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Integrating Generative Growth and Evolutionary Computation for Form Exploration

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Abstract We present a novel means of algorithmically describing a growth process that is an extension of Lindenmayer's Map L-systems. This growth process relies upon a set of rewrite rules, a map axiom and a novel geometrical interpreter which is integrated with a 3D simulated environment. The outcome of the growth process is a digital surface in 3D space which has "grown" within and in response to its environment. We have developed a complementary Evolutionary Algorithm (EA) that is able to take over the task of generating the rewrite rules set for a growth process. Using a quantitative multi-objective fitness function that evaluates a variety surface properties, the integrated system (EA + Growth Process) can explore and generate diverse and interesting surfaces with a resemblance of organic form. The algorithms have been implemented to create a design tool for architects called Genr8.

1 Introduction

In this submission we describe how we have exploited an evolutionary algorithm (EA) to explore and generate computational definitions of growth processes. The growth processes are subsequently executed interactively with a modeled physical environment and instantiated as three dimensional (3D) digital surfaces. Our EA functions as a key exploration component within an open ended design assistance tool named Genr8 [14] [13]. Genr8 is a design tool for architects for surface generation that was developed by the Emergent Design Group (EDG) at MIT. Made up of architects and computer scientists, the group has an interdiciplinary agenda. The aim of the EDG is innovation in architectural design by exploiting and exploring new algorithms from computer science. In particular, the EDG is interested

in applying ideas from Evolutionary Computation (EC) and Artificial Life (ALife) to architectural design. Both EC and ALife use Biology as an inspiration to develop novel algorithms. Our aim is to utilize these concepts to generate biologically inspired form.

Natural form has always been important to architects and designers. To many people, natural form has a strong aestethic appeal and is a highly desirable property. Moreover, most natural structures are very efficient in terms of structural capacity and economy of materials. Unlike most man-made designs, they are robust to a wide range of failures and they can fulfill multiple functions. Natural structures are the result of millions of years of evolution and as designers we would like to be able to take advantage of Nature's strategies. There is a compelling argument that best way of achieving these goals is to mimic the natural processes that give rise to natural structures.

All organisms are created by growth from a single cell. Even though this phenomenon has been studied extensively by biologists, we are still far from a complete understanding of this complex process [10]. The process takes place over multiple time scales making quantitative descriptions notoriously difficult. Many of the mathematical descriptions can be traced to the seminal work by D'Arcy Thompson [43]. In his inspirational book he uses physical principles to shed light on biological form. Another influential paradigm is that of Cellular Automata (CA) which was proposed by von Neumann as a way of describing self-replicating systems [46]. A third approach is Lindenmayer systems (L-systems) which were first introduced by biologist Aristid Lindenmayer as a phenomenological description of the growth of yeast cells. L-systems have since been successfully used to describe plant growth [33]. Unlike the work of Thompson, CAs and L-systems are algorithmic descriptions of a growth process.

Hornby and Pollack [15] have demonstrated that one can achieve more powerful results by employing a generative representation rather than a direct representation of the design. They suggest that one should evolve rules for creating an object rather than creating the object itself directly. This is in accordance with biology, where the genome should be thought of as instructions for how to build and maintain an organism rather than the exact blueprint of the organism. For this reason, L-systems are of obvious interest since they express growth using a set of rewrite rules which can be represnted by a context-free grammar.

We proceed in the following manner: In Section 2 we discuss algorithms that have been used to generate computational versions of growth and EAs in the context of architecture. In Section 3, we provide an overview of the Genr8 system architecture. In Section 4 we describe the rewrite systems and geometrical interpreter we have designed to "grow" surfaces in 3D. In Section 5 we describe how we extend Grammatical Evolution [31] so that we can exploit an EA to explore and discover rewrite systems. In Section 6, we show Genr8's surfaces that are the products of our EA and generative growth system integration.

2 Related work

The use of growth-like algorithms has been very successful in a wide range of areas in science and engineering, including but not limited to, computer graphics [38] [33] [17], neural networks [12] and analog circuit design [22]. In this brief review, we will focus on applications of growth algorithms and EAs in architecture. The use of EAs to produce artistic works has been reviewed by Johnson and Romero [19]. A recent review of aesthetic evolution of L-systems can be found in [27]. Applications of EAs to art and design are also presented in the collections edited by Peter Bentley [24] [4].

The ideas of using generative algorithms and procedures in architecture is not new. One of the early pioneers was John Frazer who began his work at the Architectural Association in the 1960's [11]. The title of his book "An Evolutionary Architecture" conveys his interest. It investigates "fundamental form-generating processes in architecture, paralleling a wider scientific search for a theory of morphogenesis in the natural world". His early projects include the Reptile structural system which incorporates notions of growth and evolution, albeit without an explicit specification from a formal language. The basic strategy was to use growth involving a small set of tiles repeatedly (rep-tile) starting from a minimal seed. His group has also evolved surfaces, although it used a mathematical description rather than growth language. The fact that his EA based form generating experiments are implemented physically allows them to be "understood in architectural terms as an expression of logic in space". Frazer's digital simulation efforts were impeded by the computing resources available at the time. Instead, many of his projects were implemented as physical devices using custom-built hardware, sensors and actuators. The Generator project uses embedded electronics in each component to create a building with distributed intelligence. The result is a reconfigurable space which can respond to varying needs. The most facinating of Frazer's many projects is the Universal Constructor, a working model of a self-organizing interactive environment. The project is a physical 3D CA, realized using custom-built cubes which can respond to the environment and interact with the user. In another example of environmental influence the group developed tools to help architects understand the impact of the geometry of the sun. The tools went beyond the standard, poorly understood stereographic projections. Frazer has also embraced the idea of creative design tools [47] [5] and in particular he has advocated the use of EC emphasizing exploration rather than optimization [18].

CAs have been used by other architecture researchers to generate form and structure. Kicinger et al [20] used a CA representation of topologies for structural steel systems in tall buildings to evolve efficient structures. CAs have also been used to generate form in more architectural applications. One example of how to interpret the result of a CA is to uses Conway's Game of Life and "stack" the planes to get obtain a 3D form [23] [7].

L-systems have also been used in previous architecture applications, in particular by the group lead by Paul Coates. They have combined L-systems and an EA to create form on an iso-spatial grid. Using an environment to simulate for example sun and wind, they evolve structures that are optimized for certain performance criteria such as enclosing space. Their work also explores how the biological concepts of symbiosis and co-evolution can be incorporated in a form-generating algorithm. They point out that evolutionary optimization would probably have had a strong appeal to modernist architects, such as Le Corbusier and Van der Rohe, since the aesthetics of the outcome is purely based on the function [6]. The EDG has also explored the potential of L-systems in the design tool MoSS [42]. A more design-oriented investigation of L-systems was pursued by Hornby and Jackson [15]. They developed a generative design system called GENRE which has been applied to table and robot design. Moreover, they conduct quantative experiments to demonstrate the advantage of generative over non-generative encodings. The generative approach is more compact and it makes it easier to evolve re-usable design modules.

