Sequence to Better Sequence: Continuous Revision of Combinatorial Structures

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Abstract

We present a model that, after learning on observations of (sequence, outcome) pairs, can be efficiently used to revise a new sequence in order to improve its associated outcome. Our framework requires neither example improvements, nor additional evaluation of outcomes for proposed revisions. To avoid combinatorial-search over sequence elements, we specify a generative model with continuous latent factors, which is learned via joint approximate inference using a recurrent variational autoencoder (VAE) and an outcome-predicting neural network module. Under this model, gradient methods can be used to efficiently optimize the continuous latent factors with respect to inferred outcomes. By appropriately constraining this optimization and using the VAE decoder to generate a revised sequence, we ensure the revision is fundamentally similar to the original sequence, is associated with better outcomes, and looks natural. These desiderata are proven to hold with high probability under our approach, which is empirically demonstrated for revising natural language sentences.

Introduction

The success of recurrent neural network (RNN) models in complex tasks like machine translation and audio synthesis has inspired immense interest in learning from sequence data (Eck & Schmidhuber, 2002; Graves, 2013; Sutskever et al., 2014; Karpathy, 2015). Comprised of elements $s_t \in \mathcal{S}$, which are typically symbols from a discrete vocabulary, a sequence $x = (s_1, \dots, s_T) \in \mathcal{X}$ has length T which can vary between different instances. Sentences are a popular example of such data, where each s_j is a word from the language. In many domains, only a tiny fraction of $\mathcal{X}$ (the set of possible sequences over a given vocabulary) represents sequences likely to be found in nature (ie. those which appear realistic). For example: a random sequence of words will almost never form a coherent sentence that reads naturally, and a random amino-acid sequence is highly unlikely to specify a biologically active protein.

In this work, we consider applications where each sequence x is associated with a corresponding outcome $y \in \mathbb{R}$. For example: a news article title or Twitter post can be associated with the number of shares it subsequently received online, or the amino-acid sequence of a synthetic protein can be associated with its clinical efficacy. We operate under the standard supervised learning setting, assuming availability of a dataset $\mathcal{D}_n = \{(x_i, y_i)\}_{i=1}^n \overset{iid}{\sim} p_{XY}$ of sequence-outcome pairs. The marginal distribution p_X is assumed as a generative model of the natural sequences, and may be concentrated in a small subspace of $\mathcal{X}$. Throughout this paper, p denotes both density and distribution functions depending on the referenced variable.

After fitting models to $\mathcal{D}_n$, we are presented a new sequence $x_0 \in \mathcal{X}$ (with unknown outcome), and our goal is to quickly identify a revised version that is expected to have superior outcome. Formally, we seek the revised sequence:

$$x^* = \underset{x \in \mathcal{C}_{x_0}}{\operatorname{argmax}} \ \mathbb{E}[Y \mid X = x] \tag{1}$$

Here, we want the set $\mathcal{C}_{x_0}$ of feasible revisions to ensure that x^* remains natural and is merely a minor revision of x_0. Under a generative modeling perspective, these two goals are formalized as the following desiderata: $p_X(x^*)$ is not too small, and x^* and x_0 share similar underlying latent characteristics. When revising a sentence for example, it is imperative that the revision reads naturally (has reasonable likelihood under the distribution of realistic sentences) and retains the semantics of the original.

This optimization is difficult because the constraint-set and objective may be highly complex and are both unknown (must be learned from data). For many types of sequence such as sentences, standard distance measures applied directly in the space of $\mathcal{X}$ or $\mathcal{S}$ (eg. Levenshtein distance or TF-IDF similarity) are inadequate to capture meaningful similarities, even though these can be faithfully reflected by a simple metric over an appropriately learned space of continuous latent factors (Mueller & Thyagarajan, 2016). In

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this work, we introduce a generative-modeling framework which transforms (1) into a simpler differentiable optimization by leveraging continuous-valued latent representations learned using neural networks. After the generative model has been fit, our proposed procedure can efficiently revise any new sequence in a manner that satisfies the aforementioned desiderata (with high probability).

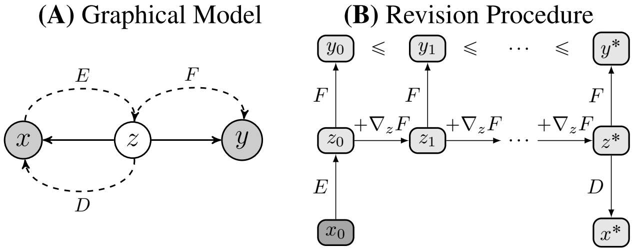
Related Work

Unlike imitation learning, our setting does not require availability of improved versions of a particular sequence. This prevents direct application of a sequence-to-sequence model (Sutskever et al., 2014). Similar to our approach, Gómez-Bombarelli et al. (2016) also utilize latent autoencoder representations in order to propose novel chemical structures via Bayesian optimization. However, unlike sequential bandit/reinforcement-learning settings, our learner sees no outcomes outside of the training data, neither for the new sequence it is asked to revise, nor for any of its proposed revisions of said sequence (Mueller et al., 2017). Our methods only require an easily-assembled dataset of sequence-outcome pairs and are thus widely applicable.

Combinatorial structures are often optimized via complex search heuristics such as genetic programming (Zaefferer et al., 2014). However, search relies on evaluating isolated changes in each iteration, whereas good revisions of a sequence are often made over a larger context (ie. altering a phrase in a sentence). From the vast number of possibilities, such revisions are unlikely to be found by search-procedures, and it is generally observed that such methods are outperformed by gradient-based optimization in high-dimensional continuous settings. Unlike combinatorial search, our framework leverages gradients in order to efficiently find good revisions at test time. Simonyan et al. (2014) and Nguyen et al. (2015) also proposed gradient-based optimization of inputs with respect to neural predictions, but work in this vein has been focused on conditional generation (rather than revision) and is primarily restricted to the continuous image domain (Nguyen et al., 2016).

Methods

To identify good revisions, we first map our stochastic combinatorial optimization problem into a continuous space where the objective and constraints exhibit a simpler form. We assume the data are generated by the probabilistic graphical model in Figure 1A. Here, latent factors $Z \in \mathbb{R}^d$ specify a (continuous) configuration of the generative process for X, Y (both sequences and outcomes), and we adopt the prior $p_Z = N(0, \mathbf{I})$. Relationships between these variables are summarized by the maps F, E, D which we parameterize using three neural networks $\mathcal{F}$, $\mathcal{E}$, $\mathcal{D}$ trained to enable efficient approximate inference under this model.


(A) Graphical Model

(B) Revision Procedure

Figure 1. (A) Assumed graphical model (shaded nodes indicate observed variables, dashed arrows are learned neural network mappings). (B) Procedure for revising a given x_0 to produce x^* with superior expected outcome.

The first step of our framework is to fit this model to $\mathcal{D}_n$ by learning the parameters of these inference networks: the *encoder* $\mathcal{E}$, the *decoder* $\mathcal{D}$, and the *outcome-predictor* $\mathcal{F}$. A good model that facilitates high-quality revision under our framework will possess the following properties: (1) Y can efficiently be inferred from Z and this relationship obeys a smooth functional form, (2) the map D produces a realistic sequence x given any z with reasonable prior probability, (3) the distribution of natural sequences is geometrically simple in the latent Z-space. We explicitly encourage (1) by choosing $\mathcal{F}$ as a fairly simple feedforward network, (2) by defining D as the most-likely x given z, and (3) by endowing Z with our simple $N(0, \mathbf{I})$ prior.

Another characteristic desired of our Z-representations is that they encode meaningful sequence-features such that two fundamentally similar sequences are likely to have been generated from neighboring z-values. Applied to image data, VAE models similar to ours have been found to learn latent representations that disentangle salient characteristics such as scale, rotation, and other independent visual concepts (Higgins et al., 2016). The latent representations of recurrent architectures trained on text (similar to the models used here) have also been shown to encode meaningful semantics, with a strong correlation between distances in the latent space and human-judged similarity between texts (Mueller & Thyagarajan, 2016). By exploiting such simplified geometry, a basic shift in the latent vector space may be able to produce higher-quality revisions than attempts to directly manipulate the combinatorial space of sequence elements.

After fitting a model with these desirable qualities, our strategy to revise a given sequence $x_0 \in \mathcal{X}$ is outlined in Figure 1B. First, we compute its latent representation $z_0 = E(x_0)$ using a trained encoding map. As the latent representations z are continuous, we can employ efficient gradient-based optimization to find a nearby local optimum z^* of $F(z)$ (within a simple constraint-set around z_0 defined later on). To z^*, we subsequently apply a simple decoding map D (defined with respect to our learned model) in order to obtain our revised sequence x^*. Under our assumed model, the optimization in latent representation-space attempts to identify a generative configuration which produces large values of Y (as inferred via F). The subsequent decoding step seeks the most likely sequence produced by the optimized setting of the latent factors.

