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Wavelets on Triangulated Surfaces

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Abstract

The graphics community initially developed multi-resolution representations of surfaces in order to solve two problems: editing and compressing of surfaces. Peter Schroder's group at Caltech, in conjunction with Wim Sweldens at Bell Labs, introduced a multi-resolution representation which they call wavelets[1].

We review the extension of wavelets from uniformly sampled data to non-uniformly sampled surfaces [2]. We examine the implementation in Schroder's group as well as a completely different style of multi-resolution representation by Leif Kobbelt [3]. We argue both pragmatically and mathematically that while these implementations are effective, they are not actually wavelet expansion.

We finish by looking at the applications for which these methods are used. Those applications show much of the power of the representations. However, there is an opportunity for a new research direction. As far as we are aware, no one has tried to analyze the wavelet (detail) coefficients. We believe that analysis of the statistics of the wavelet coefficients across a class of objects could lead to an effective deformable model for segmentation and possibly a feature detection algorithm.

1 Introduction

Polygonal meshes are regularly used to represent surfaces in three dimensions. Animated characters, medical organs, range sensing data, and objects in surface finite element simulations are examples of items that can be represented in this way. The communities that manipulate these objects have needs to process the polygonal meshes.

Unfortunately, the powerful processing tools of regularly sampled data do not easily transfer to irregularly sampled triangulated meshes. For example, while down-sampling and up-sampling techniques are well understood for regularly sampled data, it is not obvious how to down-sample or up-sample a mesh. The name Digital Geometry Processing (DPG) has been given to the set of signal processing algorithms that have been implemented on polygonal meshes. In this document we examine the extension of one such method: the extension of wavelets from regularly sampled data to irregularly sampled data.

We begin by reviewing the lifting method, which is a method used to extend wavelets from regularly to irregularly sampled data. This method can be used to create wavelets on spheres [4], and to find wavelet bases to solve partial differential equations [5]. We then examine the multi-resolution methods of Schroder's group at Caltech [1] and Kobbelt's group at the Max-Planck-Institute for Information [3]. We examine the effectiveness of the implementations as well as the issue of whether they are actually "wavelets". We examine the applications of the work of Kobbelt and Schoder, noting that potential applications analyzing wavelet coefficients have not been pursued. We finish by examining applications that might analyze this information.

2 Extending Wavelets

Wavelets on irregularly sampled data have been named "Second Generation Wavelets" [2]. These wavelets maintain some but not all the properties of "first generation" wavelets. We briefly review the lifting method for the formation of wavelets and show how the method can be extended to irregular point sets.

2.1 The Lifting Method on Regularly Sampled Data

Wavelets are a multi-resolution representation of a function using a scaled, translated set of basis functions and a bi-orthogonal set of detail functions [6, 7]. The lifting method is a way of both calculating wavelet transforms and finding the linear filters with which to calculate those transforms [8, 9]. We review the lifting method through a demonstration.

In its simplest form, the lifting method consists of three steps: splitting, prediction and update. Consider a multi-resolution representation S where $s_{j,k}$ represents the k^{th} sample at level j. The data in level j is split into odd and even samples. The even samples will eventually form a basis of a down-sampled version of the basis function. The odd samples will become the "detail" or "wavelet" coefficients. This is the first row in Figure 1.

Often the goal of a wavelet expansion is to find a lower-resolution representation that captures the majority of the variation of the function. The goal is therefore to predict the odd samples from the even samples. We demonstrate using linear interpolation. The detail coefficients become the difference between the linear prediction of points based on their neighbors: $d_{j-1,k} = s_{j,2k+1} - \frac{1}{2}(s_{j,2k} + s_{j,2k+2})$. This is the prediction step in Figure 1.

The calculation lacks one aspect of wavelets: the mean of the down-sampled coefficients is not necessarily the same as the mean of original function. This occurs because we have thrown away half the samples. One can "update" the coefficients to correct for this problem by using the detail coefficients. It is straight forward to show that adding $\frac{1}{4}$ of each of the detail coefficients will correct for this problem. The correction is shown in the last step in Figure 1.

