Problem 1 - Pinholes
Let’s assume that all pinholes were exposed the same amount time to take a picture. In such case, (a) would capture the sharpest and darkest image. Comparing to (a), the larger radius pinhole (b) would blur the image. By widening only one direction, the effect is similar to (b) except that blurring occurs only by the direction the pinhole is widened. For example, (c) would blur the image in vertical direction, and (d) would blur the image in horizontal direction.
Problem 2 - Textons

The point of this problem is to understand how textons work. Sobel operator is one of simple operators to detect edge by approximating the gradient of the image. In this problem, it is used to compute orientation and its magnitude per pixel (to mimic texture textons). The code is provided below.

(a) $M_x$
(b) $M_y$
(c) $M$
(d) $\theta$

Bin assignment
Texton Code

function textons(im, th, k)

close all;
im = im2double(im);

if size(im, 3) > 1
    im = rgb2gray(im);
end

% sobel operator
hy = fspecial('sobel');
hx = hy';

Mx = imfilter(im, hx, 'replicate');
figure; imagesc(Mx); colorbar;
My = imfilter(im, hy, 'replicate');
figure; imagesc(My); colorbar;

% Magnitude and angle
M = sqrt(Mx.^2 + My.^2);
theta = atan2(My, Mx);

figure; imagesc(M); colorbar;
figure; imagesc(theta); colorbar;
% Assign to the histogram (into 8 bins)
bin = theta / pi * 4 + 4.5;
bin(bin > 8) = 0;
bin = floor(bin) + 1;
figure; imagesc(bin); colorbar;

% Collect features (11x11 patches per pixel)
new_matrix = zeros(size(im, 1), size(im, 2), 8);
for x = 1:size(M, 1)
    for y = 1:size(M, 2)
        new_matrix(x,y,bin(x,y)) = (M(x,y) > th) * M(x,y);
    end
end

patch_sum = zeros(size(im, 1), size(im, 2), 8);
for x = 6:size(M, 1)-5
    for y = 6:size(M, 2)-5
        patch_sum(x,y,:) = sum(sum(new_matrix(x-5:x+5, y-5:y+5, :), 1), 2);
    end
end

% K-means
patch_sum = reshape(patch_sum, [size(im, 1)*size(im, 2), 8]);
[IDX,C] = kmeans(patch_sum, k, 'emptyaction', 'singleton');

IDX = reshape(IDX, [size(im, 1), size(im, 2)]);
figure; imagesc(IDX, [1 k]); colorbar;
Problem 3 - 3D

1. Let’s suppose there is a plane $P$, and the line, $l$, on $P$. $l$ is $x(t) = x_0 + at, y(t) = y_0 + bt, z(t) = z_0 + ct$. Then, the vanishing point of $l$ is $(fa/c, fb/c, f)$. We also know that $< a, b, c > \cdot N_P = 0$ (where $N_P$ is the normal of $P$).

Now, we also know that the vanishing line of $P$ is the intersection of the projection plane ($z = f$) and the plane $P'$ that is parallel to $P$ and contains the origin. Because $P'$ is parallel to $P$, we know that $N_{P'} = N_P$. Also, because $P'$ contains the origin, we know that the vanishing line is $< x, y, f > \cdot N_{P'} = < x, y, f > \cdot N_P = 0$.

Finally, $< fa/c, fb/c, f > \cdot N_P = \frac{f}{c} < a, b, c > \cdot N_P = 0$. Therefore, $(fa/b, fb/c, f)$ is on the vanishing line of $P$.

2. i. Ellipse: this is essentially the conic section problem. So, on the typical case, the sphere will show up as the ellipse. Now, let’s show the eccentricity.

In order to compute the major axis $a$, let’s consider this figure, and assume that $\overline{O'B} = \overline{OP} = r$.

![Diagram](image)

The first step is to find $\overline{PQ}$.

$\overline{QB} : r = \overline{PQ} : Z$

$$\overline{QB} = \frac{r \overline{PQ}}{Z}$$

$$r^2 + \overline{QB}^2 = \overline{O'Q}^2$$

$$r^2 + \frac{r^2 \overline{PQ}^2}{Z^2} = (X - \overline{PQ})^2$$

by substituting (1)

$$\overline{PR} = \frac{Z^2 X^2 \pm Zr \sqrt{Z^2 + X^2 - r^2}}{Z^2 - r^2}$$

The second step is to find $\overline{PR}$.

$\overline{AR} : r = \overline{PR} : Z$

$$\overline{AR} = \frac{r \overline{PR}}{Z}$$

$$r^2 + \overline{AR}^2 = \overline{O'R}^2$$
\[ r^2 + \frac{r^2 PR^2}{Z^2} = (PR - X)^2 \quad \text{by substituting (2)} \]

\[ PR = \frac{Z^2 X^2 \pm Z \sqrt{Z^2 + X^2 - r^2}}{Z^2 - r^2} \]

Therefore, now we can get \( QR = AR - P Q \).

\[ QR = \frac{2 Z r \sqrt{Z^2 + X^2 - r^2}}{Z^2 - r^2} \]

Now, by similarity,

\[ a : QR = f : Z \]

\[ a = \frac{2 Z r \sqrt{Z^2 + X^2 - r^2}}{Z^2 - r^2} \]

Easier analogy applies to get \( b \) (on YZ-plane; basically \( Y = 0 \)). Then,

\[ b = \frac{2 Z \sqrt{Z^2 - r^2}}{Z^2 - r^2} \]

Therefore, using the definition of eccentricity,

\[ e = \sqrt{1 - \frac{b^2}{a^2}} \]

\[ = \sqrt{1 - \frac{Z^2 - r^2}{Z^2 + X^2 - r^2}} \]

\[ = \sqrt{\frac{X^2}{Z^2 + X^2 - r^2}} \]

\[ = \frac{X}{\sqrt{Z^2 + X^2 - r^2}} \]

ii. If the image plane intersects with only one of the segment \( \overline{AO} \) and \( \overline{BO} \), and \( O' \) is on the other side with \( O \), then it will form a parabola.

iii. If the image plane intersects with only one of the segment \( \overline{AO} \) and \( \overline{BO} \), and \( O' \) is on the same side with \( O \), then it will form a hyperbola.