A dominant way of describing architectural form is shape grammars [39] [30]. They have been combined with EAs [3]. EAs are a popular method to generate form and many of the previously mentioned authors have incorporated such a component in their work. Further applications include the evolution of floor plans [35] [28] and 3D design of buildings inside a CAD tool [45].

3 Overview of Genr8

One goal of the Genr8 project is to demonstrate that a combination of a growth algorithm and EA is useful for form exploration within the architectural design process. As a proof of concept, the algorithms have been implemented in C++ and thereby we have developed a software tool which can be (and is) used in educational practice. This section gives an overview of the software system, which has two main components: the HEMLS growth engine and the EA (see Figure 1). The growth engine uses the HEMLS interpreter to parse a rewrite system. It geometrically interprets the axiom and set of rewrite rules of the parsed system. A rewrite rule set (or system) is a context-free grammar. We name the combination of HEMLS interpreter with Genr8's rewrite rule set syntax and semantics a HEMLS: Hemberg Extended L-System and dub the results HEMLS surfaces. The HEMLS growth engine's growth process is strongly influenced through its interaction with a computationally simulated physical environment. This environment is the architect's means of abstractly imposing influence on the growth and representing design space elements which should interact with the growth process. It is important to emphasize that the tool can be (and has been!) used with the growth algorithm alone.

The EA is an optional, added-value feature which leverages Genr8's expressive power by automatically generating and evaluating a large number

of rewrite rule sets. Of note is the fact that Genr8 evolves *instructions* for growing surfaces rather than the surfaces themselves. The EA's selection and variation components provide the principles of design evolution. It gives the architect an additional perspective: that of parallel, ancestral traversal through a space of multiple designs. The procedural loop of the EA is standard. Iteratively a new population is generated through selection and mutation of parents of the current one, then each member is evaluated for fitness. Genr8 employs a linear genome of integers as per Grammatical Evolution [31]. The genome is mapped to a set of rewrite rules (i.e. context free grammar) with the auxiliary aid of a Backus-Naur Form representation that defines the syntax of the context-free grammar. Then the surface is grown by the growth engine in the simulated environment using the interpreted set of rewrite rules. It is the resulting surface that is evaluated for fitness.

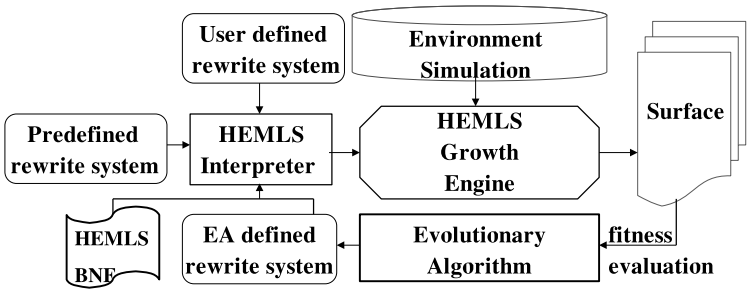

Fig. 1 An overview of the Genr8 system. At the heart of the system is the growth engine which is algorithm for growing HEMLS surfaces. The growth process is influenced by the environment. The system contains a parser (HEMLS interpreter) that interprets a rewrite system. The user can supply a hand written rewrite system or use a predefined one. In these cases, the EA component is inactive. The EA is used to automatically generate rewrite systems. The genome of each individual in the EA is mapped to a HEMLS rewrite system using Grammatical Evolution and a Backus Naur Form definition of a HEMLS rewrite system. The surface corresponding to the genome is subsequently interpreted and grown then evaluated for fitness.

We chose to implement Genr8 as a plug-in for Maya. It has a well-documented Application Programmer's Interface (API) which is easy to work with. From a development perspective, a tremendous advantage offered by Maya is the pre-existing modeling capabilities for visualization and user interaction. Its very powerful framework saved uncountable hours of development time. The advantage to Genr8's users is that it is easier to learn how to use the tool since it is part of the Maya environment. Genr8's data object level integration within Maya implies a HEMLS surface is also a regular addressable Maya surface which very advantageoulsy allows a user to manipulate it in the usual Maya ways. A compiled version of Genr8 as

well as the source code is available on the web[1]. The implementation is not Maya-specific and in principle it could be implemented as a plug-in for another 3D modeller or as a stand alone software.

4 Extending L-systems to obtain a model capable of generating surfaces

Our aim is to create surfaces with qualities that are reminiscent of those in natural forms. To achieve this goal we employ an algorithm which is an extension of L-systems. Even though L-systems have been successfully used to model trees and flowers (see Figure 2) [33], it should be pointed out that they are purely phenomenological and not derived from the physical properties of a growing plant. This is of little concern to us since we are interested in creating a tool which is useful to designers. Thus, the growth model presented in this paper should be appreciated from a metaphorical point of view and not as a faithful description of biological growth.

Fig. 2 Left, a rendered image of a flower created using L-systems. Centre, a photograph of the the gametophyte *microsorium linguaeforme*. Right, a map L-system used to model it the gametophyte in the middle [33].

We begin by presenting ordinary L-systems and show how they can be extended to create surfaces in 3D space. Special focus is placed on the context-free grammar used to describe the L-systems since it plays a central role in Genr8. First we introduce notation from formal language theory [36, 41, 33].

Definition 1 (Grammar) *A* ***grammar*** *G is a finite specification of the (possibly infinite) sets of sentences of a language. It can formally be expressed as a tuple* $G = \{N, T, S, P\}$ *where:*

- N *is a finite set of* ***non-terminals****.*
- T *is a finite set of* ***terminals****.*
- S *is a* ***start symbol*** *such that* $S \in N$*.*

[1] http://projects.csail.mit.edu/emergentDesign/genr8/

– *P is a set of* ***productions*** *(or* ***production rules*** *of the form $B \rightarrow \alpha$, where $B \in N$ and α is a string of symbols from $N \cup T$.*

There is a hierarchy of four different grammars; unrestricted grammars, context sensitive grammars, context free grammars and regular grammars [8]. The difference between the grammars is that they have increasingly stricter productions and that they can express fewer formal languages. Context free grammars are of particular interest since they are powerful enough to describe most programming languages. To express context free grammars, a syntactic metalanguage known as Backus Naur Form (BNF) is often used. BNF is useful for formal descriptions of a language since it provides a succinct representation using a limited number of rules. The BNF notation has a number of advantages that makes it useful as a syntactic metalanguage; it is concise, precise, natural, general, simple, self-describing and linear. The last point can also be considered the biggest limitation of BNFs, they can not express non-linear grammars and they are often inadequate for defining more complex grammars. Fortunately, they are relatively easy and natural to extend if one needs to handle more complex situations [16].

