{i}, \mathbf{j}, \mathbf{k})$ . This is illustated in figure 2. Vector  $\mathbf{k}$  is normal to the surface of contact of the fingers. Vector  $\mathbf{i}$  is aligned with the fingers and points outwards the hand. During a grasp, the wrist point is constrained to lie in the grasp plane, and  $\mathbf{w}_i$  is aligned with vector  $\mathbf{k}$ .



Figure 2. The frame of the hand.

As illustrated in figure 3, a specific grasp is described by x and y, the coordinates of the wrist in the grasp plane, and by the orientation of the fingers in the grasp plane. This orientation is given by  $\vartheta$ , the angle between vectors  $\mathbf{u}_i$  and  $\mathbf{i}$ .



Figure 3. Description of a grasp.

### 2.3. Fixing the Position of the Grasp Point

 $(x, y, \vartheta)$  are still too many parameters to search over. We would like a single parameter to describe all the configurations of the hand relative to the object for a given class of grasps. This can be achieved by choosing a specific location for the grasp point, a point bound to the hand lying midway between the surfaces of contact on the fingers, and located near the tips of the fingers (see figure 2). We denote it as **G**. We could select the position of **G** relative to the object so that the corresponding range of legal  $\vartheta$  angles, taking into account possible collisions between the hand and the object, is as wide as possible. This is by itself a geometric problem that we will not discuss here.

Note that restricting the position of the grasp point is not a general solution, but a reasonable approximation. This approximations remains reasonable as long as the object we manipulate is not too elongated, in which case changes in the position of the grasp point are significant. Yet, even in that case, we could select a discrete set of positions of the grasp point to represent them all, as is done in Handey [Lozano-Pérez et al 87].

Thus, selecting a class of grasps  $G_i$  will imply the choice of a location for the grasp point **G**. This point constitutes a convenient choice for the origin  $C_i$  of the standard frame of the grasp. We will call point  $C_i$  the grasp point for class  $G_i$  in the sequel. We write  $\overline{\mathbf{CC}}_i = x_i \mathbf{u}_i + y_i \mathbf{v}_i + e_i \mathbf{w}_i$ . The angle  $\vartheta$  then completely specifies the grasp. Namely we have  $x = -\Delta g \cos \vartheta$ and  $y = -\Delta g \sin \vartheta$ , with  $\Delta g$  the distance between the grasp point and the wrist.

We define the Grasp Space to be the space of pairs  $(G_i: \vartheta)$ . Summarizing:

- G<sub>i</sub> is a class of grasps given by a grasp plane of normal n<sub>i</sub>, (it implies the choice of a location for the grasp point),
- θ is the angle of the fingers in the grasp plane, in a frame bound to the object.

### 3. Placements of an Object

A candidate for placements of the object on the table is a face of its convex hull. It is given by an outward normal  $n_j$  and  $d_j$ , the distance between the face and the center of mass. We will call such a face a class of placements if the contact is stable, this is, if the projection of the center of mass in the direction normal to the face lies in its interior. We thus build a discrete collection  $P_j, j \in \{1, ..., m\}$  of classes of placements.

So far, we have not related the position of the object to an absolute frame. We will further restrict a placement to be a position of the object on a table at a fixed place in the workspace. We assume we place the object on a horizontal table. Let  $\mathbf{H}_j$ be the projection of the center of mass of the object on the face of contact  $(\mathbf{n}_j, d_j)$ . We require that the location of  $\mathbf{H}_j$  coincide with a given point  $\mathbf{H}$ , the origin of an absolute frame  $(\mathbf{H}, \mathbf{I}, \mathbf{J}, \mathbf{K})$ such that  $(\mathbf{H}, \mathbf{I}, \mathbf{J})$  represents the surface of the table and  $\mathbf{K}$  is the normal pointing upwards (Figure 4). Then, the position of the object in this absolute frame, for a given class of placements  $P_j$ , is wholly determined by the angle it is rotated around  $\mathbf{K}$ axis,  $\varphi$ .



Figure 4. Description of a Placement.

The point  $\mathbf{H}$  above, although arbitrary, should be selected so that for a wide set of positions of the wrist above it, there exists solutions for the inverse kinematics. That is, the point is in the primary (dextrous) workspace of the robot.

We define the *Placement Space* to be the space of pairs  $(P_i; \varphi)$ . Summarizing:

- P<sub>j</sub> is a class of placements defined by the normal to the face of contact, n<sub>j</sub> (it implies a specific location for the projection of the center of mass of the object on the table),
- φ is the angle of rotation of the object around a vertical axis.

