DATE 4/10/13

TITLE CHSH game - Matt Coudron

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I CHSH GAME

$x \in \{0,1\}$ → A → $a \in \{0,1\}$

no communication

shared randomness

inputs chosen uniformly at random

$y \in \{0,1\}$ → B → $b \in \{0,1\}$

Goal: $a \oplus b = xy$

Classical Strategy: $a=0$; $b=0$ → 75% success

By enumeration, can show any deterministic strategy ≤ 75% success

Shared randomness can't help.

II QUANTUM

States

"Ket": $|\psi\rangle = \sum_i^n c_i |i\rangle$ $\quad |\psi\rangle = (c_1, c_2, \ldots c_n)^T \quad |i\rangle = \vec{e}_i$

"Bra" $\langle\phi| = \sum_i^n d_i \langle i|$ $\quad \langle\phi| = (d_1, d_2, \ldots d_n)$

Ket→Bra $(|\psi\rangle)^\dagger = \langle\psi|$, $\quad \dagger$ = complex conjugate

Normalization: $\langle\phi|\psi\rangle = (\quad \phi \quad)(\psi)$, $\quad \langle\psi|\psi\rangle = \sum_i c_i^* c_i = \||\psi\rangle\|_2^2 = 1$

e.g. • $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \in \mathbb{C}^2$ $\quad |\alpha|^2$ = prob outcome $|0\rangle$, $|\beta|^2$ = prob outcome $|1\rangle$ $\quad |\alpha|^2 + |\beta|^2 = 1$

• $|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle \in \mathbb{C}^4$

$\binom{1}{0}\otimes\binom{1}{0} \quad \binom{1}{0}\otimes\binom{0}{1} \quad \binom{0}{1}\otimes\binom{1}{0} \quad \binom{0}{1}\otimes\binom{0}{1}$

Measurement: Orthonormal basis: $\{|v_i\rangle\}$ ⇒ Prob of outcome $i = |\langle v_i|\psi\rangle|^2$

If $|\psi\rangle = |\phi_A\rangle \otimes |\phi_B\rangle$ then $|\psi\rangle$ is not entangled: $\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$ is entangled

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III QUANTUM CHSH STRATEGY

Now Alice and Bob share $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$. Inputs x, y tell Alice & Bob which measurement to make:

Measurements: Alice: $\{|v_0^x\rangle, |v_1^x\rangle\}$ $\quad$ Bob: $\{|w_0^y\rangle, |w_1^y\rangle\}$

$\text{Bias} = 2\omega_q - 1 = \frac{1}{4}\sum_{x,y}\left(\Pr[\text{winning}|xy] - \Pr[\text{lose}|xy]\right)$

↑ prob winning quantum game

• Case $x=y=0$: $\text{Bias} = \frac{1}{4}\left(|\langle v_0^0|\langle w_0^0|\,|\psi\rangle|^2 + |\langle v_1^0|\langle w_1^0|\,|\psi\rangle|^2 - |\langle v_1^0|\langle w_0^0|\,|\psi\rangle|^2 - |\langle v_0^0|\langle w_1^0|\,|\psi\rangle|^2\right)$

$= \frac{1}{4}\langle\psi|\left(|v_0^0\rangle\langle v_0^0| - |v_1^0\rangle\langle v_1^0|\right)\otimes\left(|w_0^0\rangle\langle w_0^0| - |w_1^0\rangle\langle w_1^0|\right)|\psi\rangle$

$= \frac{1}{4}\langle\psi|A_0 \otimes B_0|\psi\rangle$ $\quad A_0, B_0$ have ±1 eigenvalues

• For all cases: Bias =

$\frac{1}{4}\left(\langle\psi|A_0\otimes B_0|\psi\rangle + \langle\psi|A_0\otimes B_1|\psi\rangle + \langle\psi|A_1\otimes B_0|\psi\rangle - \langle\psi|A_1\otimes B_1|\psi\rangle\right)$

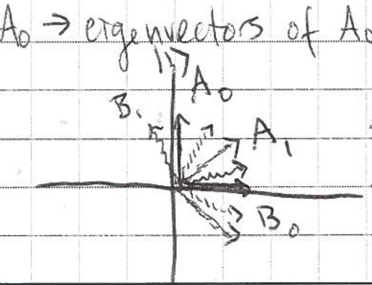
- Any $|\psi\rangle$
- Any A_i, B_j s.t. $A_i^\dagger = A_i$, $B_j^\dagger = B_j$, $A_i^2 = B_j^2 = I$

CHSH Strategy

$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ $\quad A_0 = \sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\quad A_1 = \sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$

$B_b = \frac{1}{\sqrt{2}}\left(\sigma_z + (-1)^b \sigma_x\right)$

Measurement A_0 → eigenvectors of A_0: $|v_0^0\rangle = \binom{1}{0}$, $|v_1^0\rangle = \binom{0}{1}$



Alice and Bob's measurements

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