Examining Figure 1, one can see that we have implemented the bi-orthogonal wavelet bases $(-\frac{1}{8}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, -\frac{1}{8})$, and $(-\frac{1}{2}, 1, -\frac{1}{2})$. The transform was done in place, and can easily be reversed by reversing the steps in the transform. These two bases

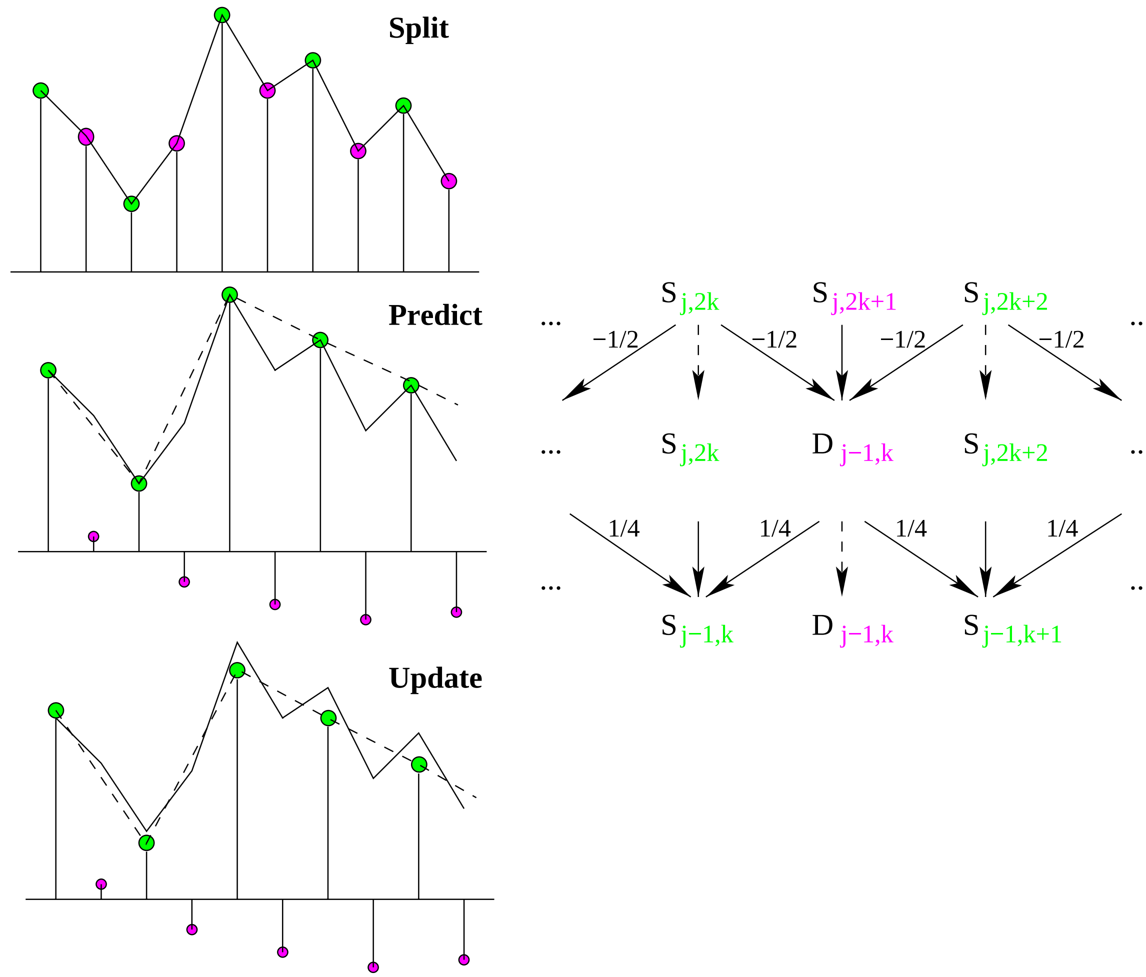

Figure 1: The three steps of a lifting transform illustrated graphically on the left, and algorithmicly on the right. Green samples become the lower-resolution samples. Magenta samples become detail coefficients. The dashed line indicates the interpolated function at the down-sampled resolution.

have all the properties of bi-orthogonal wavelet bases. Notably, the mean and the first moment of the original data are conserved in the lower-resolution coefficients while the detail coefficients filter out the mean and first moment of the data[1]. Finally, and most important, the lifting function shows that detail coefficients can be interpreted as the difference between the actual value of the function at a point and the predicted interpolation.