A BNF specification uses a number of **derivation rules** written as `<Non Terminal> ::= <String of Non-Terminals and Terminals>`. This should be interpreted in such a way that the left hand side can be replaced with whatever is on the right hand side. In 1977, Wirth [48] introduced a number of metasymbols to further facilitate the use of BNFs. The symbols that are relevant to this paper are presented in Table 1.

Table 1 Meta symbols used to describe a BNF [41].

Symbol	Meaning
|	A vertical bar is used to denote alternatives.
[ ]	Square brackets denote the optional appearance of a symbol or a group of symbols.
{ }	Curly braces indicate that we can have zero or more repetions of a symbol or a group of symbols.

4.1 Lindenmayer systems

In 1968, biologist Aristid Lindenmayer introduced L-systems as a way to describe the growth of multi-cellular organisms. L-systems are based on rewrite systems, a concept invented by Thue [2]. A rewrite system consists of a seed and a number of rewrite rules that are repeatedly applied to the string. Theoretical computer scientists soon took an interest in L-systems since they are similar to context free grammars. In this paper, we use a definition of L-systems from [33].

Definition 2 (L-system) *A **0L-system** (also called a context-free or parametric L-system) is an ordered triplet $G = \{V, \omega, R\}$ where*

- *V is an alphabet.*
- *$\omega \in V^+$ is a **seed** or axiom.*
- *$R \subset V \times V^*$ is a finite set of **rewrite rules** (these are often called productions, but we shall avoid that term here since it clashes with Definition 1). The notation $a \rightarrow \alpha$ is used for rewrite rules where the letter $a \in V$ is called the **predecessor** and the string α is called the **successor**. It is assumed that there exists at least one rewrite rule for each letter $a \in V$ and if none is specified, the identity rule $a \rightarrow a$ is implicitly assumed.*

*A 0L-system is **deterministic** iff there is exactly one successor for each predecessor.*

The string produced by L-systems can be interpreted as a topological representation of a tree-like structure. This makes them useful for modelling plants and one can capture the complex geometry of a plant in a few lines using the L-systems formalism. To produce plant images, a geometric interpretation is required. The geometric interpretation is independent of the generation of the string. The most widely used geometric interpretation based on turtle graphics [1] was introduced by Prusinkiewicz [33]. Briefly, the idea is to interpret the string of symbols generated by the L-system as instructions for an imaginary turtle moving a stylus in 3D space.

4.2 Map L-systems

To study cellular development of organisms that do not have a tree-like topology, Map L-systems were introduced by Lindenmayer and Rozenberg [26]. Map L-systems are L-system type grammars that are applied to maps. The parallel map generating systems introduced by Lindenmayer and Rozenberg are formally referred to as binary propagating map 0L-systems, or BPM0L-systems for short. To convey how Map L-systems differ from ordinary L-systems, we will give an informal definition from [25] (a formal definition of maps can be found in [44]).

Definition 3 (Map) *A map consists of **vertices**, **edges** and **regions**. Edges are ordered pairs of vertices and regions are bounded by finite sequences of edges. Each edge has one or two vertices at its end and each vertex is associated with one or more edges. All edges and vertices lie on boundaries of regions. Thus, a map is a connected structure without isolated islands or protruding vertices.*

An L-system is effectively a string rewriting system where the geometric interpretation is independent of the grammar. Map L-systems on the other hand can not be separated from the map topology and they can not be considered as one dimensional strings alone. However, only the topology is required: there is no need to specify spatial coordinates or quantitative relationships of the node and edge positions.

4.3 Hemberg Extended Map L-systems

To obtain a description of surface growth which is more suited to Genr8's needs, we have extended the Map L-systems model. HEMLS allow a wide variety of surfaces to be grown in 3D (unlike Map L-systems that are confined to 2D). From a formal perspective, HEMLS can be described as map rewriting systems. Unlike L-systems, HEMLS can not be considered simply as rewrite systems, the geometric interpretation is an integral part of the definition of HEMLS (specifying only the topology as in Map L-systems is not sufficient).

Definition 4 (Hemberg Extended Map L-systems) *A context-free HEMLS consists of*

- *A finite set of edge labels and turtle commands* Σ.
- *A* ***seed*** *or* ***axiom*** ω.
- *A set of edge* ***rewrite rules****,* R*. Each rewrite rule is of the form* $a \rightarrow \alpha$*, where the edge* $a \in \Sigma$ *is called the* ***predecessor****. The string* α *is called the successor and consists of one or more symbols from* Σ*. It is assumed that there is at least one production for each edge* $a \in \Sigma$*. If no production is explicitly specified, it is assumed that the identity production* $a \rightarrow a$ *belongs to the set of rewrite rules.*
- *Two numerical values in the interval [0, 90] that specify a* ***turn angle*** *and a* ***branch angle****. There is also a boolean variable,* ***sync****, that determines the method for joining the branches (discussed below).*

In the following, we shall use the term HEMLS rewrite system or simply rewrite system to denote a sentence specifying a seed, a set of rewrite rules and parameters (i.e. what is included in Definition 4). To grow a HEMLS surface, the seed is first placed in the starting position. Next, a number of **derivation steps** take place. Each derivation step consists of:

1. **Increase size**. A displacement vector is calculated for each vertex. The vector originates from the center of the surface. Each vertex is moved along the line defined by the displacement vector **r**. The distance moved is determined by a **scale factor** s. We allow for $s < 1$ which means that the surface can shrink as well. Vertices are also affected by the environment which may result in further modifications or truncations of the growth displacement vector (see Section 4.7).
2. **Apply rewrite rules**. Each rewrite rule in R is applied to edges with corresponding labels. The edge is replaced by the successor starting from the start vertex. The predecessor is divided into a number of new edges as indicated by the rewrite rule. This step is similar to the derivation step in Map L-systems. Edges that are drawn between one or more push-pop pairs will not have an end vertex. These edges are called **branches** and in order to restore the map topolgy, they will be removed or pairwise joined into complete edges.