Variational Autoencoder

For approximate inference in the X, Z relationship, we leverage the variational autoencoder (VAE) model of Kingma & Welling (2014). In our VAE, a generative model of sequences is specified by our prior over the latent values z combined with a likelihood function $p_D(x \mid z)$ which our decoder network $\mathcal{D}$ outputs in order to evaluate the likelihood of any sequence x given $z \in \mathbb{R}^d$. Given any sequence x, our *encoder* network $\mathcal{E}$ outputs a variational approximation $q_E(z \mid x)$ of the true posterior over the latent-values $p(z \mid x) \propto p_D(x \mid z) p_Z(z)$. As advocated by Kingma & Welling (2014) and Bowman et al. (2016), we employ the variational family $q_E(z \mid x) = N(\mu_{z|x}, \Sigma_{z|x})$ with diagonal covariance. Our revision methodology employs the encoding procedure $E(x) = \mu_{z|x}$ which maps a sequence to the maximum a posteriori (MAP) configuration of the latent values z (as estimated by the encoder network $\mathcal{E}$).

The parameters of $\mathcal{E}, \mathcal{D}$ are learned using stochastic variational inference to maximize a lower bound for the marginal likelihood of each observation in the training data:

$$\begin{aligned} \log p_X(x) &\geqslant -\big[\mathcal{L}_{\text{rec}}(x) + \mathcal{L}_{\text{pri}}(x)\big] \qquad (2) \\ \mathcal{L}_{\text{rec}}(x) &= -\mathbb{E}_{q_E(z|x)}\left[\log p_D(x \mid z)\right] \\ \mathcal{L}_{\text{pri}}(x) &= \text{KL}(q_E(z \mid x) \| \, p_Z) \end{aligned}$$

Defining $\sigma_{z|x} = \text{diag}(\Sigma_{z|x})$, the prior-enforcing Kullback-Leibler divergence has a differentiable closed form expression when q_E, p_Z are diagonal Gaussian distributions. The *reconstruction* term $\mathcal{L}_{\text{rec}}$ (ie. negative log-likelihood under the decoder model) is efficiently approximated using just one Monte-Carlo sample $z \sim q_E(z \mid x)$. To optimize the variational lower bound over our data $\mathcal{D}_n$ with respect to the parameters of neural networks $\mathcal{E}$, $\mathcal{D}$, we use stochastic gradients of (2) obtained via backpropagation and the reparameterization trick of Kingma & Welling (2014).

Throughout, our encoder/decoder models $\mathcal{E}$, $\mathcal{D}$ are recurrent neural networks (RNN). RNNs adapt standard feedforward neural networks for sequence data $x = (s_1, \dots, s_T)$, where at each time-step $t \in \{1, \dots, T\}$, a fixed size hidden-state vector $h_t \in \mathbb{R}^d$ is updated based on the next element in the input sequence. To produce the approximate posterior for a given x, our encoder network $\mathcal{E}$ appends the following additional layers to the final RNN hidden-state (parameterized by $W_\mu, W_\sigma, W_v, b_\mu, b_\sigma, b_v$):

$$\mu_{z|x} = W_\mu h_T + b_\mu \ \in \mathbb{R}^d$$

$$\sigma_{z|x} = \exp(-|W_\sigma v + b_\sigma|), \ v = \text{ReLU}(W_v h_T + b_v) \quad (3)$$

The (squared) elements of $\sigma_{z|x} \in \mathbb{R}^d$ form the diagonal of our approximate-posterior covariance $\Sigma_{z|x}$. Since $\mathcal{L}_{\text{pri}}$ is minimized at $\sigma_{z|x} = \vec{1}$ and $\mathcal{L}_{\text{rec}}$ is likely to worsen with additional variance in encodings (as our posterior approximation is unimodal), we simply do not consider $\sigma_{z|x}$ values that exceed 1 in our variational family. This restriction results in more stable training and also encourages the encoder and decoder to co-evolve such that the true posterior is likely closer to unimodal with variance $\leqslant 1$.

To evaluate the likelihood of a sequence, RNN $\mathcal{D}$ computes not only its hidden state h_t, but also the additional output:

$$\pi_t = \text{softmax}(W_\pi h_t + b_\pi) \quad (4)$$

At each position t, π_t estimates $p(s_t \mid s_1, \dots, s_{t-1})$ by relying on h_t to summarize the sequence history. By the factorization $p(s_1, \dots, s_T) = \prod_{t=1}^T p(s_t \mid s_{t-1}, \dots, s_1)$, we have $p_D(x \mid z) = \prod_{t=1}^T \pi_t[s_t]$, which is calculated by specifying an initial hidden-state $h_0 = z$ and feeding $x = (s_1, \dots, s_T)$ into $\mathcal{D}$. From a given latent configuration z, our revisions are produced by decoding a sequence via the most-likely observation, which we denote as the map:

$$D(z) = \operatorname*{argmax}_{x \in \mathcal{X}} p_D(x \mid z) \quad (5)$$

While the most-likely decoding in (5) is itself a combinatorial problem, beam search can exploit the sequential-factorization of $p(x \mid z)$ to efficiently find a good approximate solution (Wiseman & Rush, 2016; Sutskever et al., 2014). For $x^* = D(z) \in \mathcal{X}$, this decoding strategy seeks to ensure neither $p_X(x^*)$ nor $p(z \mid x^*)$ is too small.

Compositional Prediction of Outcomes

In addition to the VAE component, we fit a compositional outcome-prediction model which uses a standard feed forward neural network $\mathcal{F}$ to implement the map $F : \mathbb{R}^d \to \mathbb{R}$. It is assumed that $F(z) = \mathbb{E}[Y \mid Z = z]$ under our generative model. Rather than integrating over Z to compute $\mathbb{E}[Y \mid X = x] = \int F(z) q_E(z \mid x) \mathrm{d}z$, we employ the first-order Taylor approximation $F(E(x))$, where the approximation-error shrinks the more closely F resembles an affine transformation. To ensure this approximate-inference step accurately estimates the conditional expectation, we jointly train $\mathcal{E}$ and $\mathcal{F}$ with the loss:

$$\mathcal{L}_{\text{mse}}(x, y) = [y - F(E(x))]^2 \quad (6)$$

If the architecture of networks $\mathcal{E}$, $\mathcal{F}$ is specified with sufficient capacity to capture the underlying conditional relationship, then we should have $F(E(x)) \approx \mathbb{E}[Y \mid X = x]$ after properly learning the network parameters from a sufficiently large dataset (even F is a nonlinear map).

Enforcing Invariance

In theory, it is possible that some dimensions of z pertain solely to the outcome y and do not have any effect on the decoded sequence $D(z)$. Happening to learn this sort of latent representation would be troubling, since subsequent optimization of the inferred y with respect to z might not actually lead to a superior revised sequence. To mitigate this issue, we carefully ensure the dimensionality d of our latent Z does not significantly exceed the bottleneck capacity needed to produce accurate outcome-predictions and VAE reconstructions (Gupta et al., 2016). We explicitly suppress this undesirable scenario by adding the following loss to guide training of our neural networks:

$$\mathcal{L}_{\text{inv}} = \mathbb{E}_{z \sim p_Z}\big[F(z) - F(E(D(z)))\big]^2 \tag{7}$$

When optimizing neural network parameters with respect to this loss, we treat the parameters of $\mathcal{D}$ and the lefthand $F(z)$ term as fixed, solely backpropagating Monte-Carlo estimated gradients into $\mathcal{E}$, $\mathcal{F}$. Driving $\mathcal{L}_{\text{inv}}$ toward 0 ensures our outcome-predictions remain invariant to variation introduced by the encoding-decoding process (and this term also serves as a practical regularizer to enforce additional smoothness in our learned functions).

Joint Training

The parameters of all components of this model (q_E, p_D, and F) are learned jointly in an end-to-end fashion. Training is done via stochastic gradient descent applied to minimize the following objective over the examples in $\mathcal{D}_n$:

$$\mathcal{L}(x, y) = \mathcal{L}_{\text{rec}} + \lambda_{\text{pri}}\mathcal{L}_{\text{pri}} + \frac{\lambda_{\text{mse}}}{\sigma_Y^2}\mathcal{L}_{\text{mse}} + \frac{\lambda_{\text{inv}}}{\sigma_Y^2}\mathcal{L}_{\text{inv}} \tag{8}$$

where σ_Y^2 denotes the (empirical) variance of the outcomes, and the $\lambda \geqslant 0$ are constants chosen to balance the relative weight of each goal so that the overall framework produces maximally useful revisions. By setting $\lambda_{\text{mse}} = \lambda_{\text{inv}} = 0$ at first, we can optionally leverage a separate large corpus of unlabeled examples to initially train only the VAE component of our architecture, as in the unsupervised pretraining strategy used successfully by Kiros et al. (2015); Erhan et al. (2010).