### 4. Constructing the Grasp-Placement Space

## 4.1. Representing Conjunctions of Grasps and Placements

We can now define more formally the regrasping problem. The choice of a grasp  $(G_i; \vartheta)$  binds the object to the hand of the manipulator, and to an absolute frame once we have chosen the joint angles. A placement  $(P_j; \varphi)$  positions the object in an absolute frame on the table. Then a grasping (or equivalently ungrasping) operation is the choice of some conjunction of  $(G_i; \vartheta)$  and  $(P_j; \varphi)$ . Not all such combinations can be realized: we must be able to exhibit at least one set of joint angles for which both of the above bindings to absolute frames give identical results. More, we must take into account constraints of reachability due to

- possible intersection of the object and the hand: this restricts legal choices of (G<sub>i</sub>: θ) parameters.
- 2. possible intersection of the hand and the environment at the place where regrasping is performed: this restricts legal choices of conjunctions of  $(G_i: \vartheta)$  and  $P_j$ .
- mechanical bounds on the joint angles: this restricts legal choices of conjunctions of (G<sub>i</sub>: θ) and (P<sub>j</sub>: φ).

We define the Grasp-Placement Space to be the product of Grasp Space and Placement Space. Its topology is illustrated in figure 5 by what we call the grasp-placement table. The interesting subset of this space is the set of all legal conjunctions of grasps and placements. It is represented by the union of the sets  $\overline{\mathbf{q}_{ij}}$  of compatible  $(\vartheta, \varphi)$  parameters, for a given pair  $(G_i, P_j)$ . Sets  $\overline{\mathbf{q}_{ij}}$  are computed by sequentially pruning them with the three above constraints, as detailed in section 5. Note that for certain pairs  $(G_i, P_j)$  it can happen that  $\overline{\mathbf{q}_{ij}}$  is empty.



Figure 5. The grasp-placement table.

### 4.2. Defining a Sequence of Grasps

We now refine the description of regrasping operations. A regrasping is a transition between two grasps  $(G_{i_1}:\vartheta_1)$  and  $(G_{i_2}:\vartheta_2)$ taking advantage of a placement  $(P_j:\varphi)$  compatible with both grasps. It appears as a vertical jump in the grasp-placement table. The two propositions below give a more formal characterization of those jumps. The second one expresses the duality between grasps and placements. Proposition 1:

A transition between the grasps  $(G_{i_1}: \vartheta_1)$  and  $(G_{i_2}: \vartheta_2)$  is legal if and only if there exists a placement  $(P_j: \varphi)$  such that:

$$(\vartheta_1, arphi) \in \overline{\mathbf{q}_{i_1 j}} \ (\vartheta_2, arphi) \in \overline{\mathbf{q}_{i_2 j}}$$

This simply states that we can, with a grasp of class  $G_{i_2}$ , pick up an object that has been ungrasped with a grasp of class  $G_{i_1}$ . • Proposition 2:

Equivalently, a transition between the placements  $(P_{j_1}; \varphi_1)$  and  $(P_{j_2}; \varphi_2)$  is legal if and only if there exists a grasp  $(G_i; \vartheta)$  such that:

$$(\vartheta, \varphi_1) \in \overline{\mathbf{q}_{ij_1}}$$
  
 $(\vartheta, \varphi_2) \in \overline{\mathbf{q}_{ij_2}}$ 

This transition is indeed a move with an invariant grasp  $(G_i: \vartheta)$  between two placements of the object.

One version of the Regrasping Problem can now be stated as follows: given some initial grasp  $(G^{ini}:\vartheta^{ini})$  and some goal one  $(G^{goal}:\vartheta^{goal})$ , find a sequence of grasps  $(G_{i_0}:\vartheta_0), ..., (G_{i_n}:\vartheta_n)$ ,

such that  $(G_{i_{\alpha}}:\vartheta_0) = (G^{ini}:\vartheta^{ini}), (G_{i_n}:\vartheta_n) = (G^{goal}:\vartheta^{goal}),$ and there exists a legal transition between two consecutive grasps. If there are solutions to the problem, we want the given sequence to be of minimum length.

Such a sequence is illustrated in figure 6. Any pair of consecutive arrows represents a jump between two points of the Grasp-Placement space, either vertical in the Grasp-Placement table if it is between two grasps, or horizontal if it is between two placements. Computation of a regrasping sequence will be described with more details in section 6.

In practice, the initial and final grasp may not be completely specified. For example, the final grasp class  $G^{goal}$  may be specified and a constraint that the grasp angle be in the range  $[\vartheta^{goal}]$ . This more general version of the Regrasping Problem is the one that we have actually implemented.



Figure 6. A sequence of regrasping operations. The sequence can equivalently be read as a series of transitions between grasps through placements, or a series of transitions between placements through grasps.