3. **Join branches**. There are two modes for joining the branches, synchronous or asynchronous. The latter is default and the former is used if the keyword `sync` is included in the rewrite system. In synchronous mode, all the rewrite rules are applied before the branches are joined. If the keyword is not used, branches are joined after each rewrite rule has been applied. Branches that could not be joined are kept through the remainder of the derivation step which means that they will have several chances of getting joined. When joining branches all pairs of branches are checked to see if they fulfill the criteria for forming a new edge together. To join two branches, the following criteria must be fulfilled:
   - The branches must appear in the same region.
   - They must have the same type.
   - The scalar product of their orientation must be less than a given tolerance. The tolerance is given as an angle by the `BranchAngle` keyword.
   - The branch points are within a specified maximum distance (which can be set to infinity).

   When all rewrite rules have been applied and there has been one or more attempts to connect the branches, any remaining unconnected branches are removed.

4.4 Example of a HEMLS rewrite system

In Genr8, there are two pre-defined rewrite systems that are considered especially interesting and useful. The rewrite systems start from a square or an equilateral triangle and proceeds by subdividing the original shape into four new squares or triangles. The rewrite system for the squares is shown in Table 2. Here the first line is the seed, derived from the `<Axiom>`

Table 2 The rewrite system for the squares shown in Figure 3.

ω	A + B + A + B
A →	A [[+ B] - B] A
B →	B [[+ A] - A] B
Angle	90
BranchAngle	90

non terminal. Lines `2` and `3` are rewrite rules and the fourth line specifies the turn angle for the turtle. Figure 3 shows the geometric interpretation and application of these rules.

4.5 HEMLS variants

There are a number of variations and extensions of L-systems in the literature [33]. These can easily be applied to HEMLS as well to provide a more

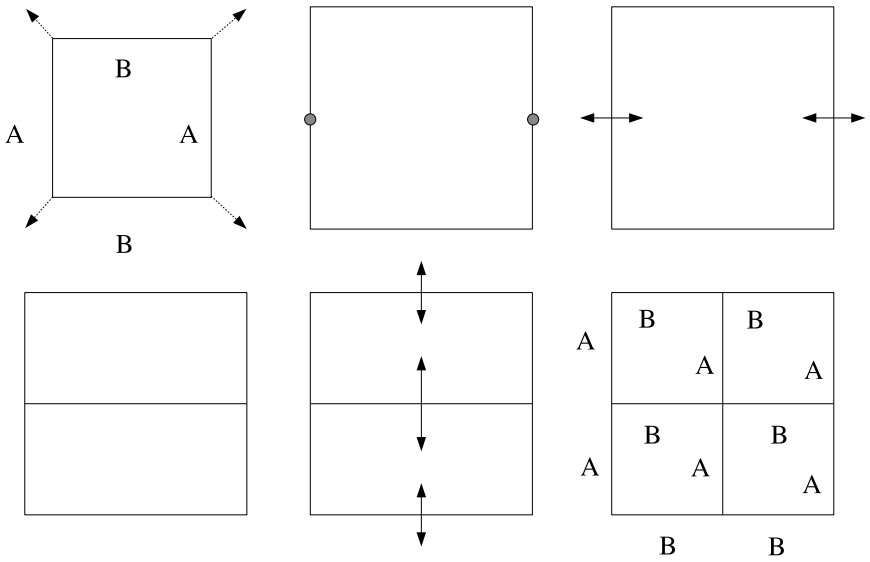

Fig. 3 One derivation step of the rewrite system in Table 2. The square to the upper left is the seed and the arrows indicate how the vertices should be moved when increasing the size. In the next step, the A edges have been split and the new vertices are indicated by circles. Next, the branches are drawn and connected. The same procedure is applied to the B edges in the middle panel on the bottom. The figure on the bottom right shows the surface after one iteration of the rewrite rules.

powerful growth model. A natural extension of L-systems is to make them context-sensitive, that is rewrite rules require a specific context in order to be activated. To represent this, we use the notation $A < B > C- > \alpha$, where A and C are edge labels that need to be connected to the start and end node respectively. When applying the rules, the directions of the edges is not taken into account.

Another interesting variant is to introduce stochasticity in such a way that there are more than one possible rewrite rule that can be applied to each edge. A probabilistic HEMLS is specified by a tuple $G = \{\Sigma, \omega, R, \pi\}$ where π is a probability distribution mapping the set of rewrite rules to the set of production probabilities. Every time a rewrite rule is applied to an edge, one of the available rewrite rules is randomly chosen based on the probability distrubtion π.

Timed L-systems allow us to change the rewrite rules that are applied to edges over time. This can for example be used to model plants that first grow a stem and then a flower. This feature is achieved by adding an **age** parameter to every edge (represented as a subscript). Again, each rewrite-rule can have multiple successors and the age of the edge determines which one is chosen.

4.6 The HEMLS BNF in Genr8

Genr8 includes a parser that can be used to grow surfaces specified by a HEMLS supplied by the user within a text file. In order to make the file easy to parse the general description presented above had to be restricted. The grammar is represented in Figure 4 using the BNF notation.

```
 (1) N = { <L-System>, <Axiom>, <RewriteRule>, <Predecessor>,
 (2)       <Successor>, <Modifier>, <Condition>, <Segment>,
 (3)       <Constant>, <Weight> }
 (4) T = { +, -, &, ^, \, /, ~, '[', ']', '<', '>', ->, Edge,
 (5)       Angle, Sync, Edge_i, Edge_i+1, Edge_i-1, If, i, '='
 (6)       BranchAngle, '0', '1', '2', ... }
 (7) S = { <L-System> }
 (8) P = {
 (9) <L-System> ::= <Axiom> ';' <RewriteRule> ';'
(10)                { <RewriteRule> ';' } Angle <Constant>
(11)                ';' [ Sync ';' ] BranchAngle <Constant>
(12) <Axiom> ::= <Segment> [ ~ ] + <Segment> [ ~ ] + <Segment>
(13)             <Segment> { [ ~ ] + <Segment> }
(14) <RewriteRule> ::= <Predecessor> -> <Successor> [ <Condition> ] |
(15)                   <Predecessor> -> <Successor> [ <Weight>
(16)                   <Constant> ] { -> <Successor> [ <Weight>
(17)                   <Constant> ] }
(18) <Successor> ::= { <Modifier> } <Segment>
(19) <Predecessor> ::= <Segment> { <Segment> } |
(20)                   <Segment> '<' <Segment> |
(21)                   <Segment> '>' <Segment> |
(22)                   <Segment> '<' <Segment> '>' <Segment>
(23) <Modifier> ::= { <Segment> } |
(24)                + <Modifier> - |
(25)                - <Modifier> + |
(26)                & <Modifier> ^ |
(27)                ^ <Modifier> & |
(28)                \ <Modifier> / |
(29)                / <Modifier> \ |
(30)                ~ <Modifier> |
(31)                <Modifier> [ <Successor> ] <Modifier>
(32) <Segment> ::= Edge | EdgeX | EdgeY | EdgeZ | Edge_i |
(33)               Edge_i+1 | Edge_i-1
(34) <Condition> ::= If i '<' <Constant> |
(35)                 If i '<' <Constant> |
(36)                 If i '=' <Constant>
(37) <Constant> ::= 0 | 1 | 2 | ...  }
```

Fig. 4 The BNF representation of the language for describing HEMLS that can be understood by Genr8's parser.