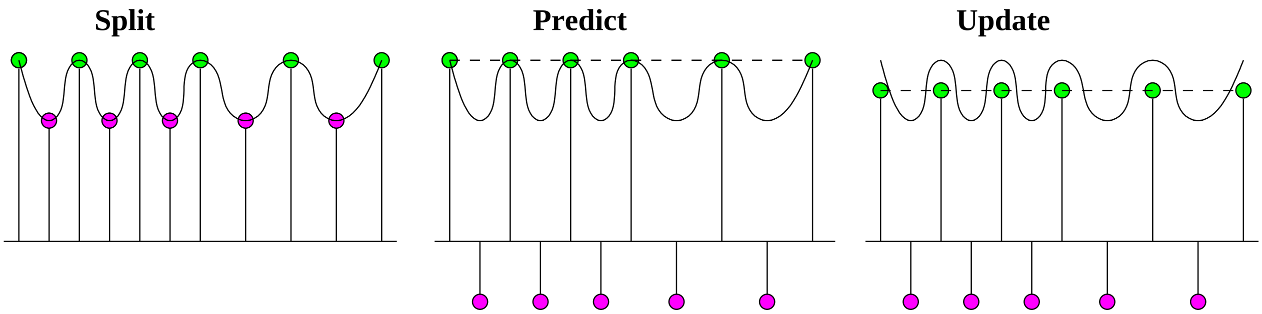

Figure 2: The three steps of a lifting transform for irregularly sampled data using a third degree polynomial interpolation (Lagrange interpolation). Samples on the left are sampled twice as often as samples on the right. The function similarly wiggles twice as fast as the left. Note that while the detail coefficients are all the same, the coefficients on the left represent a higher spatial frequency than the ones on the right.

2.2 Second Generation Wavelets in 1D

Second generation wavelets are wavelet expansions based on irregularly sampled data. The expansion can be found using the same splitting, predicting, and updating steps as shown in Figure 2. Since the points are irregularly spaced, one need not select every other point to be removed from the next lower resolution representation. Though, typically one assumes that points are roughly equally spaced so that choosing every other point as a detail coefficient is a reasonable strategy.

As before, the detail coefficients are replaced by the difference between their value and the value of the interpolating function based on neighbors. However the interpolating basis functions are no longer translation invariant. We instead fit a function to the data points. Depending on the relative locations of the neighbors, the coefficients multiplying the values of each point will be different.

The final step, the update step, is also different. To preserve the weighted average of the mean, one can still update the samples to be kept by the neighboring detail coefficients. However, the coefficients that multiply the detail coefficients will be different in each case. One determines them by integrating the interpolating function along the function. (Note that this can be pre-computed efficiently. [8]).

2.3 Second Generation Wavelet Concerns

In "Wavelets on Irregular Point Sets", Daubechies et al. [2], express concerns with the methods they developed. Most of their concerns are with up-sampling, not down-sampling. They are concerned that using second order wavelets while up-sampling a function using arbitrary point-introduction techniques may not converge or may converge to something that is not smooth (C^2 continuous). This is important in graphics where one would like sub-division processes to yield smooth functions [10]

[1] What the lifting method actually does is start with a bi-orthogonal wavelet basis, modify that basis to get a new basis, and then correct the resulting representation to have the appropriate vanishing moments.

and in wavelets for finding bi-orthogonal wavelet bases to a certain polynomial order. In a seperate paper, they show [11, 12] that for up-sampling methods using polynomial interpolation that have a bounded ratio of the lengths for neighboring sub-intervals, that these properties hold. We developed a easier, though less rigorous, way to examine and understand this problem which we describe in Appendix A.

Our first concern with second order wavelets is that the basis functions and the detail coefficients capture different information. This is guaranteed through bi-orthogonality. Bi-orthogonality for certain interpolators is shown in [11]. Roughly, for polynomial interpolators, one can show the the basis functions capture the first few polynomial orders, while the detail coefficients do not. Thus, this concern is not warranted.

Our second major concern is the interpretation of the wavelet coefficients. Figure 2 is constructed to show that detail coefficients at the same level may have extremely different meanings. The detail coefficients on the right indicate a spatial frequency half as large as the detail on the left. Thus, detail coefficients at the same level do not necessarily indicate the same spatial frequencies. Figure 2 also demonstrates that there is an issue of choosing detail coefficients in a level. For irregularly sampled data, it is not clear that choosing every other point is a good strategy. For example, choosing points with only the highest sampling rate may be better.

