Proof: $p(abe|xyz)$ non-signalling

with PR box $p(ab|xy) = \begin{cases} 1/2 : & a \oplus b = xy \\ 0 : & \text{otherwise} \end{cases}$

Compute: $P_{AE}(ae|xz) = \sum_b p(abe|xyz) \overset{N.S.}{=} \sum_b p(abe|x\tilde{y}z)$

i) choose $\tilde{y} = 0 \Rightarrow a \oplus b = x \cdot 0 = 0 \quad \forall x$

hence $P_{AE}(ae|xz) = P_{AE}(ae|z) = p(aae|00z) = p(aae|10z)$

ii) choose $\tilde{y} = 1 \Rightarrow a \oplus b = x$ with $x = 1$

$\Rightarrow P_{AE}(a\bar{a}e|11z) = p(aae|01z) = p(aae|10z)$

from i) independent of x $= p(aae|00z)$

Compute: $P_{BE}(be|yz) = \sum_a p(abe|xyz) \overset{N.S.}{=} \sum_a p(abe|\tilde{x}yz)$

$\Rightarrow$ same arguments, hence:

$p(\bar{b}be|11z) = p(bbe|00z) = \dots$

and also $P_{BE}(be|yz) = P_{BE}(be|z)$

Comparing marginals AE & BE we have:

$p(\bar{b}be|11z) = p(bbe|00z) = p(b\bar{b}e|11z)$ etc....

$\Rightarrow P_{BE}(be|z) = P_{BE}(\bar{b}e|z) \quad (*)$

From normalization: $\Rightarrow \sum_{be} P_{BE}(be|z) = 1 \quad \forall z$

$\overset{*}{\Rightarrow} \sum_b P_{BE}(bb|z) = \frac{1}{2} \quad \forall z$

$\Rightarrow P(b=e) = \frac{1}{2} \quad \square$