This description merits some comments since there are a few features that could not be captured by the BNF. The first three lines list all the non-terminals (`N`) used in the HEMLS grammar. Next is the set of terminals (`T`) and the start symbol (`S`). Lines 8-37 show the derivation rules (`P`) which describe how to construct a rewrite system. The expansion of the start symbol, `<L-system>`, gives the overall structure of a rewrite system with an `<Axiom>` that describes the seed, one or more `<RewriteRule>` and finally the parameters (the semicolons indicate end of line). Each `<RewriteRule>` consists of a `<Predecessor>` and one or more `<Successor>`. Lines `20-22` show how context sensitivity can be introduced. The successor is a string of turtle commands that are used to replace the predecessor. The turtle commands are generated using the `<Modifier>` non-terminal. There are two constraints on the grammar imposed by the geometric interpretation. First, there must be an equal number of push and pop symbols. This constraint is handled by the production rule on line `31`. Second, when rewriting an edge, the turtle must return to the position where the old edge ended. Thus, the number of left turns must equal the number of right turns, etc. Lines `24-29` makes sure that this balance is preserved.

The `<Segment>` non-terminal is used to generate the `Edge` terminals. A couple of things needs to be said about this terminal since it is not a terminal in the strict sense. The conventional L-system notation for lines is capital letters. To make it easier to parse, Genr8 uses a nomenclature where edges are identified by the keyword `Edge` followed by a non negative integer that defines the **type** (or **ID**), eg `Edge0`. All terminals starting with `Edge` require a parameter specifying the ID. There are other versions of the `Edge` terminal and they all require a type. The variations on the `Edge` terminal, `Edge_i`, `Edge_i+1` and `Edge_i-1` are used for time-varying HEMLS where the subscript represents the age of the edge. The counter is initialized to 0 when the terminal is created and can be incremented by the rewrite rules.

4.7 Environment

In nature, the environment plays an important role in the growth of an organism. The obvious way to alter the resulting surface in Genr8 is to modify the rewrite system. In practice it is often easier and more intuitive to use the alternative approach of changing the environment. One example of environmental influence on growth is *tropism*, which can be defined as the response of an organism to external stimuli. A simple example of tropism is a tree that turns its branches towards the sun as it grows. We wish to mimic this type of reactive influence of the environment in the growth of HEMLS surfaces. Again, we proceed by adopting existing ideas from the L-systems literature.

In Genr8 there are two different environmental features, **forces** and **boundaries**. These are easy and intuitive to use and by combining them one may produce non-linear and unexpected outcomes. There are three

different kinds of forces; **attractors**, **repellors** and **gravity**. The metaphor underlying attractors and repellors is a magnet which forces the surface to grow towards or away from it. When moving a vertex during the derivation step, the new position is modified by the forces in the environment. For each vertex a resultant force vector is calculated by summing the forces from all environmental features. The default in Genr8 is to have the magnitude of the force depend on the inverse square of the distance between the vertex and the attractor/repellor. The resulting vector **f** is added to the displacement vector **r** to find the new position of the vertex. Two adjacent vertices can potentially move different distances and in different directions and thereby distort the surface.

Boundaries are used to constrain the growth by preventing vertices to move through them. Boundaries can be very powerful as they can be used both as enclosures or as obstacles (as in the left panel of Figure 5). The environment can have a profound impact on the growth due to the non-linear interactions. It is almost impossible to predict the exact outcome when there are more than two elements in the environment. This is clearly seen in Figure 5 which shows two examples of the square rewrite system from Table 2.

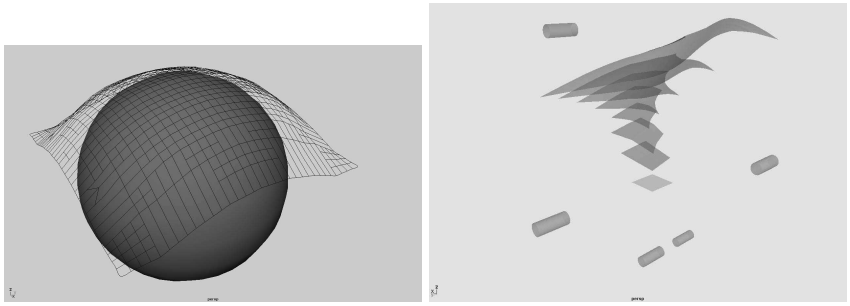

Fig. 5 The figure shows two examples of the square rewrite-system from the example in Table 2 grown in two different environments. Left, the seed was placed above the sphere and pulled down by gravity as it grew. The surface was prevented from growing through the sphere and instead it drapes the sphere. Right, the surface is pushed upwards by the two repellors beneath it. During the development it was further deformed by the repellors (the five repellors are located to a single point, but drawn as cylinders). The figure shows all derivation steps overlaid and the shape of the seventh surface is far from the original flat square.

5 Evolutionary computation

Creating a rewrite system that grows interesting surfaces by hand is a very hard task. This stems mainly from the difficulties of imagining what a given rewrite system will look like after repeated iterations. The influence of the

environment only serves to exacerbate this problem. There is also the additional complication of making sure that the rewrite system is syntactically correct. It is with these concerns in mind that we have added a an EA to Genr8. The EA will automatically create valid rewrite rules and the user can exert high-level control over the process through the fitness function. This has the benefit that the user does not have to know anything about HEMLS or how the surfaces were created. It is merely required that the user has a conceptual understanding of the nature of EAs and the growth process.