2.4 The Lifting Process on Surfaces

The lifting process theoretically generalizes to irregularly sampled 2-D surfaces in 3 dimensions. One can imagine choosing a vertex as a detail coefficient, fitting a local function to the neighboring vertices, predicting the location of the point, etc. No one has done it. In a personal communication, Wim Sweldens wrote that it was too difficult to design filter which after iteration make smooth functions.[13].

The problem appears to be a standard one with smoothing in triangulated surfaces. Vertices of the triangulated surfaces often don't over-sample the original surface. (By over-sample, we mean that interpolation don't produce the pre-triangulated surface.) Thus, after a surface is fit with a smooth function, it will be smooth but potentially have non-intuitive low-frequency range undulations. Figure 3 shows how fitting points to a curve can result in over shooting and undulations in an upsampled surface. (For subdivision schemes used to achieve smoothness, see the Siggraph 2000 notes [14])

Peter Schroder and Wim Sweldens were successful in transferring the lifting procedure to functions on spheres [4]. They show that wavelets can be very useful to represent BRDF's, which are generally very smooth, but less useful to represent highly textured data such as topographic data on the earth. Looking closely at that work, there is one key result shown in the right-most plot of Figure 11 of that paper. Wavelets from the the butterfly sub-division scheme and the "lifted" butterfly sub-division scheme represent the particular BRDF better than other wavelets. But, the "lifted version" is only moderately better that the wavelets based directly on the butterfly sub-division scheme (essentially not including the "update" step of the lifting scheme). This result suggests that for arbitrary triangulated-surfaces, it may be

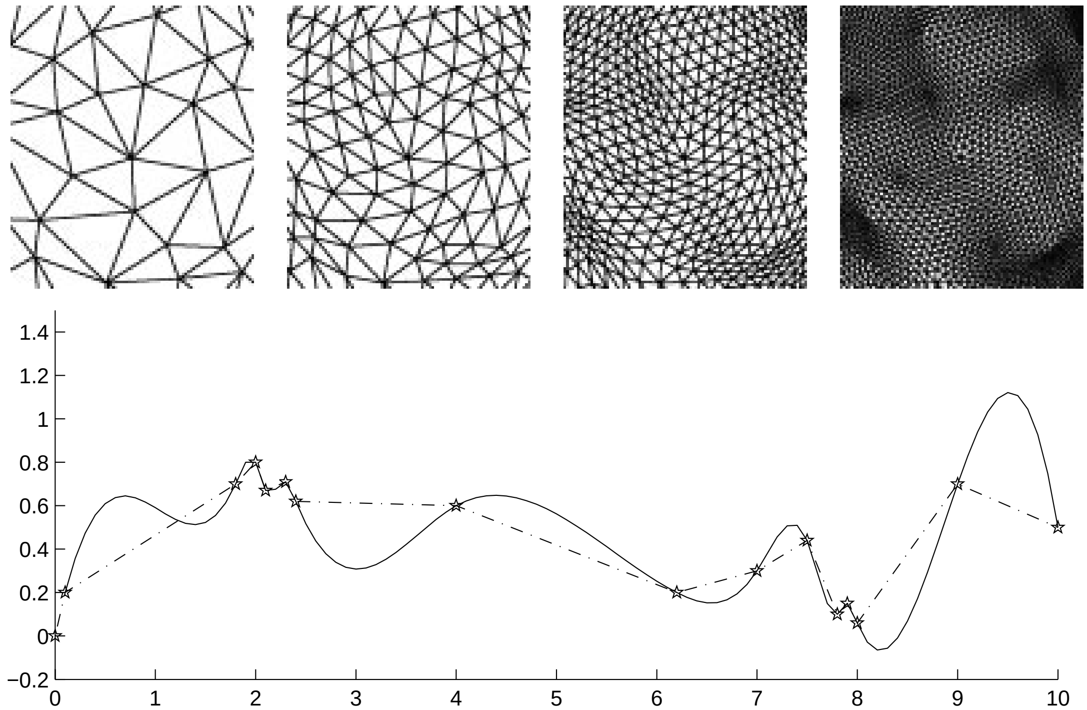

Figure 3: Examples of potential problems caused by fitting functions. In the top images, subdivision of a "smooth" surface where unwanted undulations develop. (Picture from [3]). In the bottom image, examples of unwanted undulations and overshooting using cubic splines on irregularly sampled data. A linear interpolation is shown for reference.

possible to ignore the lifting scheme and still find useful representations.

