Gaussian Process Interpolation for Uncertainty Estimation in Image Registration

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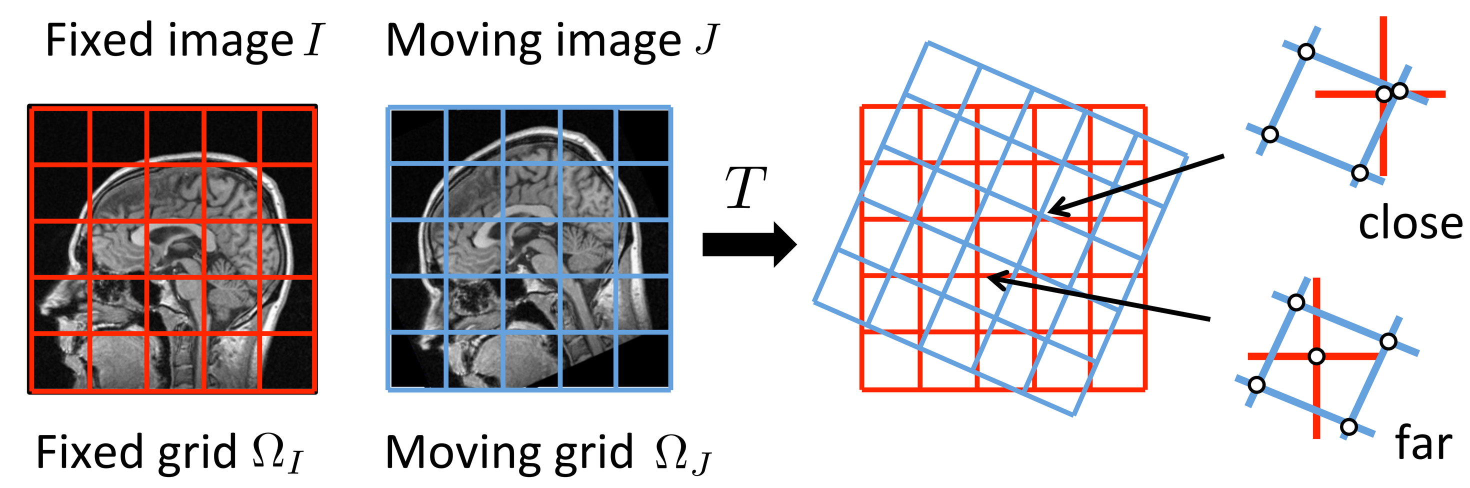
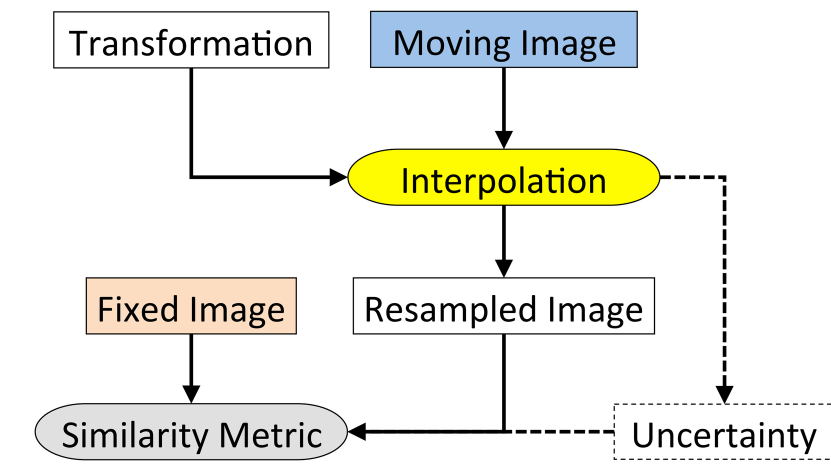
Introduction

- Intensity-based registration requires interpolation for computing the similarity of images
- Uncertainty of interpolation varies across the image
- Problem: All locations have same influence on similarity
- Goal: Estimate interpolation uncertainty and propagate uncertainty to similarity measure