5.1 Grammatical Evolution

There are many variants of the EA paradigm particularly in how they represent the genome or how they handle selection. The specific EA used in Genr8 is called Grammatical Evolution (GE). It was introduced by Ryan and O'Neill in 1997 [31] and it combines the strengths of genetic algorithms (GAs) [29] and genetic programming (GP) [21]. What makes GP such a powerful algorithm is that it allows us to evolve executable structures represented as trees directly. Unfortunately, the genetic operations often become very complicated as one has to deal with subtrees which must be ensured to be compatible when swapped. GAs on the other hand are very convenient when applying the genetic operators since the genome is represented as an array of integers. The drawback is that one is restricted to work with integers. GE uses the same representation of the genome as GAs. The significant invention in GE is that it can be applied to *any* language whose grammar can be represented in Backus-Naur Form. When interpreting a genome, the genes are used to select production rules as the sentence is expanded (see example in Section 5.2). The expansion of the genome via the BNF can be viewed as a tree similar to those used in GP. When applying genetic operators to these structures there would be considerable overhead if we had to ensure that the swapping of two subtrees results in a syntactically correct outcome. Thus, GE provides a strict separation between the genetic operations and the language that we wish to use. The constraints of the language are automatically and efficiently handled by the BNF representation of the grammar.

The BNF used by the EA is more restricted than the one that Genr8's parser can handle (Figure 4). This is because we want to facilitate the search. The full BNF provides too many options and we have found that the EA finds interesting results faster using the restricted BNF. An additional reason for restricting the BNF for the EA is that some of the non-standard features of the BNF would be quite complicated to implement with GE. Despite the simplifications, the BNF in Figure 6 is still more complicated than the ones mostly used in GE. For this reason, we had to extend the standard GE in order to handle the more complicated features of the BNF. Below we present our extensions and modifications of GE.

```
 (1) N = { <L-System>, <Axiom>, <RewriteRule>, <Predecessor>,
 (2)       <Successor>, <Modifier>, <AngleValue>, <BranchAngleValue> }
 (3) T = { +, -, &, ^, \, /, ~, '[', ']', '<', '>', ->, Edge, ';',
 (4)       Angle, Sync, EdgeX, BranchAngle, 15, 30, 45, 60, 75 }
 (5) S = { <L-System> }
 (6) P = {
 (7) <L-System> ::= <Axiom> ';' <RewriteRule> ';'
 (8)                { <RewriteRule> ';' } Angle <AngleValue>
 (9)                ';' [ Sync ';' ] BranchAngle <BranchAngleValue>
(10) <Axiom> ::= Edge [ ~ ] + Edge [ ~ ] + Edge [ ~ ] + Edge
(11)             { [ ~ ] + Edge }
(12) <RewriteRule> ::= <Predecessor> -> <Successor>
(13) <Successor> ::= { <Modifier> } <Segment>
(14) <Predecessor> ::= Edge { Edge } |
(15)                   Edge '<' Edge |
(16)                   Edge '>' Edge |
(17)                   Edge '<' Edge '>' Edge
(18) <Modifier> ::= { Edge } |
(19)                + <Modifier> - |
(20)                - <Modifier> + |
(21)                & <Modifier> ^ |
(22)                ^ <Modifier> & |
(23)                \ <Modifier> / |
(24)                / <Modifier> \ |
(25)                ~ <Modifier> |
(26)                Edge '[' '[' + EdgeX ']' - EdgeX ']' Edge |
(27)                Edge '[' '[' + + EdgeX ']' - - EdgeX ']' Edge |
(28) <AngleValue> ::= 30 | 45
(29) <BranchAngleValue> ::= 15 | 30 | 45 | 60 | 75
```

Fig. 6 The BNF for HEMLS used by the EA. It is more restricted than the one in Figure 4 to speed up the search. There are fewer non-terminals in the restricted BNF. The `<Segment>` non-terminal is no longer used since we are no longer using the option of time-dependent rewrite-rules. Furthermore, we have removed the timed and probabilistic extensions and we have restricted the choice of angle values.

In BNF notation, the symbols { ... } indicate that the string appearing between them can be repeated zero or more times (the symbols [...] have a similar function in that the string can be used once or not at all). This feature is used extensively in Genr8, for instance it allows us to have an arbitrary number of rewrite rules (line 6 in Figure 6). Because of the importance of genetic inheritance and the propagation of genetic characteristics in an EA, it is important that this optional quality is consistent through all decodings of the genotype (i.e. from one generation to the next or for multiple copies of a genotype in the population). Moreover, we want a scheme where there is no fixed upper limit on the number of times the string is

used. The best way to achieve this consistency is to use the genotype itself to determine how many times a string should be expanded. In Genr8 this is accomplished with a simple algorithm that works as follows:

1. When a { is encountered, a counter is initialized to zero.
2. Read a gene from the genome and if `gene mod ( counter + 2 ) <= counter` do not write any more of the optional symbols.
3. Write the string of optional symbols (until the }).
4. Increase the counter and go back to step 2.

The above scheme is deterministic for identical sequences of genes and there is no fixed upper limit on the number of expansions although the probability of adding to the expansion decreases with the number of strings expanded. The constant `2` in the second step above can be adjusted to change the expected number of times we will go through the loop.

The reader may recall that the `Edge` terminal requires an additional integer parameter. When the expansion of the rewrite system results in an `Edge`, we use a scheme similar to the one for multiple terminals to select the ID for the `Edge`. The system has counter, `max_ID` that keeps track of the total number of edge IDs used so far. When an `Edge`symbol is encountered, the ID is determined by the value of the next gene and `max_ID`, through the relationship `ID = geneValue mod ( max_ID + 1 )`. As for the optional number of symbols, this scheme enable the system to introduce new edge types at a marginally decreasing rate.

Another heuristic is introduced by the `EdgeX` terminals. If a production contains an `EdgeX` terminal all those terminals will get the same ID in a manner that is akin to variable binding in PROLOG. This is used in lines `26-27` where it is ensured that the branches will always come as pairs with the same ID. This makes it more likely that the branches will join in the rewrite system. Joining branches is crucial since they provide the internal structure of the surface. If the branches do not join, the surface will simply consist of a perimeter, making it very uninteresting.

When the ID of the predecessors is chosen randomly, it is likely that the rewrite system will include edge types without an explicitly defined rewrite rule. This is often undesired since these edges will not be modified during the growth process. To overcome this issue, we have introduced a repair mechanism in the Genr8 EA. When the rewrite system has been expanded, the algorithms checks to make sure that all edge types have an explicit rewrite rule. If not, the rewrite system is extended by the addition of new rewrite rules.