3 Multi-Resolution Triangulated Surfaces

Multi-resolution polygonal surface, are formed in a manner similar to the lifting method, though without the "update" step. To take a surface to the next lower resolution, one removes one or more vertices. In order to get back to the original surface, one uses a subdivision scheme to get-back the original vertices and predict their locations. One then stores the difference between the predicted and actual location as detail coefficients.

Guskov, Sweldens and Schroder form a multi-resolution representation by removing one vertex at each level [2] through a "half" edge collapse on to a neighboring vertex (Figure 4). They remove edges in increasing order of length. Their subdivision scheme consists of re-introducing the vertex in a position to minimize a smoothing function. They then smooth the neighbor vertices using the same smoothing function. (This step is roughly like an "update" step.) To undo an edge collapse, they must keep displacement vectors for the removed vertex as well as all the neighbors. The vectors are stored relative to a local tangent plane.

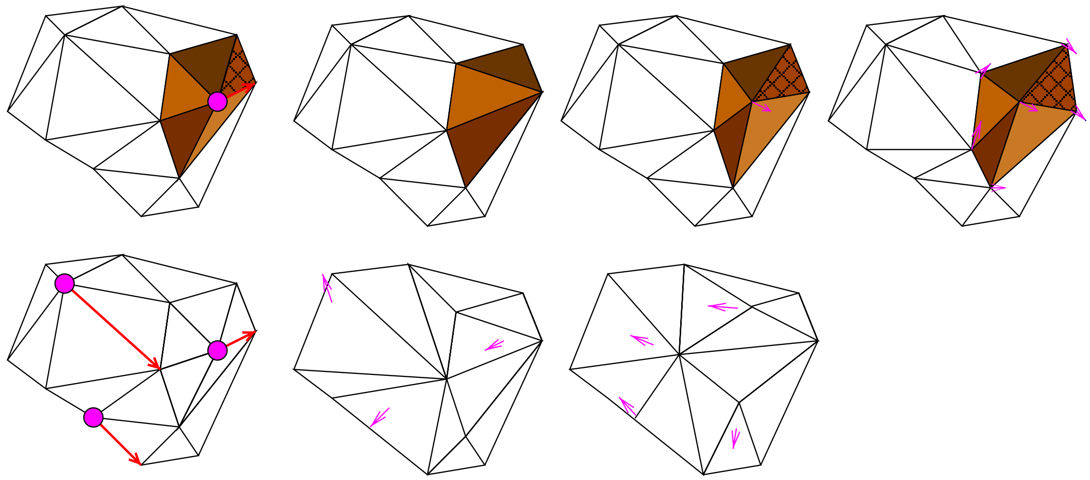

Figure 4: The top row illustrates Guskov's method. From Left to Right: Pick a vertex to be removed, collapse one vertex into the next (a half edge collapse), subdivide to get the vertex back, smooth the vertices around the new vertex. The magenta arrows indicate the detail coefficients. The bottom row illustrates Kobbelt's method. Pick vertices to be removed, perform the half edge collapses (finding a local coordinate system to make the detail coefficient), then smooth the underlying mesh. The magenta details coefficients indicate the location of the vertices perpendicular to a local coordinate system.

Their smoother is based on a linear version of the dihedral angle. For each vertex, they find the angles between the triangles containing the point and their neighbors that do not, and then smooth the angles. They call the resulting multi-resolution representation an oversampled "wavelet pyramid" [2]. They call it "oversampled" because there are several detail coefficients (vectors) for each removal of a vertex, rather than just one vector.

Kobbelt, Vorsatz and Seidel form a different multi-resolution representation [3]. They are interested in fast smoothing according to a global smoothing term, the thin plate bending energy. They are able to perform the smoothing quickly by forming a multi-resolution hierarchy, smoothing the lowest-resolution mesh, propagating the changes to the next highest level and repeating.

To form the hierarchy, they remove a fraction of the vertices at each level using half-edge collapses (See Figure 4), removing the smallest bending energy vertices first. For each vertex removed at a level, they find a local parameterization of the coarser surface. The detail coefficient for each point is the location in the parameterization and the perpendicular vector. To make the smoothed hierarchy, they begin by smoothing the lowest resolution surface and then propagating to the next highest resolution surface where they smooth again, repeating until they reach the highest level surface (which may or may not be smoothed). In each level, detail coefficients are stored for each vertex in the level.