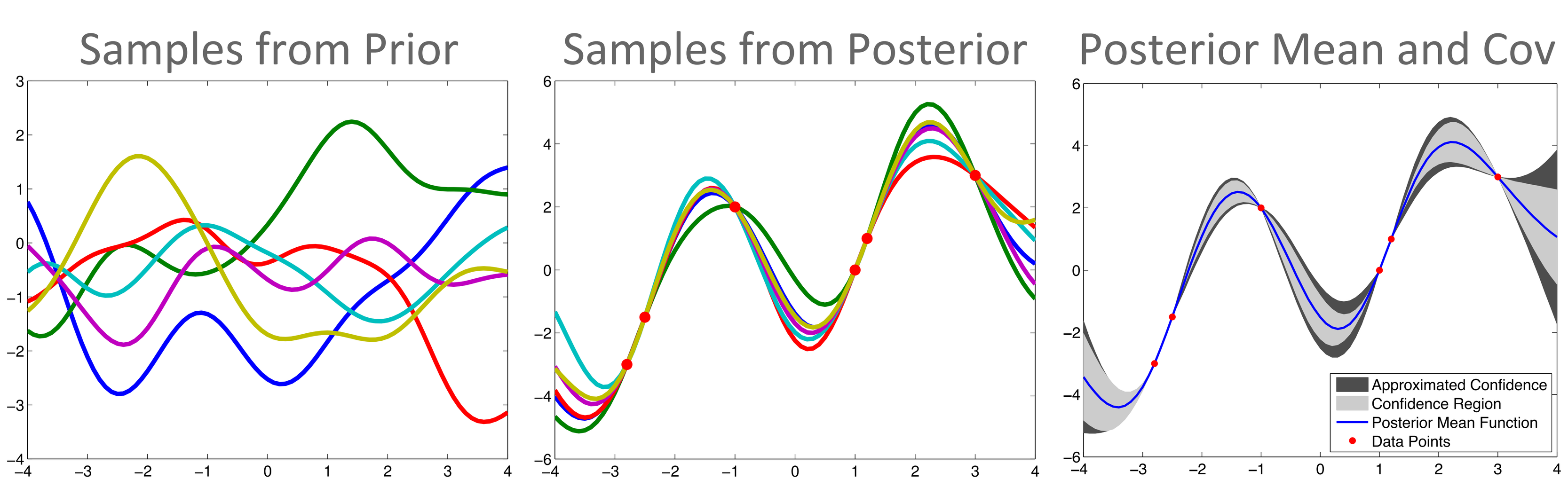


Estimate Interpolation Uncertainty with Gaussian Processes

Image resampling as Bayesian prediction

- Gaussian process over images $J \sim \mathcal{GP}(\mathbf{0}, k)$
- Predict resampled image

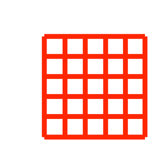
$$p(J^* \mid J; X^*, X) = \mathcal{N}(\boldsymbol{\mu}_J, \Sigma_J)$$
$$\boldsymbol{\mu}_J = k(X^*, X) \cdot [k(X,X) + \sigma_J^2 \mathbf{I}]^{-1} \cdot J$$
$$\Sigma_J = k(X^*, X^*) - k(X^*, X) \cdot [k(X,X) + \sigma_J^2 \mathbf{I}]^{-1} \cdot k(X, X^*)$$