One problem of using GE to map the genome to a rewrite system is that the expansions tend to get very long. This is not a problem that is inherent to GE, but an effect of the HEMLS grammar. In particular, it is caused by the productions for the `<Modifier>` non-terminal (lines `18-27` in Figure 6) where seven of the ten productions contain a new `<Modifier>`. To combat this hefty expansion, Genr8 includes a new mechanism for restricting the length of the expanded grammars. The expanded rewrite system can be

viewed as a tree where the leaves are terminals and the internal nodes are non-terminals (see Figure 7). We can keep track of the depth of the tree during the expansion to measure how large the tree is. When the expansion reaches a specified limit `max_depth`, the expansions are brought to a halt. This is achieved by changing the way productionas are chosen. Instead of choosing from all the production rules, only the subset of rules that do not contain any non-terminals are used For the `<Modifier>` it means that only lines `17` and `26-27` will be used. This scheme allows the user to control the size of the rewrite systems generated by the EA while at the same time making sure that the genome will be consistently mapped.

Because of the use of GE and BNF notation, it is powerful yet simple to enable the EA to evolve additional classes of rewrite rule sets. For example, we have implemented two additional grammars in Genr8 that are minor variations of the one in Figure 6. The first of these grammars allows the EA to evolve probabilistic rewrite rules. The second grammar is more constrained than the default grammar and generates symmetrically balanced surfaces. It ensures that there will always be at least one axis of symmetry if the surface is undistorted by the environment. To achieve this, the seed is enforced to be symmetric and two new terminals, `EdgeY` and `EdgeZ` analogous to `EdgeX` are introduced. The grammar enforces operations prescribed by the rewrite rules to preserve the symmetry.

5.2 Example expansion

To illustrate the mapping of a genome to a rewrite system, we shall give a brief example, showing part of the expansion procedure. The genome that we shall be mapping starts with

$$212, 187, 632, 832, 800, 517, 338, 39, 878, 185, 954, 863, ...$$

To expand a rewrite system, we begin with the start symbol `<L-system>`. There is only one production rule, so the gene value (`212`) is irrelevant in this case. The rewrite system is expanded to

```
<Axiom> ';' <RewriteRule> ';'{ <RewriteRule> ';' }[ Sync ';' ]
Angle <AngleValue> ';' BranchAngle <BranchAngleValue>
```

Next, the axiom is expanded and again, there is only one production available. We proceed to determine the type of the first `Edge` in the `<Axiom>`. When the EA is initialized, there are two edge types (with IDs `0` and `1`), so the type of the edge is determined by `632 mod 3 = 2` (thus introducing a new edge type). There are two more `Edge` terminals and their types are determined by `832 mod 4 = 0` and `800 mod 4 = 0`. The next four symbols are { `'+' 'Edge'` } indicating that the sequence `'+' 'Edge'` should occur zero or more times. A counter to keep track of how many times the string has featured is initialized to zero. The counter is used to determine

whether or not another string should be expanded. The test for adding another string is `gene mod (counter + X) > (counter + Y)` where `X` and `Y` are parameters that can be adjusted to determine the expected number of expansions. In Genr8, `X = 0` and `Y = 2` which gives `517 mod 2 = 1 > 0`, ie another `'+'` and `Edge`should be added. The type of the `Edge` is set to `338 mod 4 = 2` and testing for further expansions we find that `39 mod 3 = 0 < 1`. This yields a rewrite system of the form:

```
Edge2 + Edge0 + Edge0 + Edge2
<RewriteRule> ';'
{ <RewriteRule> ';' }
[ Sync ';' ]
Angle <AngleValue> ';'
BranchAngle <BranchAngleValue> ';'
```

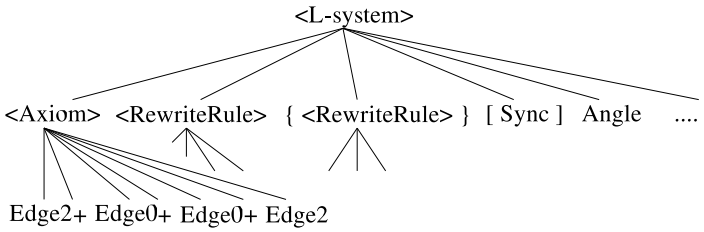

Fig. 7 An example of how the expansion of the genome via the BNF can be viewed as a tree (from Section 5.2). Internal nodes are non-terminals and the rewrite system can be obtained by reading the terminals at the leaves.

The expansion of the rewrite systems can be graphically viewed as a tree as shown in Figure 7. So far we have expanded the axiom of the rewrite system and we see that it corresponds to a square with two types of `Edge`s.

5.3 Fitness Evaluation

A crucial part of an EA is the fitness evaluation which guides the search towards better solutions. In design, there is no general way of algorithmically defining a 'good' surface. Coming up with a useful fitness evaluation scheme for design applications is still an open research question [34] [40]. The most common approach is that of Interactive Evolutionary Computation (IEC) where the user acts as fitness function and evaluate every design [40]. The main drawback of IEC is that the search space is restricted since it is tedious for the user to evaluate surfaces [37]. Another difficulty stems from the user being inconsistent in his or her evaluations [9]. An alternative approach is to try to use an artificial neural network or some other learning algorithm to learn the user's preferences. This implies mapping the complex

high-dimensional tastes of the user to a low dimensional representation. Unfortunately, this scheme usually does not work very well in practice [34].

In Genr8 we use a fitness evaluation scheme that gives the user high level control of the evolutionary search. In practice this is achieved by a multi-parametric fitness function. Each parameter represents a specific feature of the surface. The user may set target values for each parameter as well as weights to determine the importance of each criteria. The total fitness is $F_{tot} = \sum w_i F_i$, where i runs over the six different criteria and the weight w_i is a positive real number indicating the relative importance of each criterion. The user can modify both the target values F_i and the weights at any point during the run, providing for a high degree of flexibility.

- **Size**. Measures the size of the surface in the x and $y$directions. It is defined as $F_{size} = \max_{i,j} |x_i - x_j| + \max_{i,j} |y_i - y_j|$.
- **Smoothness**. A local measure of the variation in the $z-$direction. The smoothness is defined as

$$F_{smooth} = \frac{1}{2} \sum_{i=0}^{N} \sum_{j=0}^{N_i} |z_i - z_j|,$$

  where z_i is the coordinate value for node i, N is the total number of nodes and N_i the number of neighbors for node i.
- **Soft boundaries**. If this criterion is used, the surface is allowed to grow through boundaries. However, it occurs a fitness penalty for each node which is on the wrong side of the boundary.
- **Subdivisions**. This measure is defined as the number of vertices divided by the number of edges. Thus, it will be 1 for a surface which does not have any internal structures (such as the top left one in Figure 3) and lower for surfaces with many internal vertices and edges.
- **Symmetry**. This metric is a rough way of assesing the degree of symmetry of the surface. It is defined as $F_{sym} = (sym(x) + sym(y))/2$, where the function $sym(x)$ returns the ratio of the number nodes at either side of the line parallel to the x-axis running through the centre of the surface.
- **Undulation**. This measure is a global measure of the variation in the z-direction. It is defined as $F_{und} = \max_{i,j} |z_i - z_j|$.