In practice, we found the following training strategy to work well, in which numerous mini-batch stochastic gradient updates (typically 10-30 epochs) are applied within every one of these steps:

Step 1: Begin with $\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$, so $\mathcal{L}_{\text{rec}}$ and $\mathcal{L}_{\text{mse}}$ are the only training objectives. We found that regardless of the precise value specified for λ_{mse}, both $\mathcal{L}_{\text{rec}}$ and $\mathcal{L}_{\text{mse}}$ were often driven to their lowest possible values during this joint optimization (verified by training individually against each objective).

Step 2: Grow λ_{pri} from 0 to 1 following the sigmoid annealing schedule proposed by Bowman et al. (2016), which is needed to ensure the variational sequence to sequence model does not simply ignore the encodings z (note that the formal variational lower bound is attained at $\lambda_{\text{pri}} = 1$).

Step 3: Gradually increase λ_{inv} linearly until $\mathcal{L}_{\text{inv}}$ becomes small on average across our Monte-Carlo samples $z \sim p_Z$. Here, p_D is treated as constant with respect to $\mathcal{L}_{\text{inv}}$, and each mini-batch used in stochastic gradient descent is chosen to contain the same number of Monte-Carlo samples for estimating $\mathcal{L}_{\text{inv}}$ as (sequence, outcome) pairs.

Proposing Revisions

While the aforementioned training procedure is computationally intensive, once learned, our neural networks can be leveraged for efficient inference. Given user-specified constant $\alpha > 0$ and a to-be-revised sequence x_0, we propose the revision x^* output by the following procedure.

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REVISE Algorithm
Input: sequence $x_0 \in \mathcal{X}$, constant $\alpha \in (0, |2\pi\Sigma_{z|x_0}|^{-\frac{1}{2}})$
Output: revised sequence $x^* \in \mathcal{X}$
1) Use $\mathcal{E}$ to compute $q_E(z \mid x_0)$
2) Define $\mathcal{C}_{x_0} = \{z \in \mathbb{R}^d : q_E(z \mid x_0) \geqslant \alpha\}$
3) Find $z^* = \operatorname*{argmax}_{z \in \mathcal{C}_{x_0}} F(z)$          (gradient ascent)
4) Return $x^* = D(z^*)$                                  (beam search)
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Intuitively, the level-set constraint $\mathcal{C}_{x_0} \subseteq \mathbb{R}^d$ ensures that z^*, the latent configuration from which we decode x^*, is likely similar to the latent characteristics responsible for the generation of x_0. Assuming x_0 and x^* share similar latent factors implies these sequences are fundamentally similar according to the generative model. Note that $z^* = E(x_0)$ is always a feasible solution of the latent-factor optimization over $z \in \mathcal{C}_{x_0}$ (for any allowed value of α). Furthermore, this constrained optimization is easy under our Gaussian approximate-posterior, since $\mathcal{C}_{x_0}$ forms a simple ellipsoid centered around $E(x_0)$.

To find z^* in Step 3 of the REVISE procedure, we use gradient ascent initialized at $z = E(x_0)$, which can quickly reach a local maximum if F is parameterized by a simple feedforward network. Starting the search at $E(x_0)$ makes most sense for unimodal posterior approximations like our Gaussian q_E. To ensure all iterates remain in the feasible region $\mathcal{C}_{x_0}$, we instead take gradient steps with respect to a penalized objective $F(z) + \mu \cdot J(z)$ where:

$$\begin{aligned} J(z) &= \log\Big[K - (z - E(x_0))^T \Sigma_{z|x_0}^{-1}(z - E(x_0))\Big] \\ K &= -2\log[(2\pi)^{d/2}|\Sigma_{z|x}|^{1/2}\alpha] \end{aligned} \tag{9}$$

and $0 < \mu \ll 1$ is gradually decreased toward 0 to ensure the optimization can approach the boundary of $\mathcal{C}_{x_0}$. In terms of resulting revision quality, we found this log barrier method outperformed other standard first-order techniques for constrained optimization such as the projected gradient and Franke-Wolfe algorithms.

In principle, our revision method can operate on the latent representations of a traditional deterministic autoencoder for sequences, such as the seq2seq models of Sutskever et al. (2014) and Cho et al. (2014). However, the VAE offers numerous practical advantages, some of which are highlighted by Bowman et al. (2016) in the context of generating more-coherent sentences. The posterior uncertainty of the VAE encourages the network to smoothly spread the training examples across the support of the latent distribution. In contrast, central regions of the latent space under a traditional autoencoder can contain holes (to which no examples are mapped), and it is not straightforward to avoid these in our optimization of z^*. Furthermore, we introduce an adaptive variant of our decoder in §S1 which is designed to avoid poor revisions in cases where the initial sequence is already not reconstructed properly: $D(E(x_0)) \neq x_0$.

Theoretical Properties of Revision

Here, we theoretically characterize properties of revisions obtained via our REVISE procedure (all proofs are relegated to §S3 in the Supplementary Material). Our results imply that in an ideal setting where our neural network inference approximations are exact, the revisions proposed by our method are guaranteed to satisfy our previously stated desiderata: x^* is associated with an expected outcome-increase, x^* appears natural (has nontrivial probability under p_X whenever x_0 is a natural sequence), and x^* is likely to share similar latent characteristics as x_0 (since x^* is the most likely observation generated from z^* and $q_E(z^* \mid x_0) \geqslant \alpha$ by design). Although exact approximations are unrealistic in practice, our theory precisely quantifies the expected degradation in the quality of proposed revisions that accompanies a decline in either the accuracy of our approximate inference techniques or the marginal likelihood of the original sequence to revise.

Theorems 1 and 2 below ensure that for an initial sequence x_0 drawn from the natural distribution, the likelihood of the revised sequence x^* output by our REVISE procedure under p_X has lower bound determined by the user-parameter α and the probability of the original sequence $p_X(x_0)$. Thus, when revising a sequence x_0 which looks natural (has substantial probability under p_X), our procedure is highly likely to produce a revised sequence x^* which also looks natural. The strength of this guarantee can be precisely controlled by choosing α appropriately large in applications where this property is critical.

In each high probability statement, our bounds assume the initial to-be-revised sequence x_0 stems from the natural distribution p_X, and each result holds for any fixed constant $\delta > 0$. We first introduce the following assumptions:

(A1) For $\delta > 0, \alpha > 0$, there exists $0 < \gamma \leqslant 1$ such that:

i. With probability $\geqslant 1 - \delta/2$ (over $x \sim p_X$):

$$p(z \mid x) \geqslant \gamma \cdot q_E(z \mid x) \quad \text{whenever} \quad q_E(z \mid x) \geqslant \alpha$$

ii. $\Pr(Z \notin B_{R/2}(0)) \geqslant \gamma \cdot \Pr(\widetilde{Z} \notin B_{R/2}(0))$

where $Z \sim N(0, \mathbf{I})$, and $\widetilde{Z} \sim q_Z$, the *average encoding distribution* defined by Hoffman & Johnson (2016) as:

$$q_Z(z) = \mathbb{E}_{x \sim p_X}[q_E(z \mid x)] \tag{10}$$

$B_R(0) = \{z \in \mathbb{R}^d : ||z|| \leqslant R\}$ denotes the Euclidean ball centered around 0 with radius R defined here as:

$$R = \max\{R_1, R_2\} \tag{11}$$

with $R_1 = \sqrt{-8 \log[\alpha \cdot (2\pi)^{d/2}]}$

$$R_2 = \max\{\widetilde{R}_2, 2\}, \;\; \widetilde{R}_2 = \sqrt{8 - \tfrac{1}{4d} \log\left(\tfrac{\gamma\delta}{8}\right)}$$

(A2) There exists $\eta > 0$ (depends on δ) such that with probability $\geqslant 1 - \delta/2$ (over $x_0 \sim p_X$): $\quad p(z^* \mid x^*) \leqslant \eta$

This means the latent posterior is bounded at x^*, z^* (as defined in REVISE), where both depend upon the initial to-be-revised sequence x_0.

Theorem 1. *For any $\delta > 0$, (A1) and (A2) imply:*

$$p_X(x^*) \geqslant \frac{\alpha\gamma}{\eta} \cdot p_X(x_0)$$

with probability $\geqslant 1 - \delta$ (over $x_0 \sim p_X$).

Condition (A1) forms a generalization of absolute continuity, and is required since little can be guaranteed about our inference procedures if the variational posterior is too inaccurate. Equality holds in (A1) with probability 1 if the variational distributions q_E exactly represent the true posterior ($\gamma \to 1$ as the variational approximations become more accurate over the measure p_X). In practice, minimization of the *reverse* KL divergence ($\mathcal{L}_{\text{pri}}$) used in our VAE formulation ensures that $q_E(z \mid x)$ is small wherever the true posterior $p(z \mid x)$ takes small values (Blei et al., 2017).