Model details

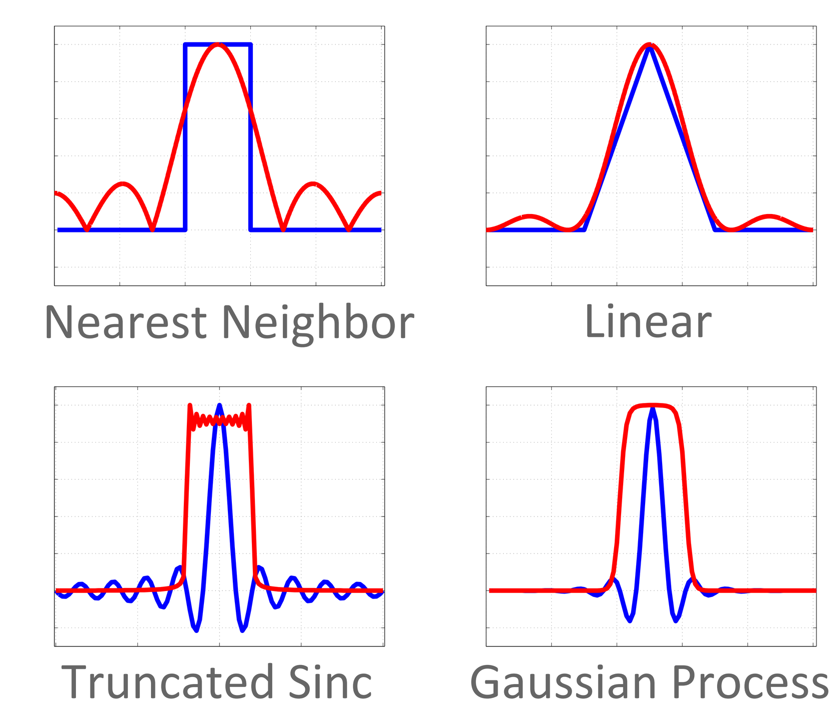
- Observations on transformed grid
$$X = T(\Omega_J)$$

- Prediction on fixed grid
$$X^* = \Omega_I$$



- Squared exponential kernel
$$k(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2 \cdot l^2}\right)$$
- Gaussian image noise
$$\varepsilon \sim \mathcal{N}(0, \sigma_J)$$

Comparison of interpolation functions in spatial and frequency domain

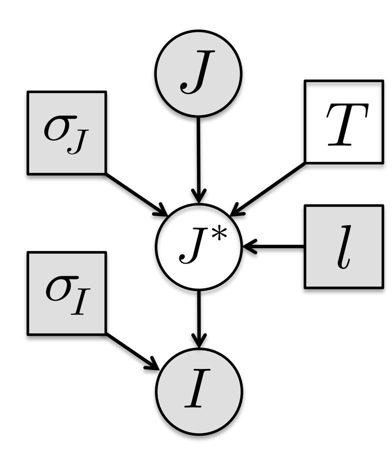


Registration with Interpolation Uncertainty Estimation

Integrate uncertainty into similarity measure

- Graphical model factorizes as

$$p(I, J, J^*; T, \sigma_J, \sigma_I, l) = p(J^* | J; T, \sigma_J, l) \cdot p(I | J^*; \sigma_I) = \mathcal{N}(\boldsymbol{\mu}_J, \Sigma_J) \cdot \mathcal{N}(J^*, \sigma_I^2 \mathbf{I})$$



- Marginalize over latent random variable

$$p(I, J; T, \sigma_I, \sigma_J, l) = \int p(I, J, J^*; T, \sigma_I, \sigma_J, l) \, \mathrm{d}J^* = \int \mathcal{N}(J^*; \boldsymbol{\mu}_J, \Sigma_J) \cdot \mathcal{N}(I; J^*, \sigma_I^2 \mathbf{I}) \, \mathrm{d}J^* = \mathcal{N}(\boldsymbol{\mu}_J, \Sigma_J + \sigma_I^2 \mathbf{I})$$

- Log-likelihood function as similarity metric

$$\log p(I, J; T, \sigma_I, \sigma_J, l) = \log[(2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{1}{2}}] - \frac{1}{2}(I - \boldsymbol{\mu}_J)^\top \Sigma^{-1} (I - \boldsymbol{\mu}_J)$$
$$\Sigma = \Sigma_J + \sigma_I^2 \mathbf{I}$$