# 5. Computing the Constraints in the Grasp-Placement Space

We will now concentrate on how to compute the  $\overline{\mathbf{q}_{ij}}$  sets, the sets of legal conjunctions of  $(G_i:\vartheta)$  and  $(P_j:\varphi)$ .

### 5.1. Interaction of the Object and the Hand

For a given choice of a class of grasps  $G_i$  (which implies a choice of the grasp point), the Configuration Space of the hand respective to the object is parameterized by the angle  $\vartheta$ . We can then compute the configuration space obstacles, that is, the set of parameters for which the hand and the object overlap. In particular, [Lozano-Pérez 86] gives an algorithm for computing the forbiden values for the angle of rotation about a given axis, for polyhedral objects. The complement of Configuration Obstacles gives the legal range of  $\vartheta$  angles  $\overline{\theta}_i^1$  such that

$$\overline{\mathbf{q}}_{ij} \subset \overline{\theta}_i^1 imes [0, 2\pi]$$

When computing the configuration obstacles, the obstacles of interest are the object itself and the table. Except, that for the first grasp we must consider any obstacles at the pick-up point and for the last grasp we must consider any obstacles at the destination.

We must also satisfy the constraint that at least one of the contacts between the hand and the object must happen on a surface of a given minimum area. In most cases, this condition will be realized for arbitrary  $\vartheta$  angles if we made an appropriate choice of the grasp point.

### 5.2. Incorporating the Kinematics of the Arm

The last set of constraints we must satisfy refer to a specific choice of a grasp  $(G_i: \vartheta)$  and of a placement  $(G_j: \varphi)$ . It expresses there must be at least a solution for the inverse kinematics of the arm for this conjunction, taking into account mechanical bounds on the joint angles. If we only specify the choice of a class of placements  $P_j$ , the object is free to rotate of an angle  $\varphi$  around a vertical axis. By simultaneously selecting a grasp  $(G_i:\vartheta)$  we constrain the wrist point  $\mathbf{W}$  to be on a circle at a height H above the surface of the table, centered above  $\mathbf{H}$ , and of radius R.

The computation of the sets  $\overline{\mathbf{q}_{ij}}$  is then achieved as follows. First we discretize the range of angles  $\vartheta$  legal for the constraints of intersection of the hand with the object and the table,  $\overline{\theta}_i^1 \cap \overline{\theta}_{ij}^2$ . We again sample the range of  $\varphi$  angles and for each choice of  $\vartheta$  and  $\varphi$  we test the inverse kinematics of the arm. We make use of the decoupling of the last three joints of the robot to first check that there is a solution for joint angles  $\theta_1, \theta_2, \theta_3$  within their range, given the position of the wrist  $\mathbf{W}$ . Then we use the inverse kinematics of the hand to check that we can realize the grasp within mechanical bounds of joint angles  $\theta_4, \theta_5, \theta_6$ . We take advantage of the smoothness of the inverse kinematics to compute the solution in the form of a collection of union of intervals of angle  $\varphi$  for each of the sample values of  $\vartheta$ .

### 6. More About The Search of a Sequence of Grasps

The search for a sequence of grasps proceeds by backward chaining from the goal grasp. The goal grasp is specified by  $(G_{i_g}: [\vartheta_g])$ . Note that we are allowing a *range* of legal grasp angles at the destination. Each step in the backwards chaining process consists of finding another grasp class  $G_{i_k}$  such that there exists placement  $(P_j: \varphi)$  and a non-null range of angles

$$[\vartheta_k] = \{ \vartheta' | \exists \varphi \exists \vartheta \in [\vartheta_g](\vartheta, \varphi) \in \overline{\mathbf{q}_{i_k j}} ext{ and } (\vartheta', \varphi) \in \overline{\mathbf{q}_{i_k j}} \}$$

If such a range exists, it means that  $P_j$  can be used as an intermediate placement between some grasp in the range  $(G_{i_g}: [\vartheta_g])$  and any grasp in the range  $(G_{i_k}: [\vartheta_k])$ . The search terminates when the initial grasp class can be reached and the range of associated angles includes the specified initial angle. The actual implementation of a backwards chaining step proceeds as illustrated in Figure 7. For each placement  $(P_j; \varphi)$ in turn, we compute the range  $[\varphi_j]$  as the union of the  $\varphi$ crossections of  $\overline{\mathbf{q}_{i_gj}}$  for  $\vartheta \in [\vartheta_g]$ . Then we identify the range  $[\vartheta_k]$  as the union of the  $\vartheta$  crossections of  $\overline{\mathbf{q}_{i_kj}}$  for  $\varphi \in [\varphi_j]$ .



Figure 7. Computing transitions between grasps.

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