While the bound in Theorem 1 has particularly simple form, this result hinges on assumption (A2). One can show for example that the inequality in (A2) is satisfied if the posteriors $p(z \mid x^*)$ are Lipschitz continuous functions of z at z^* (sharing one Lipschitz constant over all possible x^*). In general however, (A2) heavily depends on both the data distribution p_X and decoder model p_D. Therefore, we provide a similar lower bound guarantee on the likelihood of our revision x^* under p_X, which instead only relies on weaker assumption (A3) below.

(A3) There exists $L > 0$ such that for each $x \in \mathcal{X}$: $p_D(x \mid z)$ is a L-Lipschitz function of z over $B_{R+1}(0)$.

Here, L depends on δ (through R), and we assume $L \geqslant 1$ without loss of generality. (A3) is guaranteed to hold in the setting where we only consider sequences of finite length $\leqslant T$. This is because the probability output by our decoder model, $p_D(x \mid z)$, is differentiable with bounded gradients over all $z \in B_R(0)$ under any sequence-to-sequence RNN architecture which can be properly trained using gradient methods. Since $B_{R+1}(0) \subset \mathbb{R}^d$ is a closed interval, $p_D(x \mid z)$ must be Lipschitz continuous over this set, for a given value of x. We can simply define L to be the largest Lipschitz constant over the $|\mathcal{S}|^T$ possible choices of $x \in \mathcal{X}$ ($|\mathcal{S}|$ = size of the vocabulary). In the next theorem below, user-specified constant $\alpha > 0$ is defined in REVISE, and L, γ, R all depend on δ.

Theorem 2. *For any $\delta > 0$, if (A1) and (A3) hold, then with probability $\geqslant 1 - \delta$ (over $x_0 \sim p_X$):*

$$p_X(x^*) \geqslant \frac{Ce^{-R}}{L^d} \cdot \left[\gamma \cdot \alpha \cdot p_X(x_0)\right]^{d+1}$$

where constant $C = \dfrac{\pi^{d/2}}{\Gamma(\frac{d}{2}+1)} \cdot \dfrac{(d+1)^d}{(d+2)^{d+1}}$

Our final result, Theorem 3, ensures that our optimization of z^* with respect to F is tied to the expected outcomes at $x^* = D(z^*)$, so that large improvements in the optimization objective: $F(z^*) - F(E(x_0))$ imply that our revision procedure likely produces large expected improvements in the outcome: $\mathbb{E}[Y \mid X = x^*] - \mathbb{E}[Y \mid X = x_0]$. For this result, we make the following assumptions:

(A4) For any $\delta > 0$, there exists $\kappa > 0$ such that $\Pr(X \in \mathcal{K}) \geqslant 1 - \delta/2$, where we define:

$$\mathcal{K} = \{x \in \mathcal{X} : x_0 = x \implies p_X(x^*) \geqslant \kappa\} \tag{12}$$

as the subset of sequences whose improved versions produced by our REVISE procedure remain natural with likelihood $\geqslant \kappa$. Note that either Theorem 1 or 2 (with the corresponding assumptions) ensures that one can suitably define κ such that (A4) is satisfied (by considering a sufficiently large finite subset of $\mathcal{X}$).

(A5) For any $\kappa > 0$, there exists $\epsilon_{\text{mse}} > 0$ such that $\Pr(X \in \mathcal{E}_{\text{mse}}) > 1 - \kappa$, where we define:

$$\mathcal{E}_{\text{mse}} = \{x \in \mathcal{X} : |F(E(x)) - \mathbb{E}[Y|X = x]| \leqslant \epsilon_{\text{mse}}\} \tag{13}$$

(A6) For any $\delta > 0$, there exists $\epsilon_{\text{inv}} > 0$ such that:

$$|F(z) - F(E(D(z)))| \leqslant \epsilon_{\text{inv}} \quad \text{for all } z \in B_R(0) \subset \mathbb{R}^d$$

where R is defined in (11) and depends on δ.

Here, ϵ_{mse} and ϵ_{inv} quantify the approximation error of our neural networks for predicting expected outcomes and ensuring encoding-decoding invariance with respect to F. Standard learning theory implies both $\epsilon_{\text{mse}}, \epsilon_{\text{inv}}$ will be driven toward 0 if we use neural networks with sufficient capacity to substantially reduce $\mathcal{L}_{\text{mse}}$ and $\mathcal{L}_{\text{inv}}$ over a large training set.

Theorem 3. *For any $\delta > 0$, if conditions (A1), (A4), (A5), and (A6) hold, then with probability $\geqslant 1 - \delta - \kappa$:*

$$\Delta_{z*} - \epsilon \leqslant F(z^*) - F(E(x_0)) \leqslant \Delta_{z*} + \epsilon \tag{14}$$

where $$\Delta_{z*} = \mathbb{E}[Y \mid X = x^*] - \mathbb{E}[Y \mid X = x_0]$$ $$\epsilon = \epsilon_{inv} + 2\epsilon_{mse}$$

Here, $\kappa, \epsilon_{\text{inv}}$ are defined in terms of δ as specified in (A4), (A6), and ϵ_{mse} is defined in terms of κ as specified in (A5).

Experiments

All of our RNNs employ the Gated Recurrent Unit (GRU) of Cho et al. (2014), which contains a simple gating mechanism to effectively learn long-range dependencies across a sequence. Throughout, $\mathcal{F}$ is a simple feedforward network with 1 hidden layer and tanh activations (note that the popular ReLU activation is inappropriate for $\mathcal{F}$ since it has zero gradient over half its domain). Decoding with respect to p_D is simply done entirely greedily (ie. a beam-search of size 1) to demonstrate our approach is not reliant on search heuristics. §S2 contains additional details for each analysis.

Simulation Study

To study our methods in a setting where all aspects of performance can be quantified, we construct a natural distribution p_X over sequences of lengths 10-20 whose elements stem from the vocabulary $\mathcal{S} = \{A, B, \ldots, I, J\}$. Each sequence is generated via the probabilistic grammar of Table S1. For each sequence, the associated outcome y is simply the number of times A appears in the sequence (a completely deterministic relationship). Since A often follows C and is almost always followed by B under p_X, a procedure to generate natural revisions cannot simply insert/substitute A symbols at random positions.

Table 1 compares various methods for proposing revisions. Letting σ_Y denote the standard deviation of outcomes in $\mathcal{D}_n$, we evaluate each proposed x^* using a rescaled version of the actual underlying outcome-improvement: $\Delta_Y(x^*) = \sigma_Y^{-1}(\mathbb{E}[Y \mid X = x^*] - \mathbb{E}[Y \mid X = x_0])$. Except where sample size is explicitly listed, all models were trained using $n = 10,000$ (sequence, outcome) pairs sampled from the generative grammar. Wherever appropriate, the different methods all make use of the same neural network components with latent dimension $d = 128$. Other than α, all hyperparameters of each revision method described below were chosen so that over 1000 revisions, the Levenshtein (edit) distance $d(x^*, x_0) \approx 3.3$ on average.

Model	$\Delta_Y(x^*)$	$-\log p_X(x^*)$	$d(x^*, x_0)$
$\log \alpha = -10000$	**0.51** ±0.55	29.0 ±9.3	3.3 ±3.4
$n = 1000$	0.15 ±0.44	32.0 ±9.4	2.8 ±3.4
$n = 100$	0.02 ±0.30	37.0 ±9.7	4.2 ±4.0
$\log \alpha = -1$	0.20 ±0.39	**28.2** ±7.6	1.4 ±2.2
ADAPTIVE	0.47 ±0.49	28.8 ±9.0	3.1 ±3.4
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	0.05 ±0.68	30.4 ±8.4	3.3 ±3.5
SEARCH	0.45 ±0.51	29.0 ±9.4	3.2 ±1.4

Table 1. Results for revisions x^* produced by different methods in our simulation study (averaged over the same test set of 1000 starting sequences $x_0 \sim p_X$, with ± 1 standard deviation shown and the best results in bold).

Model	$\Delta_Y(x^*)$	$\Delta_L(x^*)$	$d(x^*, x_0)$
$\log \alpha = -10000$	**0.52** ±0.77	-8.8 ±6.5	2.6 ±3.3
$\log \alpha = -1$	0.31 ±0.50	**-7.6** ±5.8	1.7 ±2.6
ADAPTIVE	0.52 ±0.72	-8.7 ±6.4	2.5 ±3.3
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	0.22 ±1.03	-10.2 ±7.0	3.3 ±3.4
SEARCH	0.19 ±0.56	-7.7 ±4.2	3.0 ±1.2

Table 2. Results for revised beer-review sentences x^* produced by different methods (average $\pm$ standard deviation reported over the same held-out set of 1000 initial sentences x_0). The third column employs the definition $\Delta_L(x^*) = \log L(x^*) - \log L(x_0)$.