3.1 Critique

Our analysis of the research is dominated by two questions: "Do these multi-resolution analyses work well for their intended purpose?" and "Are these wavelet representations?". The answer to the first question is "yes". Kobbelt's work is mostly aimed at finding a fast way to smooth, and a fast way to do multi-resolution editing. The results and images in his paper show he did just that. The Caltech group is also interested in editing and smoothing. And, their resulting edits and smooth surfaces show visually-pleasing pictures as well.

From the mathematical point of view, we do not consider either of these representation wavelet based representations. We believe that wavelet representations should be critically sampled rather than over-sampled: there should be one detail coefficient per vertex removed. Second, there is no evidence of bi-orthogonality. That is, there is no evidence that the detail coefficients capture some aspect of the function that the basis functions do not capture.

From a pragmatic point of view, the representations are very useful. When we make a list of desirable characteristics of a surface wavelet-based representation, we include (1) detail coefficients perpendicular to the local surface (2) one wavelet coefficient per vertex removed (3) the position of vertices do not shift within the surface during sub-division and (4) the wavelet coefficients are useful. Based on the usefulness of the methods, one can argue that both Guskov and Kobbelt's representation satisfy (1) and (4). Gustov's representation satisfies (3); Kobbelt's method does allow small motions within the surface.

Guskov et al. claim that since they order their edge collapses by edge length, the detail coefficients are roughly in order of "spatial frequency". They can therefore increase the highest detail coefficients to create a "high frequency enhancement filter". We are very impressed with the resulting images. However, to our knowledge, there is currently no application for feature enhancement. The resulting images are nothing more than pretty pictures.

4 Existing Applications

We try to show the impact of the work with multi-resolution triangulated surfaces by reviewing some of the applications which which it has been used. The community generally views these methods as a way to reverse subdivision. With the exception of compression, the detail coefficients are ignored.

4.1 Editing

Non-uniform Rational B-splines (NURBs) have traditionally been used to model animated characters. NURBs are limited to surfaces which are topologically equivalent to a sheet, cylinder or torus. Planning and building a NURBs model is difficult. They do not allow easy local refinement. And, great care must be taken to hide the seams between NURBs patches or to constrain control points near the patches. Instead, triangulated surfaces together with sub-division techniques are replacing these methods [15]. Multi-resolution representations of meshes allow real-time editing of low-resolution surfaces. The high-resolution details can be added back in when the editing is done.

4.2 Re-Meshing

Given a set of equivalent feature points on different meshes, Schroder's group can find a equivalent re-parameterization of each mesh. Essentially, they match a particular base-mesh to each object and then do the identical subdivision[16]. Subdivision of each mesh is done separately. Therefore, it is not necessary that resulting points in the mesh will necessarily correspond. (Nonetheless, assuming they do correspond, it would be interesting to compare wavelet coefficients across shapes. Schroder's paper does not do this.)

Kobbelt's group, by contrast, uses a multi-resolution snake-like method to segment volume data. They start from a user-provided rough segmentation of the surface, and subdivide using butterfly-subdivision and Laplacian (or double Laplacian) smoothing [17].

4.3 Compression

Schroder's group comes up with the concept of "Normal Meshes"[18]. They essentially re-mesh surfaces into a multi-resolution structure with the following property: a vertex to be added/removed lies along line through a point and direction determined by the lower-resolution. Thus detail coefficients can be written as a single scalar indicating the placement along the line. By prediction of the new point, the distribution of detail coefficients becomes small [19, 20].

4.4 Medical Applications

To our knowledge, only Sylvain Jaume et al. has pursued medical applications of the hierarchical based methods [21, 22, 23]. He uses them as part of an algorithm to detect tumors in the bladder. Essentially, he has used the representation for alignment of meshes. A low resolution mesh is aligned to a target and then allowed to deform along the directions of the detail coefficients.

5 Proposed Applications

There are two parts to a wavelet scheme: the multi-resolution aspect and the detail coefficients. The graphics community has effectively exploited the multi-resolution part of the representation. They have used this aspect to speed algorithms and make them more robust. We feel there is an opportunity to develop algorithms based on analyzing detail coefficients. We propose two applications: feature-based alignment and deformable models.

5.1 Deformable Model: Automatic Segmenter

The ability to mesh different structures using the same parameterization has already been shown by Schroder's group [16] and Jaume [23] as well as through numerous other matching techniques. Once a common parameterization is formed, one often runs principle component analysis (PCA) on the node positions to determine the modes of variation.