It is important to point out that in most situations there are many different ways to attain a given fitness value. That is, the fitness function is degenerate in mapping the surfaces to a single fitness value. Consequently, there are many different surfaces which are equally good solutions for agiven set of fitness criteria. This is usually advantageous since it makes it easier to maintain a diverse population. Moreover, some of the criteria are more or less in conflict with each other. This means that the EA must negotiate a trade-off between the different criteria. These situations lead to the most interesting outcomes and also help increasing the variation in the population.

6 Genr8's Surfaces

It is important to emphasize that Genr8's pre-defined HEMLS are powerful by themselves. There has been a number of successful projects based only on these, such as the one illustrated in Figure 8. There are numerous parameters in Genr8 that affect the growth, the environment and the EA. Although they have default values which yield interesting results, there is a lot of scope for exploring the parameter space.

Fig. 8 An example of a surface grown using the square rewrite system from the "Butterfly machines" project by Steve Fuchs at the Southern Californian Institute of Architecture. The environment contains five attractors, one repellor and gravity.

We end this paper with a short presentation of two projects that have used Genr8. In a project by Michel da Costa Goncalves, the environmental setup consists of four cubes as shown in Figure 9. The first step was to calibrate the parameters and the lengths of the boundary cubes in such a way that the surface will fill the bounded space. A number of repellors as well as a gravity component were also introduced in order to provide a conceptual setup for the sketching of a house. Next, two surfaces were evolved in this environment as illustrated in Figure 9. The resulting surfaces were exported to Rhino where transformed, lofted and smoothed. The final outcome is shown in Figure 10.

A second project is a pneumatic strawberry bar designed by Achim Menges of the Architectural Association, London, UK [32]. Recognizing the difficulties of incorporating material analysis in Genr8, he worked with inflatable structures which allowed him to consider the Genr8 surface as a membrane. To analyze the performance of the strawberry bar, he called a

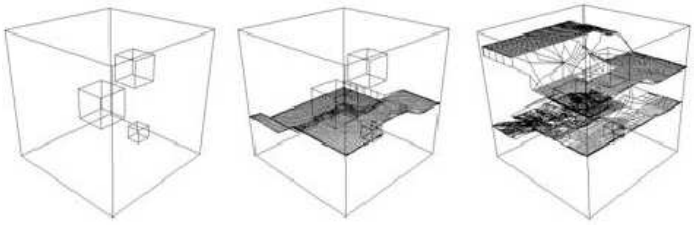

Fig. 9 The environment set up by Michel da Costa Goncalves for his Genr8 project consisting of three cubes, repellors (not shown) and gravity. The preliminary experiments were used to callibrate the parameters so that the surface would fill the bounded space.

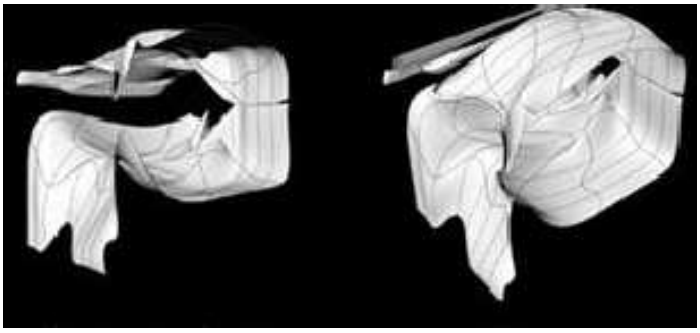

Fig. 10 The surfaces evolved in the environment in Figure 9 were exported from Maya and post-processed to form a set of lofted wire frames.

specific external software package for this type of structures during evolution. The actual bar is not one surface, but three as shown in Figure 11. Menges set up an iterative scheme whereby he would evolve a population inside a bounding box for a number of generations. Then he would use one of the evolved surfaces as boundary for the subsequent evolution, thereby gradually refining the evolved surface. Genr8's high-level fitness function allowed Menges to perform much longer runs involving hundreds of individuals over thousands of generations.

7 Conclusions

Architects have long strived to express aesthetically pleasing organic form within the character of their buildings. Due to their recognition of nature's means, they have embraced both growth processes as a means of deriving such form and evolutionary process as a means of exploring and creating ideal form. Genr8 is a creative design tool for architects which provides them with a growth model for surfaces and means of evolutionary discovery. To meet the ambitions of architects, at Genr8's technical core is the combination of HEMLS and an evolutionary algorithm. HEMLS are an extension

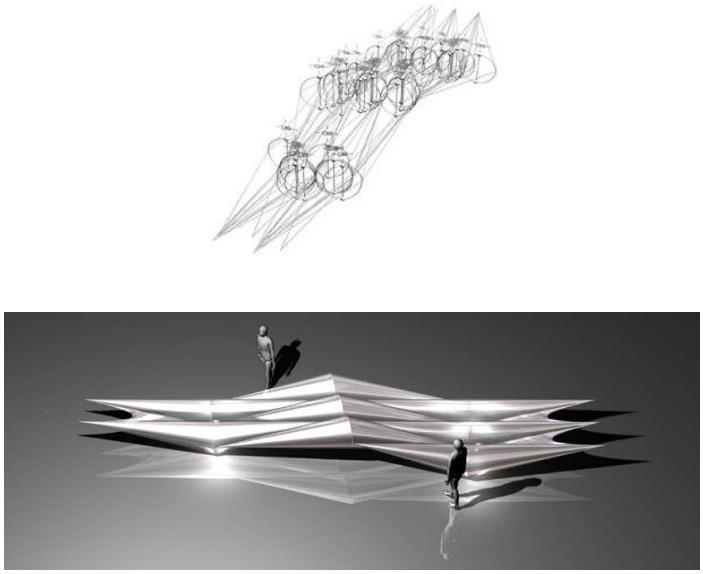

Fig. 11 Top, a screenshot from the process of creating the pneumatic strawberry bar. It shows the three different surfaces that are being connected to form the final structure. Bottom, a rendered image of the final strawberry bar.

of map L-systems that generate (or grow) 3D surfaces interactively within a bounded environment that contains attractors and repellers. HEMLS are described using context-free grammars that themselves can be described in Backus-Naur Form. Genr8's EA uses the BNF description in tandem with Grammatical Evolution to evolve rewrite systems in lieu of hand written ones. The EA's fitness function is a weighted combination of specific surface features.

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