All three results above the line in Table 1 are based on the full model described in our joint training procedure, with new sequences proposed via our REVISE algorithm (using the setting $\log \alpha = -10000$). In the latter two results, this model was only trained on a smaller subset of the data. We also generated revisions via this same procedure with the more conservative choice $\log \alpha = -1$. ADAPTIVE denotes the same approach (with $\log \alpha = -10000$), this time using the adaptive decoding D_{x_0} introduced in §S1, which is intended to slightly bias revisions toward x_0. The model with $\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$ is a similar method using a deterministic sequence-to-sequence autoencoder rather than our probabilistic VAE formulation (no variational posterior approximation or invariance-enforcing) where the latent encodings are still jointly trained to predict outcomes via F. Under this model, a revision is proposed by starting at $E(x_0)$ in the latent space, taking 1000 (unconstrained) gradient steps with respect to F, and finally applying D to the resulting z.

The above methods form an ablation study of the various components in our framework. SEARCH is a different combinatorial approach where we randomly generate 100 revisions by performing 4 random edits in x_0 (each individual edit is randomly selected as one of: substitution, insertion, deletion, or no change). In this approach, we separately learn a *language-model* RNN L on our training sequences (Mikolov et al., 2010). Sharing the same GRU architecture as our decoder model, L directly estimates the likelihood of any given sequence under p_X. Of the randomly generated revisions, we only retain those sequences x for which $L(x) \geqslant \frac{1}{|\mathcal{S}|} L(x_0)$ (in this case, those which are not estimated to be $<$ 10 times less likely than the original sequence x_0 under p_X). Finally, we score each remaining candidate (including x_0) using the outcome-prediction model $F(E(x))$, and the best is chosen as x^*.

Table 1 shows that our probabilistic VAE formulation outperforms the alternative approaches, both in terms of outcome-improvement achieved as well as ensuring revisions follow p_X. For comparison, $-\log p_X(x_0)$ had an average value of 26.8 (over these 1000 starting sequences), and changing one randomly-selected symbol in each sequence to A results in an average negative log-probability of 32.8. Thus, all of our revision methods clearly account for p_X to some degree. We find that all components used in our REVISION procedure are useful in achieving superior revisions. While individual standard deviations seem large, nearly all average differences in Δ_Y or $-\log p_X$ values produced by different methods are statistically significant considering they are over 1000 revisions.

From Supplementary Figure S1, it is clear that α controls how conservative the changes proposed by our REVISE procedure tend to be, in terms of both $-\log p_X(x^*)$ and the edit distance $d(x_0, x^*)$. The red curve in Figure S1A suggests that our theoretical lower bounds for $p_X(x^*)$ are overly stringent in practice (although only the average-case is depicted in the figure). The relationship between $\log p_X(x_0)$ and $\log p_X(x^*)$ (see Figure S1B) is best-fit by a line of slope 1.2, indicating that the linear dependence on $p_X(x_0)$ in the Theorem 1 bound for $p_X(x^*)$ is reasonably accurate. Figure S1C shows that the magnitude of changes in the latent space (arising from z-optimization during our REVISE procedure) only exhibits a weak correlation with the edit distance between the resulting revision and the original sequence. This implies that a fixed shift in different directions in the latent space can produce drastically different degrees of change in the sequence space. To ensure a high-quality revision, it is thus crucial to carefully treat the (variational) posterior landscape when performing manipulations of Z.

Improving Sentence Positivity

Next, we apply our model to ∼1M reviews from BeerAdvocate (McAuley et al., 2012). Each beer review is parsed into separate sentences, and each sentence is treated as an individual sequence of words. In order to evaluate methods using an outcome that can be obtained for any proposed revision, we choose $y \in [0, 1]$ as the VADER sentiment compound score of a given sentence (Hutto & Gilbert,

Model	Sentence	$\Delta_Y(x^*)$	$\Delta_L(x^*)$	$d(x^*, x_0)$
x_0	**this smells pretty bad.**	-	-	-
$\log \alpha = -10000$	smells pretty delightful!	+2.8	-0.5	3
ADAPTIVE	smells pretty delightful!	+2.8	-0.5	3
$\log \alpha = -1$	i liked this smells pretty.	+2.5	-2.8	3
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	pretty this smells bad!	-0.2	-3.1	3
SEARCH	wow this smells pretty bad.	+1.9	-4.6	1

Table 3. Example of a held-out beer review x_0 (in bold) revised to improve the VADER sentiment. Underneath the original sentence, we show the revision produced by each different method along with the true (rescaled) outcome improvement Δ_Y, change in estimated marginal likelihood Δ_L, and edit distance $d(x^*, x_0)$. Table S2 contains additional examples.

# Steps	Decoded Sentence
x_0	**where are you, henry??**
100	where are you, henry??
1000	where are you, royal??
5000	where art thou now?
10000	which cannot come, you of thee?
x^*	where art thou, keeper??
x_0	**you are both the same size.**
100	you are both the same.
1000	you are both wretched.
5000	you are both the king.
10000	you are both these are very.
x^*	you are both wretched men.

Table 4. Decoding from latent Z configurations encountered at the indicated number of (unconstrained) gradient steps from $E(x_0)$, for the model trained to distinguish sentences from Shakespeare vs. contemporary authors. Shown first and last are x_0 and the x^* returned by our REVISION procedure (constrained with $\log \alpha = -10000$). Table S3 contains additional examples.

2014). VADER is a complex rule-based sentiment analysis tool which jointly estimates polarity and intensity of English text, and larger VADER scores correspond to text that humans find more positive with high fidelity.

We applied all aforementioned approaches to produce revisions for a held-out set of 1000 test sentences. As p_X underlying these sentences is unknown, we report estimates thereof obtained from a RNN language-model L learned on the sentences in $\mathcal{D}_n$. Table 2 demonstrates that our VAE approach achieves the greatest outcome-improvement. Moreover, Tables 3 and S2 show that our probabilistically-constrained VAE revision approach produces much more coherent sentences than the other strategies.

Revising Modern Text in the Language of Shakespeare

For our final application, we assemble a dataset of ∼100K short sentences which are either from Shakespeare or a more contemporary source (details in §S2.3). In this training data, each sentence is labeled with outcome $y = 0.9$ if it was authored by Shakespeare and $y = 0.1$ otherwise (these values are chosen to avoid the flat region of the sigmoid output layer used in network $\mathcal{F}$). When applied in this domain, our REVISE procedure thus attempts to alter a sentence so that the author is increasingly expected to be Shakespeare rather than a more contemporary source.

Tables 4 and S3 show revisions (of held-out sentences) proposed by our REVISE procedure with adaptive decoding (see §S1), together with sentences generated by applying the adaptive decoder at various points along an unconstrained gradient-ascent path in latent Z space (following gradients of F). Since the data lack similar versions of a sentence written in both contemporary and Shakespearean language, this revision task is an ambitious application of our ideas. Without observing a continuous spectrum of outcomes or leveraging specially-designed style transfer features (Gatys et al., 2016), our REVISE procedure has to alter the underlying semantics in order to nontrivially increase the expected outcome of the revised sentence under F. Nevertheless, we find that many of the revised sentences look realistic and resemble text written by Shakespeare. Furthermore, these examples demonstrate how the probabilistic constraint in our REVISE optimization prevents the revision-generating latent Z configurations from straying into regions where decodings begin to look very unnatural.

Discussion

This paper presents an efficient method for optimizing discrete sequences when both the objective and constraints are stochastically estimated. Leveraging a latent-variable generative model, our procedure does not require any examples of revisions in order to propose natural-looking sequences with improved outcomes. These characteristics are proven to hold with high probability in a theoretical analysis of VAE behavior under our controlled latent-variable manipulations. However, ensuring semantic similarity in text-revisions remains difficult for this approach, and might be improved via superior VAE models or utilizing additional similarity labels to shape the latent geometry.

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Supplementary Material

S1. Adaptive Decoding

Limited training data in practical settings can limit the inferential accuracy of the learned autoencoder model, and we may have $x_0 \neq D(E(x_0))$ for a given to-be-revised sequence x_0 (particularly if $p_X(x_0)$ is low). In this case, even when $z^* = E(x_0)$ solves our latent-factor optimization, our REVISE procedure can return a different sequence than x_0 (despite not expecting any associated outcome-improvement).

To ensure that our methods simply return the initial x_0 when no superior revision can be identified, we replace our decoder model $p_D(x \mid z)$ with an adaptive variant $p_{D_{x_0}}(x \mid z)$ that is efficiently defined once $x_0 = \left(s_1^{(x_0)}, \ldots, s_{T_{x_0}}^{(x_0)}\right)$ is specified at test time. Like before, we write $D_{x_0}(z)$ to denote the (beam-search approximated) most-likely decoding with respect to $p_{D_{x_0}}$. Recall from our definition in (4), π_t is the vector of symbol-probabilities output by our decoder RNN $\mathcal{D}$ to compute p_D. Using the indexing notation $\pi_t[s_t]$ to denote the decoder RNN's approximation of $p(s_t, \mid s_1 \ldots, s_{t-1})$, we let $\pi_t^{(x_0)}$ denote particular conditional-probability values output by $\mathcal{D}$ when the initial hidden state is $z = E(x_0)$.