- Maximum likelihood estimation to retrieve transformation

$$\hat{T} = \arg\max_T \log p(I, J; T, \sigma_J, \sigma_I, l)$$

Parameters

- Length scale l in kernel determines smoothness of image, e.g., scale-space creation
- Gaussian noise in both images
$$\varepsilon_I \sim \mathcal{N}(0, \sigma_I^2)$$
$$\varepsilon_J \sim \mathcal{N}(0, \sigma_J^2)$$

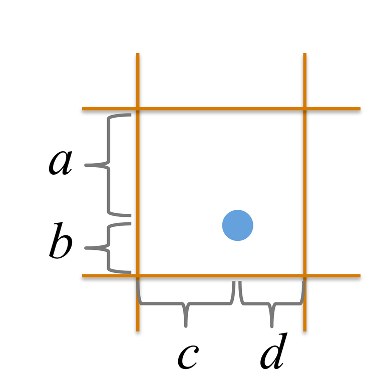
Practical considerations

- Block-wise estimation for 3D images due to large kernel size
- Approximation of the similarity measure by variances on the diagonal (closely related to sum of squared differences)

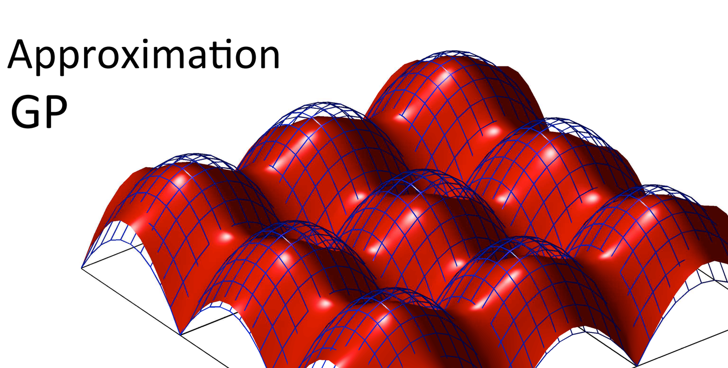
$$\log p(I, J; T, \sigma_I, \sigma_J, l) \approx -\sum_{x \in \Omega_I} \frac{(I(x) - \boldsymbol{\mu}_J(x))^2}{2 \cdot \Sigma_{xx}}$$