We argue that detail coefficients are a much more meaningful representation than vertex coordinates. We believe it is far more likely to find statistical patterns in detail coefficients than node positions. We conclude this because:

- For a class of related shaped, we expect the low resolution representations to have little variance. It may be difficult to find this small variation in point co-ordinates.
- As the wavelet coefficients are stored in local coordinate systems, relative rotations of the system are not relevant. Conversely, principle component analysis on point sets can be very sensitive to alignment.
- One can imagine a bump that is elongated and rotated in some shapes, but not on others. It is very difficult for PCA on point data to find this kind of variation because of the scatter in the vertices. Conversely, PCA on the detail coefficients should be able to find the variations in such bumps.

We propose a segmenter using a multi-resolution mesh with prior probabilities on the detail coefficients. We envision aligning a low resolution mesh to the image and then allowing the lowest resolution vertices to be introduced, using a combination of the prior on the data and the prior probability of the coefficients. We can allow more and more detail coefficients to change at each vertex, eventually resulting in a segmentation of the data.

5.2 Feature detection/alignment

It should be possible to detect features on the basis of detail coefficients. A feature should have a signature set of hierarchical detail coefficients. A small protruding bump, for example, should always have positive detail coefficients in all levels.

We suggest defining features based on the detail coefficients that lead to a smoothed surface. The result would be roughly a tree of detail coefficients, with the base of the tree being the detail and the leaves of the tree being the smallest detail.

However, feature detection across arbitrary surfaces is likely very difficult. As each surface has a different triangulation, it is not clear what sort of specificity can be attained by trying to do feature detection. But we think that given a feature with enough structure (maybe a hand, or ear) it should be possible. One would need to study how the detail coefficients change with parameterization in order to make feature detection possible.

6 Summary

We examined the extension of wavelets from regularly sampled data to irregularly sampled data. The methods could be extended to the sphere in a straight forward way. However, there are difficulties in using the same methods on arbitrary triangulated surfaces. We reviewed two implementations of “wavelet” representations on arbitrary-triangulated surfaces. We argued that those methods were extremely effective for their applications, but were not actually wavelets. We concluded by noting an opportunity for a new direction of research: analyzing the detail coefficients in these wavelike-methods.

7 Acknowledgements

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A Continuity on Irregularly Sampled Points.

In “Wavelets on Irregular Point Sets”, Daubechies et al. [2], are concerned that up-sampling a function using an arbitrary point selection scheme might not lead to a smooth function (C2 continuous or continuous first derivative). Here, we show an intuitive way to understand their concerns. We do this by mapping the irregularly sampled points on to regularly sampled points.

Given a set of irregularly sampled points with x-coordinate $\{x_i\}$, find a smoothly varying, strictly increasing function $f(x) = u$ with $f(x_i) = i$. (One we can show for a given set of points, we had find a f arbitrarily close to being smoothly varying.) We can then find the wavelets in u space where there are evenly sampled points. However, the distance between points is space-varying according to $f'(x)$, the derivative of f. The bi-orthogonality relationship changes from $\int_x \phi(x-i)\psi(x-j)dx = 0$ to $int_u \phi(u-i)\psi(u-j)du/f'(x) = 0$ where $\phi(x)$ and $\psi(x)$ are the basis functions. Note that bi-orthogonality relationships with non-uniform inner products have been used with wavelets before. In fact, Sweldens [5] uses this type to solve partial differential equations.

Imagine that all of the x_i are spaced by 1 except two of the x_i which are arbitrarily close together, less than ϵ. The function $f(x)$ will have slope greater than $1/\epsilon$ in that region. $f'(x)$ will therefore change very quickly from 1 to $1/\epsilon$. In the limit of arbitrarily small ϵ, $f'(x)$ becomes discontinuous. When we subdivide our point sequence, we can make ϵ as small as we like by selecting points close together next to points spaced far apart. The orthogonality relationship in u space forces either at least one of $\phi(u)$ or $\psi(u)$ to be discontinuous. Since $f(x)$ is still continuous, when mapping $\psi(u)$ on to $\psi(x)$ or $\phi(u)$ to $\phi(x)$, the discontinuity remains. Thus, we have shown how an irregular sampling scheme could lead to discontinuous wavelet functions.

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