For any $x = (s_1, \ldots, s_T) \in \mathcal{X}$, we define:

$$p_{D_{x_0}}(x \mid z) = \prod_{t=1}^{T} \widetilde{\pi}_t[s_t] \quad \text{where for } t = 1, \ldots, T, \ s \in \mathcal{S}: \ \ \widetilde{\pi}_t[s] = \begin{cases} \pi_t[s] + \beta_t^{(x_0)} & \text{if } s = s_t^{(x_0)} \\ \pi_t[s] - \frac{1}{|\mathcal{S}|}\beta_t^{(x_0)} & \text{otherwise} \end{cases} \tag{15}$$

$$\text{and } \beta_t^{(x_0)} = \max_{s \in \mathcal{S}} \pi_t^{(x_0)}[s] - \pi_t^{(x_0)}[s_t^{(x_0)}] \geqslant 0 \quad \text{for } t = 1, \ldots, T_{x_0}$$

At each time step, the $\beta_t^{(x_0)}$ measure any probability gap between the most likely symbol under p_D and the actual sequence x_0 when our decoder model $\mathcal{D}$ is applied to $E(x_0)$. Thus, the definition in (15) ensures $D_{x_0}(E(x_0)) = x_0$. When revising sequences using this adaptive decoding procedure, we compute all $\beta_t^{(x_0)}$ by first decoding from $E(x_0)$ before beginning the latent z-optimization in the REVISE procedure. These values are stored so that we can subsequently decode from the optimal latent-configuration z^* with respect to $p_{D_{x_0}}$ rather than p_D.

According to our adaptive decoding definition, x_0 is more likely than any other sequence under $p_{D_{x_0}}(x \mid E(x_0))$, and $p_{D_{x_0}}$ is very easy to derive from p_D (no additional model besides our original $\mathcal{D}$ is needed). Furthermore, the (beam-search) maximizer of $p_{D_{x_0}}$ can be used to decode from any latent z values, resulting in a mapping that is slightly more biased toward x_0 than decoding with respect to p_D. Finally, we note that if x^* is produced by D_{x_0} rather than D, Theorem 3 continues to hold if we replace D with D_{x_0} in assumption (A6). Theorems 1 and 2 remain valid without any change, since:

$$p_{D_{x_0}}(x^* \mid z^*) \geqslant p_{D_{x_0}}(x_0 \mid z^*) \ \text{ and } \ p_{D_{x_0}}(x_0 \mid z^*) - p_D(x_0 \mid z^*) \geqslant p_{D_{x_0}}(x^* \mid z^*) - p_D(x^* \mid z^*)$$

together imply that $p_D(x^* \mid z^*) \geqslant p_D(x_0 \mid z^*)$, as required for expression (16) in our original proofs.

S2. Experiment Details and Additional Results

Automatic differentiation in TensorFlow is used to obtain gradients for both our revision procedure and the (stochastic) learning of neural network parameters. Throughout our applications, the GRU input is a vector-representation of each symbol in the sequence, taken from a dictionary of embeddings that is learned jointly with the neural network parameters via the Adam optimization algorithm of Kingma & Ba (2015). To ensure the decoder can actually generate variable-length sequences, a special <End> symbol is always included in $\mathcal{S}$ and appended at the end of each sequence in the training data. Note that all α-values stated in the text were actually first rescaled by $(2\pi)^{-d/2}$ before the REVISE procedure (to avoid confounding from the choice of latent-dimensionality d in the relationship between the listed α and characteristics of the resulting revisions).

S2.1. Simulation Study

When sampling a sequence for this simulation, we first draw its length uniformly from the range [10,20], and subsequently draw the symbols at each position following the probabilistic grammar of Table S1. Before its quality is evaluated, any proposed sequence whose length violates the [10,20] range is either truncated or extended via repeated duplication of the last symbol. In all models we apply, the encoder/decoder GRUs operate on input-embeddings of size 8, and the outcome-prediction model $\mathcal{F}$ is a feedforward network with one tanh hidden layer of size 128.

Rule	Probability
$s_t = A \mid s_{t-1} = C$	0.50
$s_t = B \mid s_{t-1} = A$	0.95
$s_t = D \mid s_{t-3} = D$	0.95
$s_t = E \mid s_{t-5} = E$	0.95
$s_t = J \mid s_{t-2} = H, s_{t-1} = I$	0.95
$s_t = I \mid s_{t-2} = I, s_{t-1} = H$	0.95
$s_t = B \mid s_{t-3} = B, s_{t-2} = C$	0.95
$s_t = F \mid s_{t-1} = F, t \geqslant 11$	0.95
$s_7 = G \mid s_6 = F$	0.95
$s_8 = G \mid s_7 = F$	0.50
$s_5 = C$	0.50
$s_{10} = C$	0.50
$s_{15} = C$	0.50
$s_{20} = C$	0.50

Table S1. Probabilistic grammar used to generate sequences $(s_1, \ldots, s_T)$ in our simulation. All events not listed here are assumed to occur randomly (uniformly among the remaining probability mass). When one or more conditioning statements are valid at a given t, we renormalize the probabilities for $s_t \mid s_1, \ldots, s_{t-1}$ before sampling the next character.

In the SEARCH procedure, evaluating 100 candidates took similar computation time as a typical run of our REVISE algorithm. Note that in this small scale simulation study, SEARCH is able to examine a nontrivial subset of the possible sequences around x_0. However, exponentially more randomly generated revisions would be needed to retain the performance of this SEARCH approach under longer sequences with larger vocabularies, whereas the computational complexity of our REVISE procedure scales linearly with such increases. Whereas the SEARCH method changes nearly every given initial sequence by a relatively similar amount, our REVISE procedure tends to either make larger changes or no change at all. As is desirable, our approach (particularly with adaptive decoding) tends to favor no change for x_0 where the corresponding latent posterior has high uncertainty, both because the VAE training objective urges all decodings in a large region around $E(x_0)$ to heavily favor x_0 and the invariance term $\mathcal{L}_{\text{inv}}$ encourages F to be more flat in such regions.

S2.2. Improving Sentence Positivity

For simplicity, our analysis of the beer reviews only considers sentences that are short ($\leqslant$ 30 words) and entirely composed of words that appear in $\geqslant 100$ other sentences. This restricts the size of the vocabulary to $|\mathcal{S}| \approx 5,500$. In this analysis, the SEARCH procedure is allowed to score 1000 candidate sequences, which is now far slower than our REVISE algorithm. In our models, GRUs $\mathcal{E}$ and $\mathcal{D}$ employ an embedding layer of size 128, the latent representations (and GRU hidden states h_t) have $d = 256$ dimensions, and $\mathcal{F}$ is feedforward network with one hidden layer of the same size (and tanh activations)

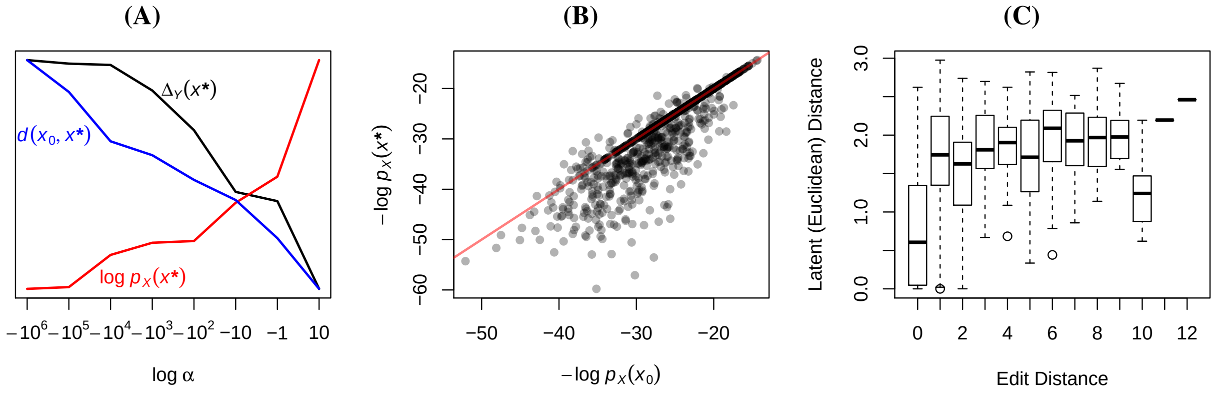

Figure S1. Behavior of the REVISE procedure in our simulation study. **(A)** Relationship between α and properties of revised sequence (averaged over same 1000 initial sequences $x_0 \sim p_X$, with units rescaled so that all curves share the same range): outcome improvement (black), edit distance (blue), marginal log-likelihood (red). **(B)** Likelihood of each original sequences vs. its revised version, when $\log \alpha = -10000$. The diagonal red line depicts the identity relationship $y = x$. **(C)** Boxplot of $||z^* - E(x_0)||_2$ values for each resulting value of $d(x_0, x^*)$ observed when $\log \alpha = -10000$. Note there were very few revisions where $d(x_0, x^*) > 8$.

followed by a sigmoid output layer. The language model L shares the same GRU architecture as our decoder network $\mathcal{D}$.