- Approximation of variance by weights from linear interpolation

$$V = ab + cd$$



Blue: Approximation
Red: GP

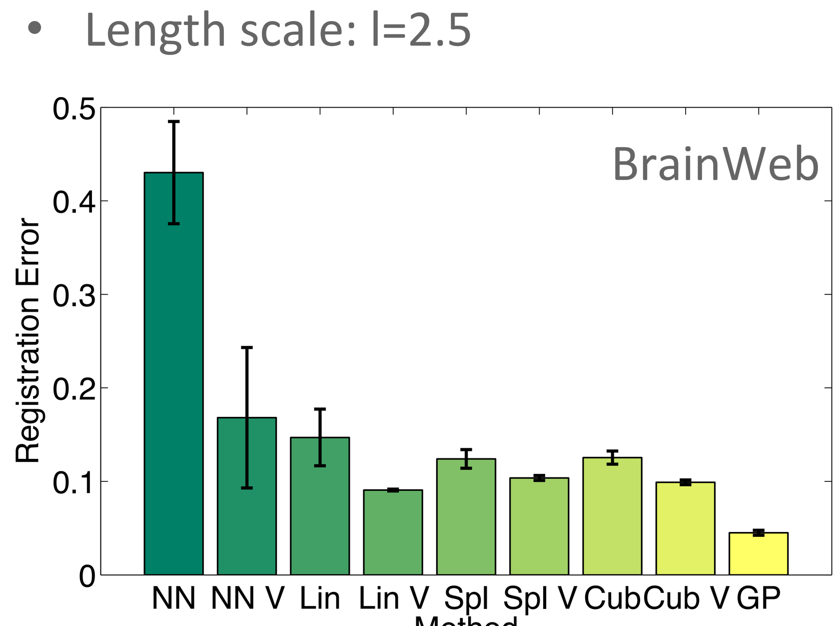


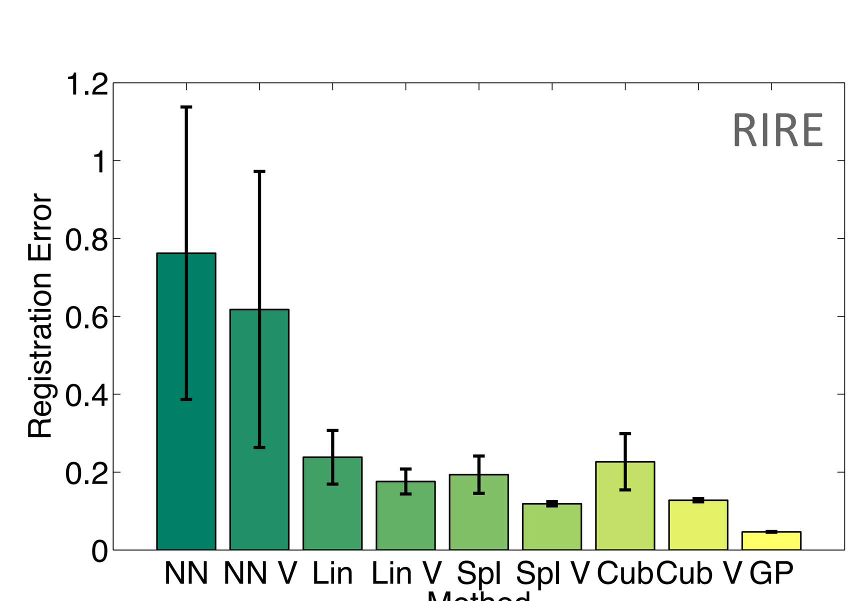
Registration Experiments:

- Three MR datasets: (i) BrainWeb, (ii) RIRE, (iii) Head
- Comparison of registration errors for different interpolation schemes: nearest neighbor, linear, spline, cubic, GP ($\sigma^2_I = \sigma^2_J = 0.1$)
- Variance approximation indicated with 'V'
- Statistics over 50 runs from random initial transformations

2D registration results

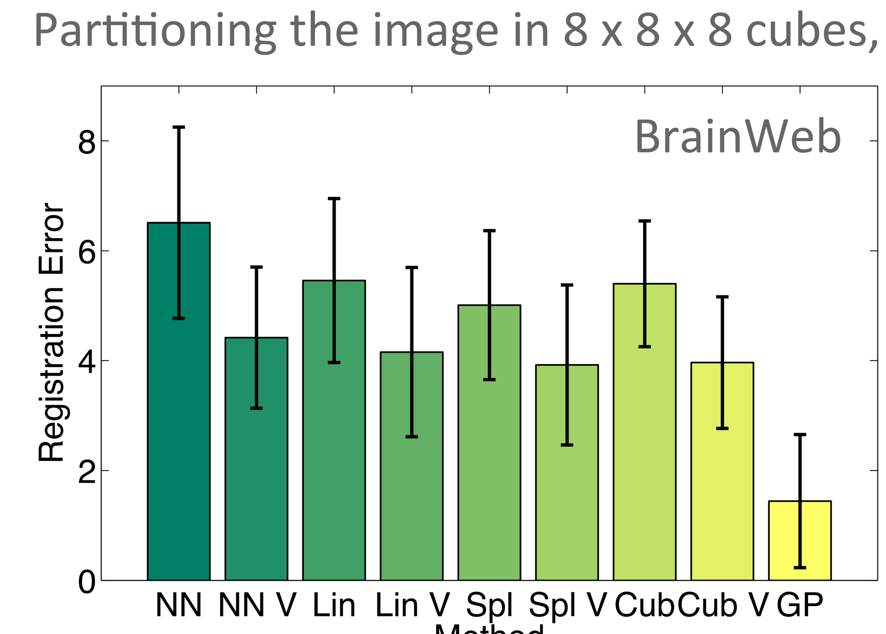
- Full covariance matrix
- Length scale: l=2.5