Examining the REVISE output, we find that punctuation patterns are quite often perfectly preserved in revisions (this is interesting since all punctuation characters are simply treated as elements of the vocabulary in the sequences). There exist many initialization-points where if unconstrained gradient ascent is run for a vast number of iterations with a large step-size, the resulting decoding produces the sentence: “excellent excellent excellent excellent excellent excellent excellent.”, which is has near-optimal VADER sentiment but low marginal likelihood. Starting from other z-initializations, the decoding which results from a massive shift in the latent space often reverts to repetitions of a safe choice where each decoded word has high marginal likelihood, such as: “the the a the the the a the” or “tasting tasting tasting tasting tasting tasting tasting ”.

S2.3. Revising Modern Text in the Language of Shakespeare

Sentences used in this analysis were taken either from the concatenated works of Shakespeare (Karpathy, 2015) or from various more contemporary texts (non-Shakespeare-authored works from the Brown, Reuters, Gutenberg, and FrameNet corpora in Python’s NLTK library (Bird et al., 2009)). Here, we use the same architecture for networks $\mathcal{F}$, $\mathcal{E}$, $\mathcal{D}$ as in the previous beer-reviews application.

Model	Sentence	$\Delta_Y(x^*)$	$\Delta_L(x^*)$	$d(x^*, x_0)$
x_0	**caramel, fruit, sweetness, and a soft floral bitterness.**	-	-	-
$\log \alpha = -10000$	caramel, fresh, sweetness, quite soft and a good bitterness.	+1.88	-5.1	6
ADAPTIVE	caramel, fresh, sweetness, quite soft and a good bitterness.	+1.88	-5.1	6
$\log \alpha = -1$	caramel, fruit sweetness, and a soft floral nose.	+1.17	+0.2	1
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	caramel, fruit sweetness, and a soft floral and tangy nose.	+1.17	-16.4	3
SEARCH	caramel, fruit sweetness, and a soft floral, cocoa.	+ 1.17	-7.0	2
x_0	**i like to support san diego beers.**	-	-	-
$\log \alpha = -10000$	i love to support craft beers!	+0.5	+1.6	4
ADAPTIVE	i like to support san diego beers.	0	0	0
$\log \alpha = -1$	i like to support craft beers!	+0.1	+2.6	3
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	i like to support you know.	0	+3.7	3
SEARCH	i like to super support san diego.	+0.7	-2.9	2
x_0	**good carbonation makes for a smooth drinking experience.**	-	-	-
$\log \alpha = -10000$	good carbonation makes a great smooth drinking stuff.	+1.1	-1.1	3
ADAPTIVE	good carbonation makes a great smooth drinking stuff.	+1.1	-1.1	3
$\log \alpha = -1$	good carbonation makes for great smooth drinking.	+ 1.1	+3.0	2
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	good carbonation makes for a smooth drinking like experience.	+0.7	-9.2	1
SEARCH	good carbonation makes for a drinking nice experience!	+0.9	-4.1	3
x_0	**i’m not sure how old the bottle is.**	-	-	-
$\log \alpha = -10000$	i definitely enjoy how old is the bottle is.	+3.0	-3.6	4
ADAPTIVE	i definitely enjoy how old is the bottle is.	+3.0	-3.6	4
$\log \alpha = -1$	i’m sure not sure how old the bottle is.	+2.5	-6.8	1
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	i’m sure better is the highlights when cheers.	+3.3	-9.2	6
SEARCH	i ’m not sure how the bottle is love.	+2.3	-3.3	2
x_0	**what a great afternoon!**	-	-	-
$\log \alpha = -10000$	what a great afternoon!	0	0	0
ADAPTIVE	what a great afternoon!	0	0	0
$\log \alpha = -1$	what a great afternoon!	0	0	0
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	what a great afternoon lace!	0	-8.2	1
SEARCH	what a solid great!	+0.19	-7.1	2
x_0	**the finish is a nice hoppy bitter, with ample spice.**	-	-	-
$\log \alpha = -10000$	the finish is a nice hoppy plant, with ample spice and great mouthfeel.	+2.5	-6.4	4
ADAPTIVE	the finish is a nice hoppy plant, with ample spice.	+1.3	-0.8	1
$\log \alpha = -1$	the finish is a nice hoppy plant, with ample spice.	+1.3	-0.8	1
$\lambda_{\text{inv}} = \lambda_{\text{pri}} = 0$	the finish is a nice hoppy bitter, with ample spice.	0	0	0
SEARCH	the finish is a nice hoppy bitter best, with ample spice.	+2.0	-7.9	1

Table S2. Additional examples of held-out beer reviews x_0 (in bold) revised to improve their VADER sentiment. Underneath each sentence, we show the revision produced by each different method along with the true outcome improvement $\Delta_Y(x^*) = \mathbb{E}[Y \mid X = x^*] - \mathbb{E}[Y \mid X = x_0]$ (rescaled by the standard deviation of outcomes in the training data), estimated change in marginal likelihood $\Delta_L(x^*) = \log L(x^*) - \log L(x_0)$, and Levenshtein (edit) distance $d(x^*, x_0)$.

# Steps	Sentence
x_0	**you find the evidence of that in the chart on this page.**
100	you find the evidence of that in the chart on this page.
1000	you find the chart of action in this page.
5000	you find the chart of the chart that page of action in this page.
10000	find you in this page of the way of your highness.
x*	you speak of the chart in this page of the lord.
x_0	**somewhere, somebody is bound to love us.**
100	somewhere, somebody is bound to love us.
1000	courage, honey, somebody is bound to love us!
5000	courage man; 'tis love that is lost to us.
10000	thou, within courage to brush and such us brush.
x*	courage man; somebody is bound to love us.
x_0	**the story of the fatal crash is not fully known**
100	the story of the injured is not known.
1000	the story of our virtue is not yet known.
5000	the story of our virtue is not given me yet.
10000	the virtue of our story is not yet.
x*	the story of our virtue is not yet known.
x_0	**this is the root issue for which the united states should stand.**
100	this is the root issue which is an issue on the united states.
1000	the root issue is that the dialogue itself should stand provided.
5000	the general is for the root chief held for which is thy tale.
10000	this the shallow is sworn thee. shallow for thee.
x*	the root issue is the national dialogue from thine.
x_0	**there is no such magic in man-made laws.**
100	there is no such magic of man in such magic.
1000	there is no magic of man in such magic.
5000	there is no magic question with such a man in man.
10000	there is no magic in revolution and made no such india.
x*	there is no magic in such noble birth;
x_0	**check the quality of the water.**
100	check the quality of the water.
1000	check the quality of thy water.
5000	check the quality of thy quality.
10000	check the king of gloucester.
x*	check the quality of thy water.
x_0	**what are you doing here?**
100	what are you doing here?
1000	what are you doing here?
5000	cardinal what does thou live here?
10000	cardinal what does thou live here?
x*	does thou live here?

Table S3. Adaptive decoding from various latent Z configurations encountered at the indicated number of (unconstrained) gradient steps from $E(x_0)$, for the model trained to distinguish sentences from Shakespeare vs. contemporary authors. Shown first and last are the initial sequence x_0 and the revision x^* returned by our REVISION procedure (constrained with $\log \alpha = -10000$).

S3. Proofs and Auxiliary Lemmas

Proof of Theorem 1.

By the definition of x^*, we have:

$$\begin{aligned} & p_D(x^* \mid z^*) \geqslant p_D(x_0 \mid z^*) && (16) \\ \Longrightarrow\ & p_X(x^*) \geqslant \frac{p(z^* \mid x_0)}{p(z^* \mid x^*)} \cdot p_X(x_0) && \text{by Bayes' rule} \\ & \geqslant \frac{\gamma q_E(z^* \mid x_0)}{p(z^* \mid x^*)} \cdot p_X(x_0) \text{ with probability} \geqslant 1 - \delta \end{aligned}$$

by assumptions (A1) and (A2) combined via the union bound. Finally, from the definitions in REVISE, we have that $z^* \in \mathcal{C}_{x_0}$, which implies $q_E(z^* \mid x^*) \geqslant \alpha$. □

Lemma 1. *If (A1) holds, then for* z^* *defined in* REVISE*:* $z^* \in B_R(0)$ *with probability* $\geqslant 1 - \frac{\delta}{2}$ *(over* $x_0 \sim p_X$*).*