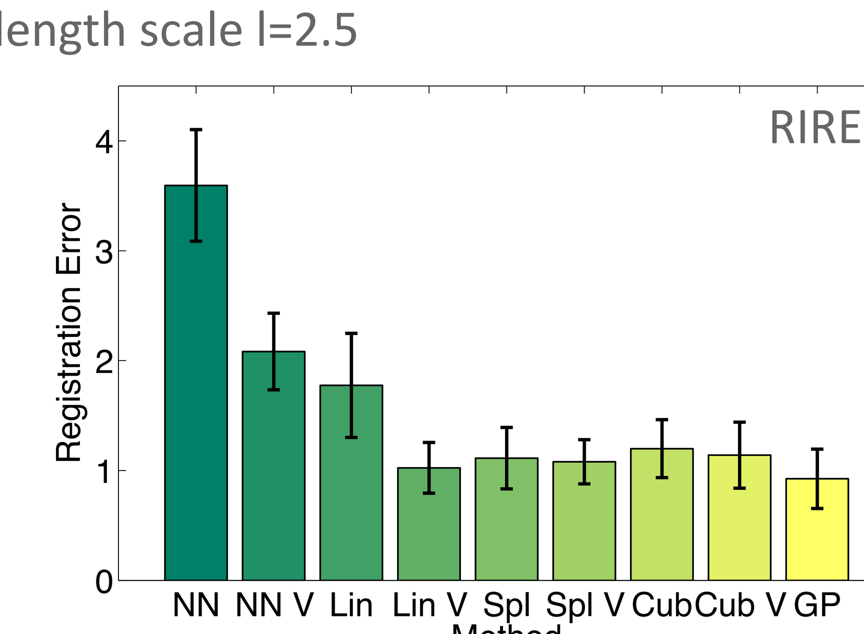




3D registration results

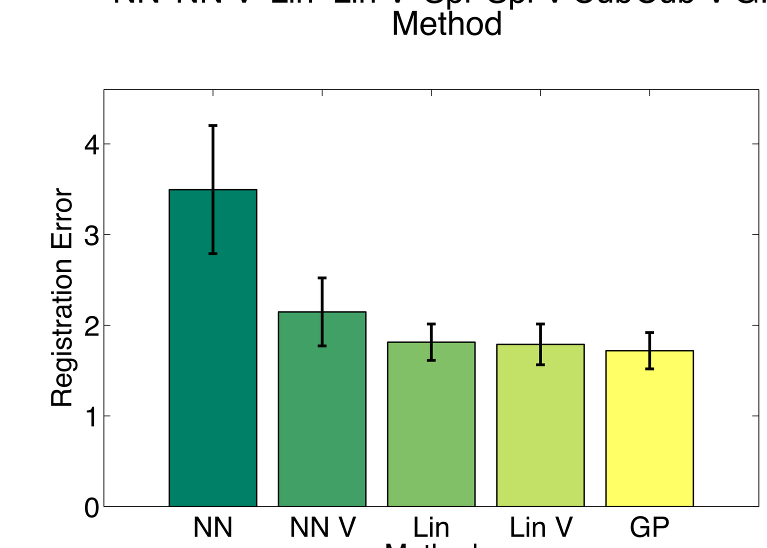
- Diagonal entries of covariance matrix
- Partitioning the image in 8 x 8 x 8 cubes, length scale l=2.5





Head results

- Diagonal entries of covariance matrix
- Partitioning the image in 8 x 8 x 8 cubes
- Length scale l=2.5
- MR acquisitions on two different grids



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