Proof. Recall that $B_R(0)$ is defined as the Euclidean ball of radius R centered around 0. We show:

$$||z^* - E(x_0)|| \leqslant \frac{1}{2}R \qquad (17)$$

and with probability $\geqslant 1 - \frac{\delta}{2}$:

$$||E(x_0)|| \leqslant \frac{1}{2}R \qquad (18)$$

Subsequently, the triangle inequality completes the proof.
To prove (17), we recall that from our definition in (3): $q_E(z \mid x_0)$ is a Gaussian distribution with mean $E(x_0)$ and diagonal covariance $\Sigma_{z|x}$ where each entry is $\leqslant 1$. Furthermore, the definitions in REVISE ensure $z^* \in \mathcal{C}_{x_0} \implies q(z^* \mid x_0) \geqslant \alpha$. Defining $K = -2\log[(2\pi)^{d/2}|\Sigma_{z|x}|^{1/2}\alpha]$ which specifies the level-α isocontour of the $N(0, \Sigma_{z|x})$ density, we have:

$$\begin{aligned} & q(z^* \mid E(x_0) \geqslant \alpha \\ \Longrightarrow\ & (z^* - E(x_0))^T \Sigma_{z|x}^{-1} (z^* - E(x_0)) \leqslant K \\ \Longrightarrow\ & ||z^* - E(x_0)|| \leqslant \sqrt{K \cdot \lambda_{\max}(\Sigma_{z|x})} \leqslant \frac{1}{2}R_1 \end{aligned}$$

where $\lambda_{\max}(\Sigma_{z|x})$ is the largest eigenvalue of $\Sigma_{z|x}$ and $\lambda_{\max}(\Sigma_{z|x}) \leqslant 1,\ |\Sigma_{z|x}|^{1/2} \leqslant 1$ for our $q_E(z \mid x)$.

Now, define $\mathcal{R} = \{x \in \mathcal{X} : E(x) > \frac{1}{2}R\}$, and let $\widetilde{Z} \sim q_Z$ as defined in (10). To prove (18), we note that for all $x \in \mathcal{R}$: $q_E(z \mid x)$ is a diagonal Gaussian distribution centered around $E(x)$ which has norm $> R/2$. Thus:

$$\begin{aligned} \frac{\gamma}{4} \cdot p_X(\mathcal{R}) <& \gamma \sum_{x \in \mathcal{R}} \int_{||z|| \geqslant \frac{1}{2}R} q_E(z \mid x) \, \mathrm{d}z \, p(x) = \gamma \cdot \Pr\left(||\widetilde{Z}|| \geqslant \frac{1}{2}R\right) \\ \leqslant & \Pr\left(||Z|| \geqslant \frac{1}{2}R\right) && \text{by the second condition in (A1)} \\ \leqslant & \Pr\left(||Z|| \geqslant \frac{1}{2}R_2\right) && \text{as we defined } R \geqslant R_2 \end{aligned}$$

Since $Z \sim N(0, \mathbf{I})$ under our prior, $||Z||^2 \sim \chi_d^2$.
Applying the Chernoff bound to the tail of the χ^2 distribution (Dasgupta & Gupta, 2002), we thus obtain:

$$\Pr\left(||Z||^2 \geqslant \frac{1}{4}R_2^2\right) \leqslant \left[\frac{1}{4}R_2^2 \cdot \exp\left(1 - \frac{1}{4}R_2^2\right)\right]^{d/2} \leqslant \left[\exp\left(1 - \frac{1}{16}R_2^2\right)\right]^{d/2}$$

which implies $p_X(\mathcal{R}) < \delta/2$ by our definition of R_2. □

Proof of Theorem 2.

For $\epsilon \in (0, 1]$, let $B_\epsilon(z)$ denote the ϵ-ball centered at z. We have:

$$
\begin{aligned}
& p_X(x^*) = \int p_D(x^* \mid z) p_Z(z) \, \mathrm{d}z \\
\geqslant & \Pr(Z \in B_\epsilon(z^*)) \left[p_D(x^* \mid z^*) - L\epsilon \right] \\
& \text{assuming } z^* \in B_R(0) \text{, which occurs with probability} \geqslant 1 - \delta/2 \text{ by Lemma 1} \\
\geqslant & \Pr(Z \in B_\epsilon(z^*)) \left[p_D(x_0 \mid z^*) - L\epsilon \right] && \text{by (16)} \\
= & \Pr(Z \in B_\epsilon(z^*)) \left[\frac{p(z^* \mid x_0)}{p_Z(z^*)} p_X(x_0) - L\epsilon \right] \\
\geqslant & \Pr(Z \in B_\epsilon(z^*)) \left[\gamma \frac{q_E(z^* \mid x_0)}{p_Z(z^*)} p_X(x_0) - L\epsilon \right] \\
& \text{assuming } z^* \in B_R(0) \text{ and } x_0 \text{ satisfies the (A1) inequality, which occurs with probability} \geqslant 1 - \delta \text{ by the union bound} \\
\geqslant & \frac{\Pr(Z \in B_\epsilon(z^*))}{p_Z(z^*)} \left[\gamma \alpha p_X(x_0) - L\epsilon \right] && \text{since } p_Z(z^*) < 1 \text{ and } z^* \in \mathcal{C}_{x_0} \implies q_E(z^* \mid x_0) \geqslant \alpha \\
\geqslant & \frac{\min\limits_{||\Delta|| = \epsilon} p_Z(z^* + \Delta)}{p_Z(z^*)} \mathrm{Vol}(B_\epsilon(z^*)) \left[\gamma \alpha p_X(x_0) - L\epsilon \right] && \text{where } \mathrm{Vol}(\cdot) \text{ denotes the Lebesgue measure} \\
\geqslant & \exp\left(-\frac{1}{2} \left[||z^*||\epsilon + \epsilon^2 \right] \right) \mathrm{Vol}(B_\epsilon(z^*)) \left[\gamma \alpha p_X(x_0) - L\epsilon \right] \\
& \text{by exploiting the fact that } p_Z = N(0, I) \text{ and subsequent application of the Cauchy-Schwarz inequality} \\
\geqslant & \exp\left(-\frac{||z^*|| + 1}{2} \right) \cdot \mathrm{Vol}(B_\epsilon(z^*)) \cdot \left[\gamma \alpha p_X(x_0) - L\epsilon \right] && \text{for any } \epsilon \in (0, 1] \\
\geqslant & \exp\left(-\frac{R + 1}{2} \right) \cdot \mathrm{Vol}(B_\epsilon(z^*)) \cdot \left[\gamma \alpha p_X(x_0) - L\epsilon \right] && \text{since we already assumed } z^* \in B_R(0).
\end{aligned}
$$

We conclude the proof by selecting $\epsilon = \frac{\gamma\alpha(d+1)}{L(d+2)} p_X(x_0)$ which maximizes the lower bound given above. □

Proof of Theorem 3.

Suppose for $x_0 \in \mathcal{R}$, the corresponding revision $x^* \notin \mathcal{E}$. Then:

$$
\begin{aligned}
\Pr(X \in \mathcal{E} \cap \mathcal{R}) &\leqslant 1 - p_X(x^*) - \Pr(X \in \mathcal{E} \backslash \mathcal{R}) \\
&\leqslant 1 - \kappa - \Pr(X \in \mathcal{E} \backslash \mathcal{R})
\end{aligned}
$$

Since (A5) implies $\Pr(X \in \mathcal{E}^C) < \kappa$, we also have:

$$
\begin{aligned}
\Pr(X \in \mathcal{E} \cap \mathcal{R}) &= 1 - \Pr(X \in \mathcal{E}^C) - \Pr(X \in \mathcal{E} \backslash \mathcal{R}) \\
&> 1 - \kappa - \Pr(X \in \mathcal{E} \backslash \mathcal{R})
\end{aligned}
$$

which is a contradiction. Thus, we must have $x^* \in \mathcal{E}$ if $x_0 \in \mathcal{R}$, which occurs with probability $\geqslant 1 - \delta/2$.

Lemma 1 ensures that under (A1): $z^* \in B_R(0)$ with probability $\geqslant 1 - \delta/2$, implying $|F(z^*) - F(E(D(z^*)))| \leqslant \epsilon_{\text{inv}}$ with the same probability. Consequently, we have:

$$
\begin{aligned}
& F(z^*)) - F(E(x_0)) \leqslant F(E(D(z^*)) - F(E(x_0)) + \epsilon_{\text{inv}} && \text{with probability} \geqslant 1 - \tfrac{\delta}{2} \\
\leqslant & F(E(x^*)) - \mathbb{E}[Y \mid X = x_0] + \epsilon_{\text{inv}} + \epsilon_{\text{mse}} && \text{with probability} \geqslant 1 - \tfrac{\delta}{2} - \kappa \text{ by the union bound} \\
\leqslant & \mathbb{E}[Y \mid X = x^*] - \mathbb{E}[Y \mid X = x_0] + \epsilon_{\text{inv}} + 2\epsilon_{\text{mse}} && \text{with probability} \geqslant 1 - \tfrac{\delta}{2} - \kappa - \tfrac{\delta}{2} \text{ by the union bound}
\end{aligned}
$$

The inequality in the other direction is proved via similar reasoning. □

Additional References for the Supplementary Material

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